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Estimation of Strength Parameters of Limestone Using Artificial Neural Networks and Regression Analysis

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Abstract: The accurate determination of geomechanical properties such as uniaxial compressive strength and shear strength requires considerable time in collecting appropriate samples, their preparation and laboratory testing. To minimize the time and cost, a number of empirical relations have been reported which are widely used for the estimation of complex rock properties from more easily acquired data. The purpose of this study is developing a model for estimation of UCS and shear strength of intact rocks. In this study, two mathematical methods, Artificial Neural Networks (ANNs) and regression analysis, were used to predict the uniaxial compressive and shear strength. The point load index, the P-wave velocity, the slake durability index and dry density were used as inputs for both methods. The multilayer perceptrons (MLPs) neural networks with two outputs (UCS and τ) improved the coefficients of determination to more acceptable levels of 0.861 for UCS and to 0.835 for τ. The regression equations show that the relationship between P-wave velocity, point load index, slake durability index and dry density input sets with uniaxial compressive and shear strength under conditions of linear relation has determination coefficients of (R^2) of 0.716 and 0.714, respectively. The results show that the proposed ANN methods can be applied as a new acceptable method for the prediction of uniaxial compressive and shear strength of intact rocks.

Key words: Artificial neural network; Levenberg-Marquardt, Linear regression, Uniaxial compressive strength, Shear strength.

INTRODUCTION

Rock is a natural and suitable material that is used for constructions of dams, roads, buildings and other engineering constructions such as wave breakers. Due to the distribution of outcrops of limestone's through the Iran, this type of sedimentary rocks is more used in engineering constructions, particularly in dam construction.

The geo-engineering characteristics of rock are complex and ill-defined due to the varied physical processes associated with the formation of these materials in time and space (Jaksa 1995; Singh *et al*. 2005). Some of the important design parameters which are difficult to establish can be indirectly determined, using the relationship between the static and dynamic properties of rock (Sarkar *et al*. 2009). Inoue and Ohomi (1981) suggested a relationship between the uniaxial compressive strength (UCS), elastic wave velocity and density of weak rocks while Singh and Dubey (2000) and Singh *et al*. (2004) suggested empirical relationships between UCS and P-wave velocity, mainly for Coal Measure strata.

2. Previous Studies:

Artificial Neural Networks (ANNs) have received considerable development in recent years, with a wide range of applications in Rock Mechanics and Engineering Geology.

Yang and Zhang (1997) investigated the point load testing with ANN. Cai and Zhao (1997) used ANN for tunnel design and optimal selection of the rock support measure and to ensure the stability of the tunnel. Singh *et al*. (2001) predicted the strength property of schistose rocks by neural network. The stability of waste dump from dump slope angle and dump height is investigated by Khandelwal and Singh (2002). They found very realistic results as compared to the other analytical approaches. Maity and Saha (2004) assessed the damage in structures from changes in static parameters by neural network. Singh *et al*. (2004) predicted the P-wave velocity and anisotropic properties of rocks by neural network. Maulenkamp and Grima (1999) developed a model by which uniaxial compressive strength can be predicted from Equotip hardness. It has been reported that the prediction of uniaxial compressive strength by ANN is closer from the measured values. Dehghan *et al*. (2010) Predicted uniaxial compressive strength and modulus of elasticity for Travertine samples, using regression and artificial neural networks. Sarkar *et al*. (2010) estimated the strength parameters of rocks using artificial neural networks.

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MATERIALS AND METHODS

3-1. Preparation of Samples and Data Analysis:

The database was obtained by a series of laboratory tests on limestone samples. 120 samples of limestone were collected from different locations in the Hamadan province located in the West part of Iran (Fig. 1). The UCS, P-wave velocity, point load index, slake durability index and dry density were determined following ISRM (1981, 1985). The results in Table 1 are the average of three tests.

Fig. 1: Location of sampling at Hamedan province in west part of Iran.

Statistical description of examined parameters is given in Table 2. It can also be seen from this table that all parameters distribution of mean and average values is close together. This object represented that distribution of this parameter for experimented samples is normal. As shown in Table 2, the measured value of UCS ranges from at least 56.15 MPa to a maximum 103.79 MPa. The mean and Std. deviation values of UCS are 86.97 MPa and 12.097, respectively. The shear strength values range from at least 6.08 MPa to a maximum 21.39 MPa, with a mean value of 14.038 (MPa) and 3.39 Std. deviation.

A correlation matrix was produced to investigate the strength of the linear relationships between the variables included in this study. For this purpose, the correlation matrix was produced applying bivariate correlation technique to the original data to define the degree of linear relationships between all variables. In correlation analysis, Pearson's correlation coefficients between UCS and τ, being the dependent variables, and the other selected rock's properties, being independent variables. Pearson's correlation coefficients (r-values) are given in Table 3. According to bivariate correlation analysis, point load index has influence on UCS and dry density. P-wave velocity and point load index have influence on shear strength (Table 3). Point load index and P-wave velocity are the most significant properties affecting to UCS and shear strength of rocks with r-value of 0.741 and 0.773, respectively.

4. Artificial Neural Networks:

The artificial neural network (ANN) is a new branch of intelligence science that has developed rapidly since 1980s. Nowadays, ANN is considered to be one of the intelligent tools to understand the complex problems. Neural networks have the ability to learn from the pattern acquainted before. Once the network has been trained, with sufficient number of sample datasets, it can make predictions, on the basis of its previous learning, about the output related to new input dataset of similar pattern (Khandelwal, 2004). They are networks with many simple processing units, which are called nodes, or neurons, with dense parallel interconnections. Each neuron receives weighted inputs from other neurons and communicates its outputs to the other neurons by using an activation function. Thus, information is represented by massive cross-weighted interconnections. Neural networks might be single or multilayered. The present study utilizes multilayer architecture for the prediction. Neural networks, with their remarkable ability to derive meaning from complicated or imprecise data, can be used to extract patterns and detect trends that are too complex to be noticed by other computer techniques due to adaptive learning. Therefore, artificial neural network can be used for a particular problem when deviation in the available data is expected and accepted, when a defined methodology is not available.

Table 1: Results of experimental tests for determining τ and UCS.

No.	P-wave velocity, V (m/s)	Point load index, Is_{50} (MPa)	Density, γ(gr/cm^3)	Slake durability index (SDI%)	Uniaxial compressive strength, UCS (MPa)	Shear strength, τ (MPa)
1	6697.21	5.37	2.741	99.69	103.99	18.39
2	6532.06	5.301	2.73	99.58	101.61	17.561
3	6367.19	5.19	2.721	99.39	98.031	17.31
4	6108.34	4.903	2.703	99.23	100.62	17.02
5	5978.32	4.86	2.694	98.89	96.02	16.76
6	5785.19	4.59	2.69	98.46	92.23	15.231
7	5706.61	4.51	2.67	98.31	89.31	15.114
8	4674.09	4.11	2.671	98.09	82.11	15.034
9	4202.35	3.81	2.67	98.14	83.59	14.68
10	4324.76	3.55	2.663	98.11	65.09	11.101
11	6410	4.86	2.689	99.5	99.31	17.023
12	5401.23	4.79	2.681	99.48	95.07	16.73
13	5389.26	4.61	2.679	99.36	94.74	16.23
14	5309.64	4.53	2.673	99.4	93.321	15.31
15	5231.09	5.46	2.668	99.32	92.123	14.84
16	6029.2	4.29	2.66	99.012	97.023	17.47
17	5901.11	2.16	2.664	99.11	92.141	15.321
18	5876.04	3.92	2.653	98.904	88.41	16.75
19	5939.19	3.78	2.641	98.79	85.02	14.41
20	5437.19	2.65	2.63	98.65	88.25	13.035
21	6440.38	3.66	2.767	99.71	97.98	17.35
22	6332.03	4.6	2.761	99.7	95.324	16.032
23	6283.32	3.62	2.75	99.66	91.651	15.53
24	6176.12	3.39	2.754	99.68	88.021	13.42
25	5812.09	3.51	2.73	99.65	89.542	14.24
26	5276.14	3.42	2.71	99.63	87.45	15.601
27	5217.62	2.76	2.72	99.60	81.491	14.121
28	5187.02	2.58	2.691	99.62	76.123	12.34
29	5032.11	2.37	2.685	99.54	71.302	11.83
30	4882.17	1.52	2.682	99.50	66.76	10.35
31	4322.14	3.84	2.71	99.45	93.13	16.32
32	4307.31	3.61	2.7	99.4	90.42	15.231
33	4311.09	3.56	2.682	99.32	89.241	12.65
34	4278.34	3.35	2.671	99.41	92.34	13.73
35	4236.54	3.41	2.66	99.27	97.32	14.61
36	4295.42	3.13	2.64	99.11	89.532	8.43
37	3983.28	3.22	2.635	98.54	67.25	6.72
38	3761.03	2.75	2.62	98.87	63.72	6.23
39	3832.13	2.56	2.61	98.65	59.41	5.41
40	3478.03	1.19	2.59	98.17	56.15	6.082

Table 2: Statistical analysis of limestone samples.

Variables	Minimum	Maximum	Median	Average	Std. deviation
$I_{s(50)}$ (MPa)	2.19	5.67	3.64	3.73	1.037
V_p (m/s)	3478.03	6697.21	5349.45	5268.6	903.66
γ_d (gr/cm^3)	2.52	2.771	2.68	2.68	0.041
SDI (%)	98.17	99.78	99.34	99.14	0.508
τ(MPa)	6.082	21.39	15.07	14.038	3.4
UCS (MPa)	56.15	103.99	89.98	86.97	12.097

Table 3: Correlation matrix for original data set.

parameters	I_s	γ_d	V_p	SDI	UCS	τ
I_s	1	0.421	0.566	0.157	0.741	0.718
γ_d	-	1	0.64	0.689	0.586	0.660
V_p	-	-	1	0.455	0.692	0.773
SDI	-	-	-	1	0.475	0.394
UCS	-	-	-	-	1	0.867
T	-	-	-	-	-	1

The investigations demonstrate that neural network model have superiority in solving problems in which many complex parameters influence the process and results, when process and results are not fully understood and where historical or experimental data are available. The prediction of strength parameters are also of this type. In this study, a multilayer feedforward network and regression analyses have been used to estimate UCS and τ.

4-1. MLPs:

Multilayer perceptrons (MLPs) are layered feedforward networks typically trained with static backpropagation. These networks have found their way into countless applications requiring static pattern classification. Their main advantage is that they are easy to use, and that they can approximate any input/output map. The key disadvantages are that they train slowly, and require lots of training data (typically three times more training samples than network weights). These models consist of three layers of neurons (Heidari *et al*., 2010):

Input layer is used to present data to the network. In fact, the input layer receives the data from different sources. Hence, the number of neurons in the input layer depends on the number of input data sources.

Hidden layer(s) are used to act as a collection of feature detectors. In ANN algorithms, construction of network architecture requires both optimum number of hidden layers between the input and output layers and optimum number of neurons in each layer. This is one of the most important and difficult tasks, since there is no unified theory for the optimal architecture (Shahin *et al*., 2001; Güllü and Erçelebi, 2007). The number of hidden layers and their neurons are often determined by trial and error (Kanungo *et al*, 2006).

Output layer is used to produce an appropriate response to the given input. The output layer contains a single neuron representing UCS or shear strength (τ).

In the present study, the various algorithms (i.e. Levenberg–Marquardt, Delta-Bar-Delta, Step, Momentum, Conjugate Gradient and Quickprop) were applied in order to identify the one which best trains the network. A schematic of the MLPs is shown in Fig. 2. In this neural model, an activation function has been used that is hyperbolic tangent. Furthermore, Levenberg-Marquardt is used as a learning rule. A three-layer artificial neural network with the description of input and output nodes, employed in this study is shown in Fig. 2.

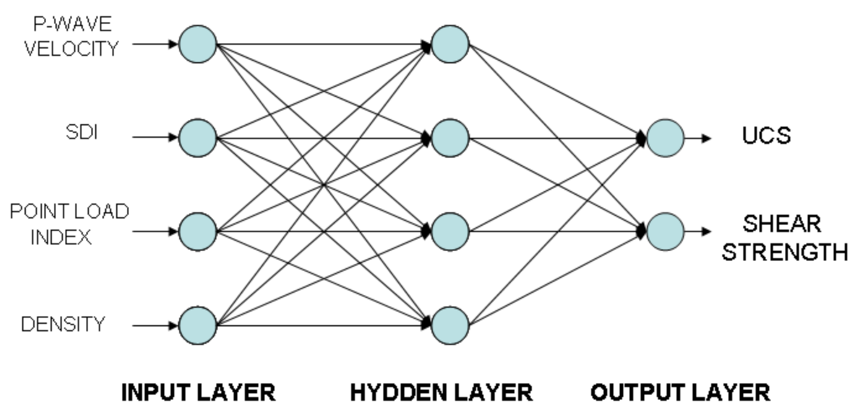

Fig. 2: Architecture of MLPs.

5. Regression:

For established predictive models among the relevant mechanical properties of rocks, regression analysis is the traditional method. Many simple models, using the dry density value, the P-wave velocity, the point load index and other properties were applied to estimate UCS & shear strength (Dehghan *et al*. 2010). However, for those who had established simple models only, R^2, i.e., the proportion of variation explained by the dependent variable from the information obtained via the independent variables was considered as sufficient criterion. This is not adequate for use in mining and civil engineering applications, because the R^2 based on simple models cannot explain the total variation introduced by the independent variables. In the other words, these are less reliable models. Therefore, the use of a multiple regression models will be more accurate in appraising elastic properties of intact rocks. Multiple regression analysis is a powerful modeling technique. This method can be useful in those cases where complex relations are involved. In addition, multiple regression analysis can be the right method where more than one variable affects a rock property (Karakus *et al*. 2005).

In this study, a linear regression has been used. Multivariate linear regression model (MLR) relates one dependent variable Y to k independent variables or predictors Xi (i = 1,…, k). The result is an equation, which can be used for estimating Y as a linear combination of the predictors, namely

$$Y = a_0 + a_1 X_1 + a_2 X_2 + ... + a_k X_k$$

Where Y is an estimate and the a_i' s are the regression coefficients. The coefficients a_is are determined such that the sum of the squares of the estimation error is a minimum on the developmental sample of size n, i.e., Min e (a), where $\min e(\underline{a}) = \sum_{j=1}^{n} \left[yi - \widehat{Y}(\underline{a}) \right]^2$

where $\underline{a} = \left[a_0 a_1 ... a_k \right]^T$. Using the least square method, the solution for the coefficients in matricial form is given by:

$$\underline{a} = \left(\underline{\underline{X'X}} \right) \underline{\underline{X'}} Y$$

Where a is a vector of length N+1 that contains the estimates of the coefficients, with N being the number of predictors (Xi; i = 1; . . .; N). The matrix $\underline{\underline{X'X}}$ contains the sums of squares and cross products matrix and $Y = \left[y_1 y_2 ... y_n \right]^T$. The (j, k) element of the matrix is

$$\sum_{i=1}^{n} X_{ji} X_{ik}, \quad i=1,\dots,N$$

For a sample of size N (Tabari *et al*. 2009, Valverde Ramirez *et al*. 2005)

RESULTS AND DISCUSSION

6-1. Artificial Neural Network Procedure:

As mentioned earlier, in this research, MLP algorithm has been used. By means of trial and error, an optimum network and parameter configuration for network was derived. For estimation of the UCS and τ, γ_d, V_p, $I_{s(50)}$ and *SDI* have been used as inputs for the neural networks. A total of 40 sets of data were used to predict UCS and τ by ANNs, of which 28 data sets were used for training and 12 sets for testing the network. Training and testing of the networks were accomplished on NeuroSolutions version 6. In order to optimize the architectural parameters, several runs have been performed with different architectural configurations. The best network was proposed with hyperbolic tangent activation function, Levenberg-Marquardt learning rule with 4-4-2 architecture neurons for hidden layers.

The graph plotting predicted and determined values indicates a good correlation and confirms the applicability of ANN. As it is clear, the R^2 values for testing the set are 0.861 for UCS and 0.835 for τ (Figs. 3, 4). These results also are indicated in the bar charts (Figs. 5, 6) for uniaxial compressive strength and shear strength for different data set. The error distribution for the 12 individual test data for UCS and shear strength has also been shown in Figs.7, 8. It must be noted that in all ANN models for prediction of both UCS and τ, Levenberg-Marquardt is the best learning rule.

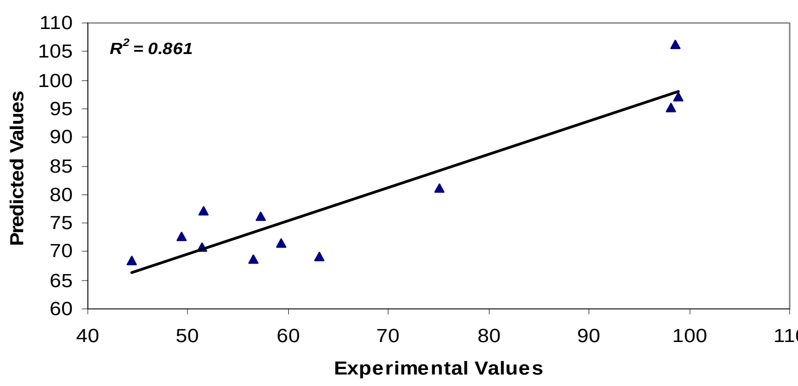

Fig. 3: Correlation between experimental and predicted values of UCS from ANN model.

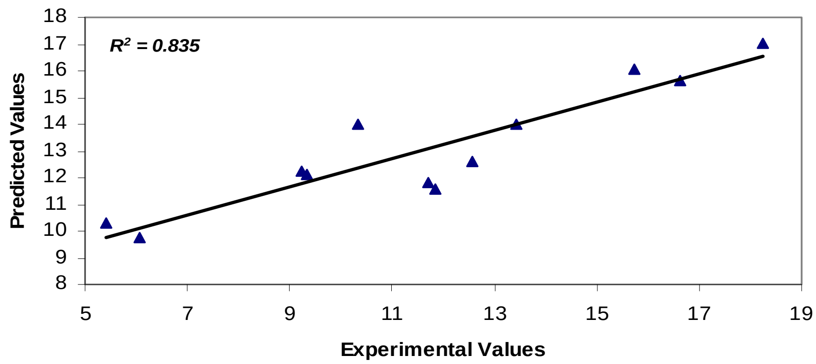

Fig. 4: Correlation between experimental and predicted values of τ from ANN model.

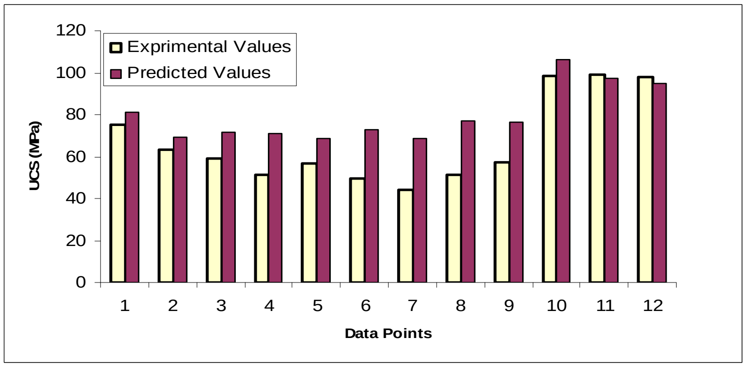

Fig. 5: Bar chart of UCS for data set numbers 1–12.

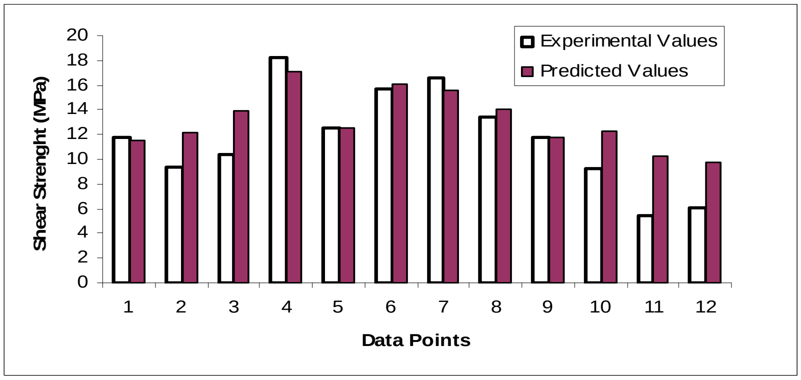

Fig. 6: Bar chart of τ for data set numbers 1–12.

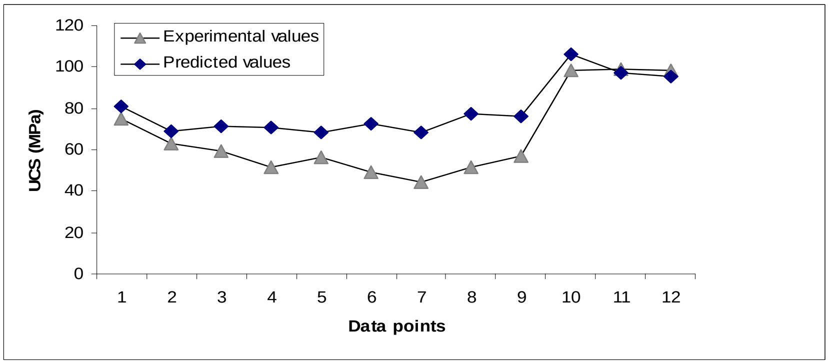

Fig. 7: The error distribution for the 12 individual tests data for UCS.

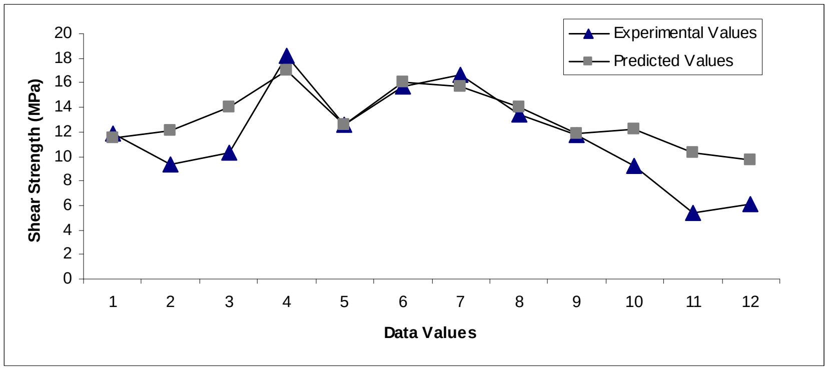

Fig. 8: The error distribution for the 12 individual tests data for shear strength.

6-2. Regression Analysis:

For this purposes a series of equation fitting was performed. Finally, two multivariable regression equations were developed for the prediction of the shear and uniaxial compressive strength, shown as follows:

$$UCS=6.638(SDI)-3.74(\gamma_d)+6.749(I_{s50})-0.003(V_p)-603.527 \qquad R^2=0.716 \qquad (1)$$

$$\tau=22.014(\gamma_d)+0.845(I_{s50})+0.001(V_p)-0.04(DSI)-52.158 \qquad R^2=0.714 \qquad (2)$$

Where UCS, τ and I_{s50} are in MPa, V_p in m/s, γ_d in (gr/cm^3) and SDI in percentage. Figs. 9, 10 show the relationship between the experimented and predicted values of UCS and τ. The distribution of the differences between UCS and τ predicted from Eqs.(1) and (2) and actual determined amounts of UCS and τ are shown in Figs. 11, 12.

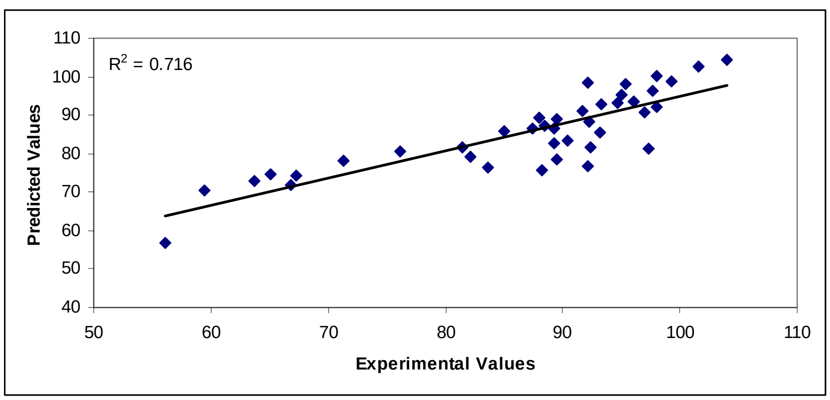

Fig. 9: Relation between experimental and predicted UCS from MLR analysis (Eqs. 1).

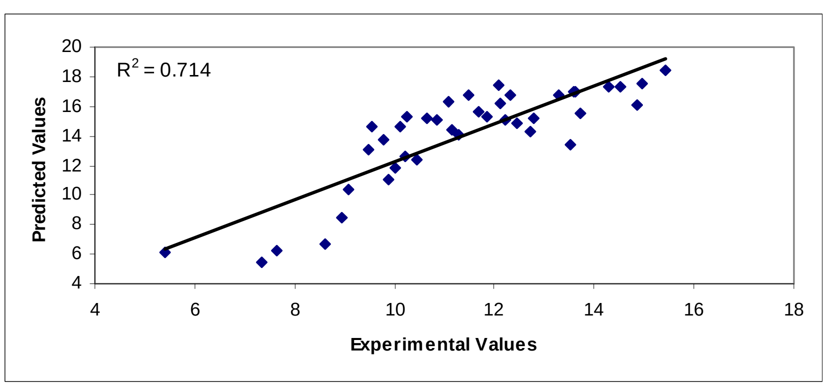

Fig. 10: Relation between experimental and predicted of shear strength from MLR analysis (Eqs. 2).

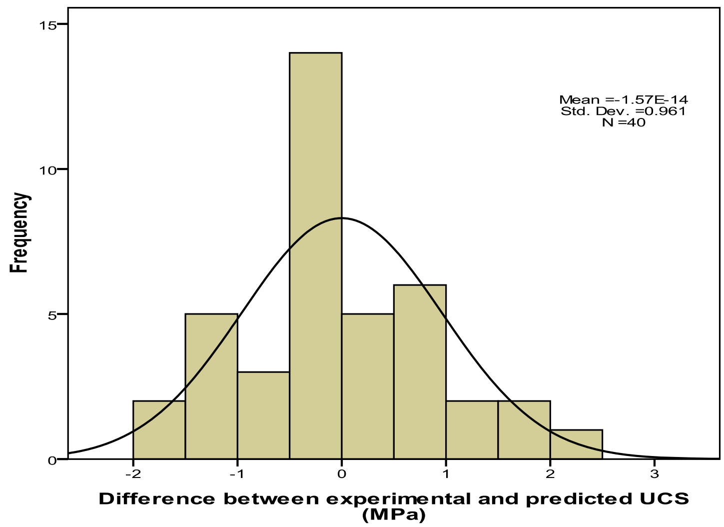

Fig. 11: Difference between experimented and predicted of UCS from MLR analysis.

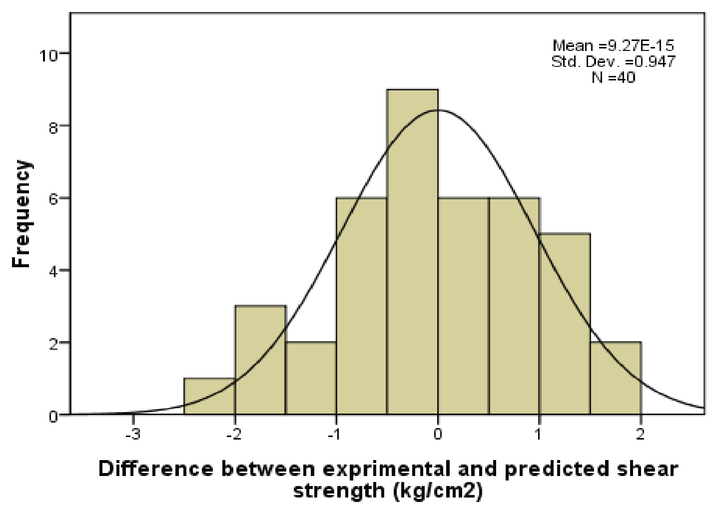

Fig. 12: Difference between experimented and predicted of shear strength from MLR analysis.

7. Conclusions:

In this research, artificial neural networks, multivariate linear regression capability for prediction of UCS and shear strength of limestone were investigated. For this purpose, the ANN model including MLP was used. The best network was proposed with Multilayer Perceptron model, hyperbolic tangent activation function, Levenberg-Marquardt learning rule with 4-4-2 architecture. The P-wave velocity in m/s (V_p), the point load index in MPa ($I_{s(50)}$), the slake durability index (SDI) and the dry density (γ_d) in the ANNs and regression analysis were used as input variables in order to estimate uniaxial compressive strength (UCS) and shear strength (τ). The multivariable regression equation predicted the uniaxial compressive strength and shear strength with correlation coefficients (R^2) of 0.716 and 0.714, respectively. In general, the highest correlation coefficient (R^2) that obtained for ANN method for estimation of UCS and τ are 0.816 and 0.835, respectively. We can conclude that, in general when the ANN method was used, the results are better than those obtained from the regression analyses. The predictions made by using a multilayer perceptrons (MLPs) artificial neural network method seems to be more reliable than those performed by equations. The AAN and MLR models that proposed in this research can be used for overcome to the problems in measurement of UCS and shear strength of intact rocks.

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