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Estimating the Survival Times of HIV Patients Using a Markov Chain

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Abstract: In this paper, real-world data are used to estimate CDC (Center for Disease Control) stage transition times for patients diagnosed with HIV using statistical methods. The method that will be used for these estimations will be an analysis of a finite state Markov chain. For the purposes of this thesis, two populations will be analyzed. The two populations are men and women. A one-step transition matrix will be derived for both of the populations. Using this one-step transition matrix, a steady-state probability vector will also be derived. The data and the corresponding results will then be used to make a comparison between the two populations. The data were collected by observing individuals infected with HIV. Individuals who were active in the study were observed every six months. During these observations, each individual's CDC stage was recorded. Involvement in the study was voluntary resulting in many individuals only being observed once which meant that no data concerning their transition from one CDC stage to another could be acquired. Even with the large number of individuals who discontinued their participation in the study, there were still a large number of individuals who maintained their participation throughout the duration of the study and ensured that the variance and error estimates of the results would be negligible.

Key words: CDC stage, HIV, Statistical methods, Markov chain,

INTRODUCTION

In this thesis, a Markov chain was used to analyze data from two different populations. The two populations under consideration were men who have been diagnosed with HIV and women who have been diagnosed with HIV. Data were collected by observing individuals from both populations to determine which CDC (Center for Disease Control) stage of HIV they were in. The data were on a list presenting the patient ID number, the visit number, and the CDC stage that was observed for that patient during that particular visit. The data also had the men and women separated. For example, if one was to examine the data for men for visit number one, one might note that patient number one was in CDC stage 2, patient number two was in CDC stage 3, etc. After listing the CDC stage for each person for stage number 1, the next lines of data would be the CDC stage for each person still participating in the study for stage number 2, and so on.

The data presented gave three different possibilities for each person as to CDC stage. The individual was either listed as being in CDC stage 1, CDC stage 2, or CDC stage 3. When researching what characterized each CDC stage, it was discovered that most sources would list four different CDC stages. Most of these sources would add an additional CDC stage 1, which is essentially the primary HIV infection stage. In this stage, a person has recently acquired HIV and may feel several effects of the virus being introduced into the body like glandular fever, neurological problems, and some opportunistic infections. However, many people will never feel any of these effects. Although the received data classified each individual as being in CDC stage 1, 2, or 3, these stages actually correspond to what many sources consider to be CDC stages 2, 3, and 4. Essentially, the received data did not consider this primary CDC stage as stage 1.

CDC stage 2, which the data in use calls CDC stage 1, is characterized by individuals who have contracted HIV but are not showing any physical signs of the illness. These individuals are considered to be asymptomatic. The next stage, CDC stage 3, also known as CDC stage 2, is characterized by a below normal CD4 T-cell count. The individuals in this stage are largely asymptomatic. Some of the symptoms that can occur in this stage are an enlargement of the lymph nodes, tiredness, excessive sweating, and aches and pains in the muscles and joints. The final stage associated with HIV, CDC stage 4, also labeled CDC stage 3, is generally characterized by individuals having severe illnesses. Some individuals may be in CDC stage 4 and still be relatively well, but some of the common, serious medical conditions that may appear in this stage are: an extended fever (a month or longer), weight loss of more than 10% of body weight, neurological problems,

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cognitive and/or motor dysfunction, loss of memory, infections associated with AIDS, recurring serious infections, and cancers associated with HIV. For consistency, and for easy use of the data, from this point forward the CDC stages will be labeled in the same way as the study data labeled them. That is, from this point on when CDC stage 1 is mentioned, the researcher is referring to what is commonly known as CDC stage 2, CDC stage 2 will be the more commonly known CDC stage 3, and CDC stage 3 will be CDC stage 4.

2. Converting the data into a Markov chain:

As has been explained previously, the data were arranged in lines, such that, each line gave an individual's patient number, the number of the visit, and the individual's CDC stage at the time of the visit. All of the data for visit one was listed first, followed by the data for visit two, followed by the data for visit three, etc.

To properly analyze the data, the data were arranged in a table. The first column of the table was patient ID numbers. The second column was for visit one, the third column was for visit two, etc. The entries that were placed in the appropriate places in the rows and columns would be the CDC stage number. So when the table was finished, one could easily determine patient number one and the appropriate stage that the patient was in for visit number one, visit number two, and so on.

Because a purpose of this thesis was to identify the probability of a transition from one CDC stage to another CDC stage, any patient who was only observed for one visit was ignored. If a patient was only observed once then there was never an opportunity for that person to be observed transitioning from one CDC stage to another and so the data for that patient was of no use for the purposes of this thesis.

If an individual was observed for more than one visit then the patient could be observed in a CDC stage transition or remaining in the previous stage. All patients with more than one visit were, therefore, considered whenever the study matrix was created.

Viewing each CDC stage as a separate state of a process and observing that these states fluctuated with time as a result of random events acting upon the system, it was easy to note that this process was an example of a stochastic process. A stochastic process is a collection of random variables denoted $\{X(n) \mid n \in N\}$. Generally, the index set N refers to time. In this thesis N refers to the visits. $X(n)$ represents measurements or observations on a system at time n. The possible values of $X(n)$ are called the states of the process. In this thesis, the states of the process are CDC stage 1, CDC stage 2, and CDC stage 3. It is also assumed that the process in question for this thesis has the Markov property. In a stochastic process having the Markov property means that each outcome depends only upon the outcome immediately preceding it. More precisely, if $\{X(n)\}$ denotes a stochastic process and $\{s(n)\}$ denotes a collection of the states of the process, then the process is said to satisfy the Markov property if:

$$P\left[X(n+1)=s(n+1) \mid X(n)=s(n), X(n-1)=s(n-1), \ldots, X(0)=s(0)\right]$$
$$=P\left[X(n+1)=s(n+1) \mid X(n)=s(n)\right]$$

The left hand side of the equation shows that one can determine the probabilities for the outcomes of $X(n+1)$ when given all of the previous outcomes. The right hand side of the equation shows that this answer is equivalent to the probabilities for the outcomes of $X(n+1)$ when only given the outcome immediate preceding it. Such processes are called Markov chains. Therefore, it is safe to say that the process referred to in this thesis, is a Markov chain.

Markov chains are easy to represent using a matrix. The first step in determining the values that will be in the matrix is determining the one-step transition probabilities for each entry in the matrix. There are three stages in the process resulting in a 3x3 matrix. In a 3x3 matrix there are nine different one-step transition probabilities to be derived. To derive these values, one would need to examine the data for patients who made more than one visit. For each person that made more than one visit, it could determined how many times that person transitioned from stage 1 to stage 1, from stage 1 to stage 2, from stage 1 to stage 3, from stage 2 to stage 1, from stage 2 to stage 2, from stage 2 to stage 3, from stage 3 to stage 1, from stage 3 to stage 2, and from stage 3 to stage 3. This process was repeated for every patient that had more than one visit. Once these occurrences were documented, the following determinations were made: the probability that if a patient is in stage 1, then that patient will be in stage 1 for the next visit; the probability that if a patient is in stage 1, then that patient will be in stage 2 for the next visit, and continue in this manner until the probability that if a patient is in stage 3, then that patient will be in stage 3 for the next visit is calculated. The notation that is used to represent the probability that a person transitions from stage 1 to stage 2 is *P*[1->2]. The nine calculations are used to derive the one-step transition probabilities and to derive the one-step transition matrix.

The One-step Transition Matrix for Men:

The nine one-step transition probabilities for the male population of the data are as follows:

P[1->1] = 0.815789
P[1->2] = 0.173684
P[1->3] = 0.010526
P[2->1] = 0.004228
P[2->2] = 0.961945
P[2->3] = 0.033827
P[3->1] = 0
P[3->2] = 0.036364
P[3->3] = 0.963636

The nine computations create a one-step transition matrix.

$$P_{men} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.815789 & 0.173684 & 0.010526 \\ 0.004228 & 0.961945 & 0.033827 \\ 0 & 0.036364 & 0.963636 \end{bmatrix} \end{array}$$

This type of matrix is called a one-step transition matrix because each entry represents the probability of a patient being in the stage given by the row for one visit and then being in the stage given by the column the next visit. So each visit is being viewed as one step. For example, the entry in the first row and first column represents the probability that a patient who was in stage 1 on the previous visit will be in stage 1 on the next visit. The entry in the second row and third column represents the probability that a patient who was in the second stage on the last visit will be in the third stage on the next visit. This matrix represents the Markov chain.

As is the case with all matrices that represent Markov chains, the sum of the elements in each row is 1. Obviously if one is in stage one on the first visit, then one has to move to some other stage or remain in stage 1 for the second visit. So the probability of staying in stage 1, plus the probability of moving to stage 2, plus the probability of moving to stage 3 is 1. With each of the patients, there is actually another option. The option that is not considered is the possibility that a patient succumbs to the illness between visits. The patients who did succumb to the illness between visits were not a part of the data, and this option was ignored for the purposes of this thesis. It is important to note that the probabilities in this matrix actually represent the probability of being in a certain stage based upon the previous stage given that the patient did not succumb to their illness.

The matrix P_{men} shows nine probabilities. One determination from this matrix is that if one is in a particular stage on a previous visit then there is a high probability of being in that same stage on the next visit. If on a male patient's previous visit he was in stage 1 then the probability that he will still be in stage 1 on the next visit is approximately 81.58%. He has approximately a 17.37% chance of moving to stage 2 and a very small 1.05% chance of moving to stage 3 on the next visit.

If a male patient was in stage 2 on a previous visit, then he had a 96.14% chance of being in stage 2 on the next visit. Male patients in stage 2 had a miniscule 0.42% chance of moving from stage 2 to stage 1 from one visit to the next, and males had approximately a 3.38% chance of moving from stage 2 to stage 3 from one visit to the next.

Male patients in stage 3 would remain in stage 3 approximately 96.36% of the time. Patients in stage 3 would move to stage 2 approximately 3.64% of the time. No male patient who was in stage 3 on a previous visit was found in stage 1 on the next visit. So the probability for this was 0. All of the probabilities are approximately correct assuming that a patient does not die between visits.

4. The One-step Transition Matrix for Women:

The nine one-step transition probabilities for the female population of the data are as follows:

$P[1\text{->}1] = 0.84$
$P[1\text{->}2] = 0.16$
$P[1\text{->}3] = 0$
$P[2\text{->}1] = 0.098039$
$P[2\text{->}2] = 0.862745$
$P[2\text{->}3] = 0.039216$
$P[3\text{->}1] = 0.045455$
$P[3\text{->}2] = 0.136364$
$P[3\text{->}3] = 0.818182$

The nine computations create the following one-step transition matrix:

$$P_{women} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.84 & 0.16 & 0 \\ 0.098039 & 0.862745 & 0.039216 \\ 0.045455 & 0.136364 & 0.818182 \end{bmatrix} \end{array}$$

This matrix should be viewed in the same way that the matrix for men was viewed. Row 1 represents the probability of a patient being in stage 1 on a previous visit and then being in one of the respective stages on the next visit. Row 2 represents the probability of being in stage 2 on a previous visit and then being in one of the respective stages on the next visit. Row 3 represents the probability of being in stage 3 on a previous visit and then being in one of the respective stages on the next visit. It is again important to note that these probabilities actually represent the probability of transitioning from one stage to another given that the patient does not die between visits.

The matrix P_{women} shows nine probabilities. Viewing this matrix, it is again apparent that if one is in a particular stage during a previous visit, that person will more than likely be in the same stage on the next visit. If a female patient was in stage 1 on a previous visit, there is approximately an 84% chance that she will be in stage 1 on the next visit. The patient had a 16% chance of moving to stage 2, if she was in stage 1 previously. In the data, there was no female who transitioned from stage 1 to stage 3 over one visit. Therefore, this probability was 0.

If a female patient was in stage 2 on a previous visit, then there was approximately an 86.27% chance of that patient remaining in stage 2 for the next visit. The patient had approximately a 9.8% chance of moving from stage 2 to stage 1 and approximately a 3.92% chance of moving from stage 2 to stage 3.

If a female patient was in stage 3 on a previous visit, then there was approximately an 81.81% chance she would remain in stage 3 for the next visit. Female patients who were in stage 3 also had a 13.64% chance of moving to stage 2 and a 4.55% chance of moving to stage 1. All of these probabilities are approximately assuming a patient does not die between visits.

5. Comparing the Men's and Women's One-step Transition Matrices:

$$P_{men} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.815789 & 0.173684 & 0.010526 \\ 0.004228 & 0.961945 & 0.033827 \\ 0 & 0.036364 & 0.963636 \end{bmatrix} \end{array} \qquad P_{women} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.84 & 0.16 & 0 \\ 0.098039 & 0.862745 & 0.039216 \\ 0.045455 & 0.136364 & 0.818182 \end{bmatrix} \end{array}$$

Comparing the one-step transition matrix for men and the one-step transition matrix for women shows that there are some interesting similarities and some interesting differences. One of the similarities is the first row. It is easy to see by comparing the two matrices side by side, that row 1 of the men's matrix is very similar to row 1 of the women's matrix. Each entry in the men's row 1 is within 3% of the entries in the women's first row. Therefore, there is approximately the same chance for a man to remain in stage 1 given he was in stage 1 on a previous visit as there is for a woman. The genders also approximately share the same chance of moving from stage 1 to stage 2 and the same chance of moving from stage 1 to stage 3.

By comparing and analyzing row 2, major differences between men and women become apparent. The probability that a male moves from stage 2 to stage 3 is approximately the same as the probability that a female moves from stage 2 to stage 3.

The major difference is the probability of someone moving from stage 2 to stage 1. With men this move occurs 0.42% of the time or approximately once for every 236 men previously in stage 2. With women this move occurs 9.80% of the time or approximately once for every 10 previously in stage 2. In other words, women are about 23 times more likely to transition from stage 2 to stage 1 than men.

Examining row 3, it can be determined that there are major differences present there also. It can be observed that 96.36% of all men would remain in stage 3 if they were in stage 3 during a previous visit. For women, this percentage is only 81.82%. The male patients in this study would go from stage 3 to stage 2 only 3.64% of the time, and there were no men who went from stage 3 to stage 1. Women went from stage 3 to stage 2 approximately 13.64% of the time and from stage 3 to stage 1 about 4.55% of the time. Therefore, although there were no men in the study who did so, approximately 1 in every 22 women who were in stage 3 would be in stage 1 on the next visit. About 1 in every 27 men would go from stage 3 to stage 2, but women would go from stage 3 to stage 2 approximately 1 in every 7. Comparing these two one-step transition matrices side by side, it is obvious that the women were more likely than the men to see an improvement in their condition from one visit to the next.

Steady-State vectors:

All of the observed matrices to this point have been one-step transition matrices. That is, they show the probability of transitioning from one stage to the next over one unit of time (in this case one visit).

Another interesting observation that can be made using a Markov chain is to analyze a k-step transition matrix. That is, given a starting stage, what are the probabilities of being in certain stages in k-steps. Such matrices are found by taking powers of the one-step transition matrix. For example, P^2_{men} would be the two-step transition matrix for men, P^3_{women} would be the three-step transition matrix for women, and P^k_{men} would be the k-step transition matrix for men. A regular Markov chain is defined as a finite state Markov chain whose k-step transition matrix has all non-zero entries for some value of k>0.

An interesting development can occur to the k-step transition matrix of a regular Markov chain when k becomes large. If the chain is regular, then as k grows large all of the rows will approach identical values. Whenever a k-step matrix has reached the point at which its rows are nearly identical, then it is said that the system has attained the steady-state condition. This condition is more formally written as: Let $X(n)$, n= 0,1,2,..., be a regular Markov chain with one-step transition matrix P. Then there exists a matrix Π. having identical rows with nonzero entries such that $\lim_{k\to\infty} P^k = \Pi$. This statement indicates that if the k-step transition matrix of a finite-state Markov chain has all nonzero entries at some point in time, then the chain will eventually attain the steady-state condition. It is common to denote each of the identical rows of the limiting matrix Π by the row vector π. Because the elements of π constitute a probability distribution, π is called the steady-state probability vector and the entries of π are called the steady-state probabilities.

One method that could be used to find this steady-state probability vector would be to use large values of k and find P^k where P is the one-step transition matrix. This method will give accurate approximations for the steady-state vector, and the powers of P are not that difficult to find using mathematical software such as Derive. There is actually a more precise method for finding this probability vector that requires more effort. The method is given as: Let $X(n)$, n=0,1,2,.., be a regular Markov chain with one-step transition matrix P. Then the steady-state probability vector $\pi = (\pi_1, \pi_2, ..., \pi_s)$ may be found by solving the system of equations $\begin{cases} \pi P = \pi \\ \pi_1 + \pi_2 + ... + \pi_s = 1 \end{cases}$. The first equation in this system was found by examining the equation $P^{k-1}P=P^k$. This finding is true because of how matrix multiplication is defined. Since $\lim_{k\to\infty} P^{k-1} = \lim_{k\to\infty} P^k = \Pi$, then $\Pi P=\Pi$. Therefore, $\pi P=\pi$. The second equation in the system is, of course, a requirement of a Markov chain.

7. The Men's Steady-state Vector:

It would be helpful to analyze the data using steady-state probability vectors. In order to be able to calculate the steady-state probability vector for men, one must first verify that the matrix for men is indeed an example of a regular Markov chain. Upon examination of the one-step transition matrix for men, one can discover an element in the matrix that is zero. However, when the two-step transition matrix, P^2_{men} is calculated the following matrix can be observed:

$$P_{men}^{2}=\begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & & & \\ 2 & & & \\ 3 & & & \end{array}\begin{bmatrix}0.666246 & 0.309147 & 0.024605\\ 0.007516 & 0.927303 & 0.065181\\ 0.000154 & 0.070022 & 0.929824\end{bmatrix}$$

Clearly, there are no zero elements in this matrix. Therefore, the finite state Markov chain for men is indeed an example of a regular Markov chain. This finding implies that ome can solve the system of equations $\begin{cases}\pi P_{men}=\pi\\ \pi_1+\pi_2+\pi_3=1\end{cases}$ to determine the steady-state probability vector for men. The actual system of equations used to calculate the steady-state probability vector for men was $\begin{cases}0.815799\pi_1+0.004228\pi_2+0\pi_3=\pi_1\\ 0.173684\pi_1+0.961945\pi_2+0.036364\pi_3=\pi_2\\ 0.010526\pi_1+0.033827\pi_2+0.963636\pi_3=\pi_3\\ \pi_1+\pi_2+\pi_3=1\end{cases}$

Solving this system of four equations with three variables, it was determined that π_1 = 0.011718, π_2 = 0.510250, and π_3 = 0.478042In other words, as k approaches infinity, the following k-step transition matrix develops:

$$\lim_{k\to\infty}P_{men}^{k}=\begin{array}{c} \\ 1\\ 2\\ 3\end{array}\begin{array}{c}\begin{array}{ccc}1 & 2 & 3\end{array}\\ \begin{bmatrix}0.011718 & 0.510250 & 0.478042\\ 0.011718 & 0.510250 & 0.478042\\ 0.011718 & 0.510250 & 0.478042\end{bmatrix}\end{array}$$

This matrix makes several points about the male patients from the study. To make these points, it is again not considered that a patient will succumb to the illness between visits. One point gleaned from the matrix is that no matter in which stage a male patient begins, as time progresses, the patient will have a 1.17% chance of being in stage 1. No matter the beginning stage a male patient will have a 51.025% chance of being in stage 2 and a 47.804% chance of being in stage 3 as time progresses. These probabilities can be written in the vector π_{men} as follows:

$$\Pi_{men}=\begin{array}{c}1\\ 2\\ 3\end{array}\begin{bmatrix}0.011718\\ 0.510250\\ 0.478042\end{bmatrix}$$

The Women's Steady-state Vector:

For the purposes of comparison, it is also important to be able to calculate the women's steady-state vector. In order to make this calculation one must again verify that the matrix for women is an example of a regular Markov chain. In examining the one-step transition matrix for women, an element in the matrix is observed to be zero. However, we calculating the two-step transition matrix, P^2_{women} the following observation is made:

$$P_{women}^{2}=\begin{array}{c} \\ 1\\ 2\\ 3\end{array}\begin{array}{c}\begin{array}{ccc}1 & 2 & 3\end{array}\\ \begin{bmatrix}0.721286 & 0.272439 & 0.006275\\ 0.168718 & 0.765363 & 0.065919\\ 0.088742 & 0.236491 & 0.674769\end{bmatrix}\end{array}$$

Clearly, there are no zero elements in this matrix. The matrix for women is, therefore, an example of a regular Markov chain. This finding implies that one can solve the system of equations $\begin{cases}\pi P_{women}=\pi\\ \pi_1+\pi_2+\pi_3=1\end{cases}$ to determine the steady-state probability vector for women. The actual system of equations used to calculate the steady-state probability vector for women was $\begin{cases} 0.84\pi_1 + 0.098039\pi_2 + 0.045455\pi_3 = \pi_1 \\ 0.16\pi_1 + 0.862745\pi_2 + 0.136364\pi_3 = \pi_2 \\ 0\pi_1 + 0.039216\pi_2 + 0.818182\pi_3 = \pi_3 \\ \pi_1 + \pi_2 + \pi_3 = 1 \end{cases}$. By solving this system of four equations with three variables, it was determined that π_1 = 0.356679, π_2 = 0.52918 and π_3 = 0.114138. In other words, as k approaches infinity, the following k-step transition matrix develops:

$$\lim_{k\to\infty} P^k_{women} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.356679 & 0.52918 & 0.114138 \\ 0.356679 & 0.52918 & 0.114138 \\ 0.356679 & 0.52918 & 0.114138 \end{bmatrix} \end{array}$$

This matrix allows the creation of a few assumptions about the female patients in the study. To make these assumptions, the possibility that a patient will succumb to the illness between visits is again not considered. One point that can be made from this matrix is that no matter in which stage a female patient begins, as time progresses that patient will have a 35.67% chance of being in stage 1. No matter the beginning stage of a female patient, that patient has a 52.92% chance of being in stage 2 and a 11.41% chance of being in stage 3 as time progresses. These probabilities can be written in the vector π_{women} as follows:

$$\pi_{women} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} 0.356679 \\ 0.52918 \\ 0.114138 \end{bmatrix}$$

Comparing the Men's and Women's Steady-state Vectors:

$$\pi_{men} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} 0.011718 \\ 0.510250 \\ 0.478042 \end{bmatrix} \qquad \pi_{women} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} 0.356679 \\ 0.52918 \\ 0.114138 \end{bmatrix}$$

By viewing the men's steady-state vector next to the women's steady-state vector, it is possible to note one element that is approximately equal. The probabilities located in row 2 are the values of 0.510250 for the men and 0.52918 for the women. These values describe the probability for a male and female, respectively, to be in stage 2 if they continue to make visits for an extended period of time. So if the possibility that a patient dies during the study is excluded both men and women have a slightly better than 50% chance of being in stage 2 as time progresses.

By examining row 1 and row 3 of the two vectors side by side, major differences for the two sexes become apparent. As time progresses men have a 1.17% chance of being in stage 1; whereas women have a 35.67% chance of being in stage 1. In other words, women are approximately 30 times more likely to be in stage 1 in time than men. This major difference is directly related to some of the large gaps in the one-step transition matrices for men and women. The most notable of these differences is the probability of transitioning from stage 2 to stage 1. In the one-step transition matrix the probability of transitioning from stage 2 to stage 1 is 23 times more likely to occur for women than it is for men. Such a large difference will greatly affect the steady-state probabilities. Another major difference in the one-step transition matrices that will have a large impact upon the steady-state probabilities is the probability of transitioning from stage 3 to stage 2. This is approximately 4 times more likely to occur for women than it is for men.

Conclusion:

The data and subsequent findings presented in this thesis yield some interesting results. It is obvious that the data strongly suggest that women with HIV are more likely than men to improve their condition and move to a lower CDC stage. One would think that this would imply that women with HIV would have a longer estimated survival time than men with HIV.

The data and subsequent findings presented in this thesis also lead to several ideas regarding future work with this topic. First and foremost, an individual working with this topic in the future would like to be able to include the probability that a patient succumbs to their illness between visits. If this probability were to be included the data would then yield a Markov chain with an absorbing state. There are several applications of a Markov chain with an absorbing state. The most relevant of these is finding the mean time to absorption. The mean time to absorption in this case would actually be the average survival time of a patient based upon their current stage. Needless to say this would be a very interesting finding and one that could be used to again analyze the differences between men and women.

REFERENCES

Bai, D.S., M.S. Kim and S.H. Lee, 1989. Optimum simple step-stress accelerated life tests with censoring. IEEE Trans. Reliability; 38: 528-532.

Khamis, I.H., J.J. Higgins, 1996. Optimum three-stress tests . IEEE Trans Reliability; R-32: 59-65.

Khamis, I.H., 1997. Comparison between constant and step-stress tests for Weibull models. International Journal of Quality & Reliability Management., 14(1): 74-81.

Meeker, W.Q., G.J. Hahn, 1977. Asymptotically optimum over-stress tests to estimate the survival probability at a condition with a low expected failure probability. Technometrics, 19(4): 381-399.

Meeker, W.Q., W.B. Nelson, 1975. Optimum accelerated life tests for Weibull and extreme value distributions. IEEE Trans. Reliability, R-24: 321-332.

Miller, R. and W.B. Nelson, 1983. Optimum simple step-stress plans for accelerated life testing. IEEE Trans. Reliability, R-32: 59-65.