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A Promoter Operator for Defuzzification Methods

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Abstract: In this paper, we first present an expansion of our previous work on fuzzy ranking ($Mag-$method). The extended approach can rank all normal and non-normal fuzzy numbers. Since we loose some information in many defuzzification methods, the obtained results are inaccurate and may be unreasonable. Therefore, in order to increase the ability of the defuzzification methods we also construct an operator so called promoter operator. It is based on the ambiguity of fuzzy numbers. Particularly, capability of the operator is considerable when two fuzzy numbers have identical centroid points or much more while they have the same ranking order. Additionally, a number of numerical examples illustrate the novel ranking approaches and the ability of promoter operator.

Key words: Defuzzification; Magnitude of fuzzy numbers; Promoter operator; Ranking; Trapezoidal fuzzy numbers.

INTRODUCTION

In many applications, ranking of fuzzy numbers is an important component of the decision process. L-R fuzzy number as the most general form of fuzzy number has been used extensively. A key issue in operationalzing fuzzy set theory is how to compare fuzzy numbers. Various approaches have been developed for ranking fuzzy numbers. In the existing research, the commonly used technique is to construct proper maps to transform fuzzy numbers into real numbers so called defuzzification. These real numbers are then compared. Herein, in approaches (Chen, S. J. and Chen, S. M., 2003; Deng, Y. and Liu, Q., 2005; Deng, Y., Zhu, Z.F. and Liu, Q., 2006; Abbasbandy, S. and Asady, B., 2006; Chen, S. J. and Chen, S. M., 2007; Abbasbandy, S. and Hajjari, T., 2009; Wang, Z.-X., Liu, Y.-J, Fan, Z.-P. and Feng, B., 2009; Chen, S.-M. and Chen, J.-H., 2009; Asady, B., 2010; Abbasbandy, S. and Hajjari, T., 2011; Hajjari T. and Abbasbandy, S. 2011; Hajjari, T., 2011; Hajjari, T., 2011) a fuzzy number is mapped to a real number based on the area measurement. In approaches (Chen, L. H. and Lu, H. W., 2001; Chen, L. H. and Lu, H. W., 2002; Liu, X. W. and. Han, S. L, 2005), $\alpha-$cut set and decision-maker's preference are used to construct ranking function. On the other hand, another commonly used technique is the centroid-based fuzzy number ranking approach (Cheng, C. H., 1998; Chu, T. and Tsao, C., 2002; Wang, Y.J. and Lee, H. Sh., 2008). It should be noted that with the development of intelligent technologies, some adaptive and parameterized defuzzification methods that can include human knowledge have been proposed. Halgamuge *et al*. (Halgamuge, S., Runkler, T. and Glesner, M. 1996) used neural networks for defuzzification. Song and Leland (Song, Q. and Leland, R. P., 1996) proposed an adaptive learning defuzzification technique. Yager (Yager, R.R., 1996) proposed knowledge based on defuzzification process, which becomes more intelligent. Similar to methods of Filve and Yager (Filev, D.P. and Yager, R.R., 1991), Jiang and Li (Jiang, T. and Li, Y., 1996) also proposed a parameterized defuzzification method with Gaussian based distribution transformation and polynomial transformation, but in fact, no method gives a right effective defuzzification output. The computational results of these methods are often conflict.

We often face difficultly in selecting appropriate defuzzification, which is mainly based on intuition and there is no explicit decision making for these parameters. For more comparison details on most of these methods, readers can see the reviews (Leekwijck, W.V. and Kerre, E. E., 2001; Roychowdhury, S. and Pedrycz, W., 2001).

In 2009, we presented a method for ranking of trapezoidal fuzzy numbers, which measured the magnitude of normal trapezoidal fuzzy numbers so called $Mag-$method. In this method, the symmetric trapezoidal (triangular) fuzzy numbers with identical mode or with identical centroid points have the same ranking order. As it is already mentioned "Mag method" was given for normal trapezoidal fuzzy numbers, in this new contribution we will extend our previous work ($Mag-$method) as the new one can rank all normal and non-normal fuzzy numbers. On the other hand, in order to overcome the weakness of defuzzification methods such as $Mag-$method, Cheng's distance, Chau and Tsao's method and any defuzzification method, the authors are

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motivated to find a way to promote the defuzzification methods so called "promoter operator". The promoter operator can rank fuzzy numbers more accurately.The reminder of this paper is organized as follows: Section 2 contains some basic notation. Section 3 contains a review and an expansion on the magnitude of a trapezoidal fuzzy number. A promoter operator for defuzzification methods will be presented in Section 4. A number of numerical examples demonstrate the application of the promoter operator. Finally, concluding remarks are given in Section 5.

2. Background Information:

In this section, we briefly review some basic concepts of generalized fuzzy numbers and some existing methods for ranking fuzzy numbers.

Basic Notations and Definitions:

In general, a generalized fuzzy number A is described as any fuzzy subset of real line R, whose membership $\mu_A(x)$ can be defined as (Dubios, D. and Prade, H., 1978).

$$\mu_A(x)=\begin{cases} L_A(x) & a \le x \le b \\ \omega & b \le x \le c \\ R_A(x) & c \le x \le d \\ 0 & otherwise, \end{cases} \tag{1}$$

where $0 \le \omega \le 1$ is a constant, and $L_A:[a,b]\to[0,\omega]$, $R_A:[c,d]\to[0,\omega]$ are two strictly monotonical and continuous mapping from R to closed interval $[0,\omega]$. If $\omega=1$, then A is a normal fuzzy number; otherwise, it is a trapezoidal fuzzy number and is usually denoted by $A=(a,b,c,d,\omega)$ or $A=(a,b,c,d)$ if $\omega=1$.

In particular, when $b=c$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number denoted by $A=(a,b,d,\omega)$ or $A=(a,b,d)$ if $\omega=1$.Therefore, triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers.

Since L_A and R_A are both strictly monotonical and continuous functions, their inverse functions exist and should be continuous and strictly monotonical. Let $L_A^{-1}:[a,b]\to[0,\omega]$ and $R_A^{-1}:[a,b]\to[0,\omega]$ be the inverse functions of $L_A(x)$ and $R_A(x)$, respectively. Then $L^{-1}{}_A(r)$ and $R^{-1}{}_A(r)$ should be integrable on the close interval $[0,\omega]$. In other words, both $\int_0^{\omega} L_A^{-1}(r)dr$ and $\int_0^{\omega} R_A^{-1}(r)dr$ should exist. In the case of trapezoidal fuzzy number, the inverse functions $L^{-1}{}_A(r)$ and $R^{-1}{}_A(r)$ can be analytically expressed as

$$L_A^{-1}(r)=a+(b-a)r/\omega \qquad 0 \le \omega \le 1 \tag{2}$$

$$R_A^{-1}(r)=d-(d-c)r/\omega \qquad 0 \le \omega \le 1 \tag{3}$$

The set of all elements that have a nonzero degree of membership in A, it is called the support of A, i.e.

$$Supp(A)=\{x\in X \mid \mu_A(x)\succ 0\} \tag{4}$$

The set of elements having the largest degree of membership in A, it is called the core of A, i.e.

$$Core(A)=\left\{x\in X \mid \mu_A(x)=\sup_{x\in X} L_A(x)\right\} \tag{5}$$

In the following, we will always assume that A is continuous and bounded support $Supp(A)=(a,d)$. The strong support of A should be $\overline{S(A)}=[a,d]$.

Definition 2.1:

A function $f:[0,1]\to[0,1]$ is a reducing function if is s increasing and $f(0)=0$ and $f(1)=1$. We say that s is a regular function if $\int_0^1 f(r)dr = 1/2$.

Definition 2.2:

If A is a fuzzy number with r-cut representation, $\left(L_A^{-1}(r), R_A^{-1}(r)\right)$ and s is a reducing function, then the value of A (with respect to s); it is defined by

$$Val(A)=\int_0^1 f(r)[L_A^{-1}(r)+R_A^{-1}(r)]dr. \tag{6}$$

Definition 2.3:

If A is a fuzzy number with r-cut representation $\left(L_A^{-1}(r), R_A^{-1}(r)\right)$, and s is a reducing function then the ambiguity of A (with respect to s) is defined by

$$Amb(A)=\int_0^1 f(r)[R_A^{-1}(r)-L_A^{-1}(r)]dr. \tag{7}$$

Let also recall that the expected interval $EI(A)$ of a fuzzy number A is given by

$$EI(A)=\left[\int_0^1 L_A^{-1}(r)dr, \int_0^1 R_A^{-1}(r)dr\right]. \tag{8}$$

Another parameter is utilized for representing the typical value of the fuzzy number is the middle of the expected interval of a fuzzy number and it is called the expected value of a fuzzy number A i.e. number A is given by (Bodjanova, S., 2005).

$$EV(A)=\frac{1}{2}\left[\int_0^1 L_A^{-1}(r)dr+\int_0^1 R_A^{-1}(r)dr\right]. \tag{9}$$

3. A Review and Expantion on the Magnitude of a Trapezoidal Fuzzy Number (Mag Method):

In 2009, Abbasbandy and Hajjari (Abbasbandy, S. and Hajjari, T., 2009) presented a new approach to compute the magnitude of a trapezoidal fuzzy number, which is called $Mag-$method. It was given for normal trapezoidal fuzzy numbers as follows.

For an arbitrary trapezoidal fuzzy number $A=(a,b,c,d)$ with parametric form $A=\left(L_A^{-1}(r), R_A^{-1}(r)\right)$, , the magnitude of the trapezoidal fuzzy number A as

$$Mag(A)=\frac{1}{2}\left[\int_0^1 (L_A^{-1}(r)dr+R_A^{-1}(r)+b+c)f(r)dr\right], \tag{10}$$

where the function $f(r)$ is a non-negative and increasing function on [0, 1] with $f(0)=0$, $f(1)=1$ and $\int_0^1 f(r)dr = 1/2$. Obviously, function $f(r)$ can be considered as a weighting function. For more details we refer the reader to (Abbasbandy, S. and Hajjari, T., 2009).

Now, we would like to extend this method for non-normal trapezoidal fuzzy numbers and also for all generalized fuzzy numbers.

Let $A=(a,b,c,d)$ be a non-normal trapezoidal fuzzy number with $r-$cut representation $A=\left(L_A^{-1}(r), R_A^{-1}(r)\right)$. Consequently, from (2), (3) and (10) we have

$$Mag(A) = \frac{(3\omega^2 + 2)(b+c)}{12\omega} + \frac{(3\omega - 2)(a+d)}{12\omega}. \tag{11}$$

It is clear that for normal trapezoidal fuzzy numbers the formula (10) reduces to

$$Mag(A) = \frac{5}{12}(b+c) + \frac{1}{12}(a+d). \tag{12}$$

In the following, we use an example to illustrate the ranking process of the proposed method. Moreover, for normal fuzzy numbers we have

$$Mag(A) = \frac{1}{2}\left[\int_0^1 (L_A^{-1}(r)dr + R_A^{-1}(r) + L_A^{-1}(1) + R_A^{-1}(1))\, f(r)dr\right]. \tag{13}$$

Example 3.1:

Assume that there are two generalized fuzzy numbers $A = (1,3,4,5;0.6)$ and $B = (2,3,4,5;0.8)$ (Chen, S.-M. and Chen, J.-H., 2009) as shown in Fig. 1.

Based on Eq. (11) we can calculate $Mag(A) = 2.9278$ and $Mag(B) = 3.1500$ then the ranking order of the fuzzy numbers A and B is: $B \succ A$.

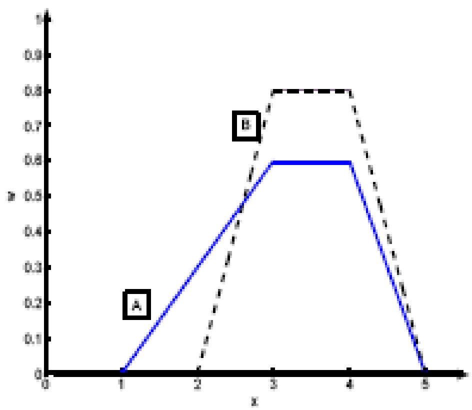


Fig. 1: Fuzzy numbers $A = (1,3,4,5;0.6)$ and $B = (2,3,4,5;0.8)$.

Example 3.2:

Consider two fuzzy numbers from (Chen, S.-M. and Chen, J.-H., 2009) $A = (1,2,5)$ and $B = (1,2,4)$ as shown in Fig. 2. The membership function of A and B are as follows:

$$\mu_B(x) = \begin{cases} x - 1 & 1 \le x \le 2, \\ \dfrac{5 - x}{3} & 2 \le x \le 5, \\ 0 & otherwise. \end{cases} \qquad \mu_B(x) = \begin{cases} \left[1 - (x-2)^2\right]^{1/2} & 1 \le x \le 2, \\ \left[1 - \dfrac{1}{4}(x-2)^2\right]^{1/2} & 2 \le x \le 4, \\ 0 & otherwise. \end{cases}$$

According to Eqs. (12) and (13) we can obtain $Mag(A) = 4.3927$ and $Mag(B) = 4.3927$. Accordingly, the ranking order of fuzzy numbers is $B \prec A$. Meanwhile, the obtained results from Wang *et al*.'s and Chu and Tsao's are consisted with the one by ours. However, our approach is simpler in calculating procedure than others. However, by Deng *et al*'s method the ranking order is $B \succ A$. From Fig. 2. We can conclude that $B \prec A$ is more consistent with human intuition.

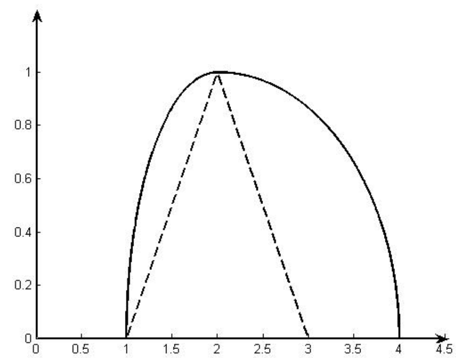


Fig. 2: Fuzzy numbers in Example 3.2.

4. A Promoter Operator for Defuzzification Methods:

In this part, we introduce an operator as a promoter for defuzzification methods in order to rank fuzzy numbers more precisely. First, we need to define inequality of ordered pairs.

Definition 4.1:

Let a_1, a_2, b_1 and b_2 are real numbers where $0 \le b_1 \le 1$ and $0 \le b_2 \le 1$. We say

1) $(a_1, b_1) \prec (a_2, b_2)$ if and only if $a_1 \prec a_2$ or ($a_1 = a_2$ and $b_1 \prec b_2$).
2) $(a_1, b_1) \succ (a_2, b_2)$ if and only if $a_1 \succ a_2$ or ($a_1 = a_2$ and $b_1 \succ b_2$).
3) $(a_1, b_1) \approx (a_2, b_2)$ if and only if ($a_1 = a_2$ and $b_1 = b_2$).

The aim of defining inequality of ordered pairs in this way is applying it to construct a promoter operator, which can rank fuzzy numbers more accurately.

Thus, we have to use the promoter operator $P : F(R) \to F(R)$, which transforms a family of all fuzzy numbers into a family of fuzzy numbers. Suppose A be a fuzzy number and $D(.)$ is a defuzzification method. We introduce the promoter operator $P : A \to P(A)$ such that

$$P(A) = \left(D(A), \frac{1}{1 + amb(A)} \right) \tag{14}$$

Consider two fuzzy numbers A and B the ranking order is based on the following situations:

If $P(A) \succ P(B)$ then $A \succ B$,

If $P(A) \prec P(B)$ then $A \prec B$,

If $P(A) \approx P(B)$ then $A \approx B$,

This promoter operator can be applied for all defuzzification methods.

Example 4.2:

Let two fuzzy numbers $A = (3,6,9)$ and $B = (5,6,7)$ from (Wang, Z.-X., Liu, Y.-J, Fan, Z.-P. and Feng, B., 2009) as shown in Fig. 3.

Through the approaches in this paper, the ranking index can be obtained as $Mag(A) = Mag(B) = 12$ and $EV(A) = EV(B) = 6$ .Then the ranking order of fuzzy numbers is $A \approx B$. Because fuzzy numbers A and B have the same mode and symmetric spread, most of existing approaches have the identical results. For instance, by Abbasbandy and Asady's approach (Abbasbandy, S. and Asady, B., 2006), different ranking orders are obtained when different index values p are taken. When $p = 1$ and $p = 2$ the ranking order is the same, i.e., $A \approx B$ Nevertheless, the same results produced when distance index, CV index of Cheng's approach and

Chu and Tsao's area are respectively used, i.e., $x_A = x_B = 6$ and $y_A = y_B = \frac{1}{3}$ then from Cheng's distance and Chau and Tsao's area we get that $R(A) = R(B) = 2.2608,\ S(A) = S(B) = 1.4142$ respectively.

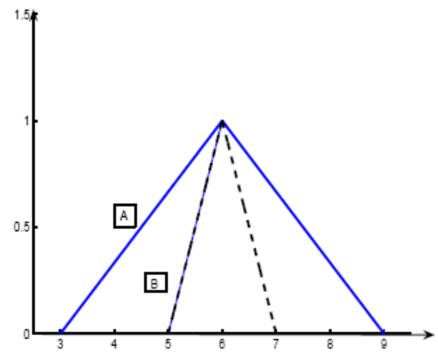


Fig. 3: Fuzzy numbers $A = (3,6,9)$ and $B = (5,6,7)$.

From the obtained results we have $A \approx B$, for two triangular fuzzy numbers $A = (3,6,9)$ and $B = (5,6,7)$. Now we review the ranking approaches by promoter operator. Since A and B have the same ranking order and the same centroid points we then compute their ambiguities. Hence, from (Deng, Y., Zhu, Z.F. and Liu, Q., 2006) it will be obtained $amb(A) = 1$ and $amb(B) = \frac{1}{3}$.

Consequently, by using Definition 4.1 and promoter operator (14) we have

$$P(A) = \left(Mag(A), \frac{1}{1+amb(A)} \right) = (12, \frac{1}{2}),\ P(B) = \left(Mag(B), \frac{1}{1+amb(B)} \right) = (12, \frac{3}{4})$$

$$P(A) = \left(EV(A), \frac{1}{1+amb(A)} \right) = (6, \frac{1}{2}),\ P(B) = \left(EV(B), \frac{1}{1+amb(B)} \right) = (6, \frac{3}{4})$$

$$P(A) = \left(R(A), \frac{1}{1+amb(A)} \right) = (2.2608, \frac{1}{2}),\ P(B) = \left(R(B), \frac{1}{1+amb(B)} \right) = (2.2608, \frac{3}{4})$$

$$P(A) = \left(S(A), \frac{1}{1+amb(A)} \right) = (12, \frac{1}{2}),\quad P(B) = \left(S(B), \frac{1}{1+amb(B)} \right) = (12, \frac{3}{4}).$$

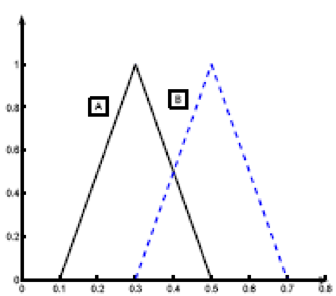
The ranking order is $A \prec B.$ Through the proposed approach by Wang *et al*., the ranking index values can be obtained as $d_1 = 0.1429$ and $d_2 = 0.1567$ Then the ranking order of fuzzy numbers is also $A \prec B.$

In the following, we use the data sets shown in Chen and Chen (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009) to compare the ranking results of the proposed approaches with Cheng method (Cheng, C. H., 1998), Chu and Tsao's method (Chu, T. and Tsao, C., 2002) and Chen and Chen (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009). The comparing of ranking results for different methods is shown in Table 1.

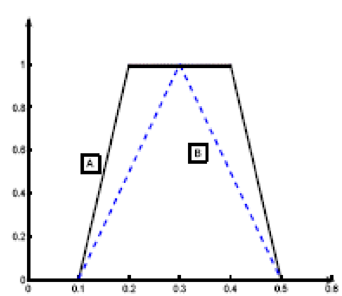
1. For the fuzzy numbers A and B shown in Set 1 of Fig. 4, Cheng's method (Cheng, C. H., 1998), Chu's method (Chu, T. and Tsao, C., 2002), Chen and Chen's method (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009) and $Mag-$method (Abbasbandy, S. and Hajjari, T., 2009) get the same ranking order $A \prec B.$

Table 1:

Methods	F.n	set 1	set 2	set 3	set 4	set 5	set 6	set 7	set 8
Cheng's (1998)	A	.583	.583	.583	.461	.424	-.583	.767	.68
	B	.707	.583	.583	.583	*	.583	.724	.726
	C	*	*	*	*	*	*	*	.746
Results		$A \prec B$	$A \sim B$	$A \sim B$	$A \prec B$	*	$A \prec B$	$B \prec A$	$A \prec B \prec C$
P.O Results		*	$A \prec B$	$A \prec B$	*	*	*	*	*
Chau's (2002)	A	.15	.15	.15	.12	.15	-.15	.287	.228
	B	.25	.15	.15	.15	*	.15	.262	.262
	C	*	*	*	*	*	*	*	.278
Results		$A \prec B$	$A \sim B$	$A \sim B$	$A \prec B$	*	$A \prec B$	$B \prec A$	$A \prec B \prec C$
P.O Results		*	$A \prec B$	$A \prec B$	*	*	*	*	*
Chen's (2007)	A	.446	.424	.446	.357	.42	.446	.413	.372
	B	.489	.446	.473	.446	.860	.747	.401	.416
	C	*	*	*	*	*	*	*	.398
Results		$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$B \prec A$	$A \prec C \prec B$
P.O Results		*	*	*	*	*	*	*	*
Chen's (2009)	A	.258	.254	.258	.254	.206	-.258	.443	.335
	B	.430	.258	.278	.258	1	.258	.404	.408
	C	*	*	*	*	*	*	*	.420
Results		$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$B \prec A$	$A \prec B \prec C$
P.O Results		*	*	*	*	*	*	*	*
Mag (2009)	A	.3	.3	.3	.27	.3	-.3	.525	.483
	B	.5	.3	.3	.3	1	.3	.575	.508
	C	*	*	*	*	*	*	*	.617
Results		$A \prec B$	$A \sim B$	$A \sim B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B \prec C$
P.O Results		*	$A \prec B$	$A \prec B$	*	*	*	*	*



Set 1
A=(0.1, 0.3, 0.3, 0.5; 1)
B=(0.3, 0.5, 0.5, 0.7; 1)



Set 2
A=(0.1, 0.2, 0.4, 0.5; 1)
B=(0.1, 0.3, 0.3, 0.5; 1)

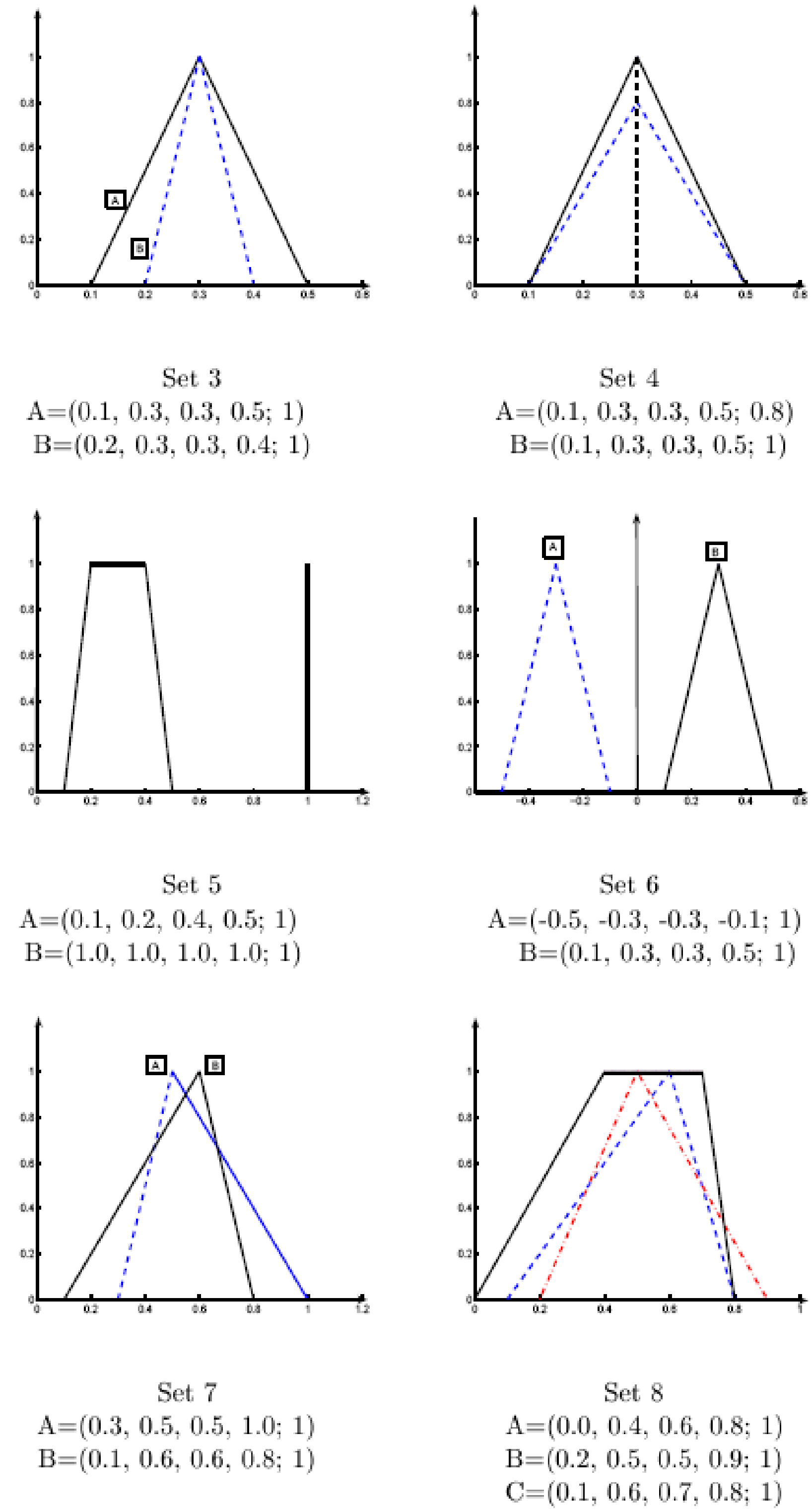


Fig. 4:

2. For the fuzzy numbers A and B shown in Set 2 of Fig. 4, Cheng's method (Cheng, C. H., 1998), Chu 's method (Chu, T. and Tsao, C., 2002) and $Mag-$ method (Abbasbandy, S. and Hajjari, T., 2009) get the same ranking order $A \approx B$, which is unreasonable. Whereas by applying the promoter operator the

ranking order is the same as Chen and Chen's method (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009), i.e. $A \prec B$.

3. For the fuzzy numbers A and B shown in Set 3 of Fig. 4, Cheng's method (Cheng, C. H., 1998), Chu and Tsao's method (Chu, T. and Tsao, C., 2002) and $Mag-$method (Abbasbandy, S. and Hajjari, T., 2009) get an inaccurate ranking order $A \approx B$ whereas by applying the promoter operator the ranking order is the same as Chen and Chen's method (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009) i.e. $A \prec B$.
4. For the fuzzy numbers A and B shown in Set 4 of Fig. 4, Cheng's method (Cheng, C. H., 1998), Chu and Tsao's method (Chu, T. and Tsao, C., 2002), Chen and Chen's method (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009) and $Mag-$method (Abbasbandy, S. and Hajjari, T., 2009) get the same ranking order: $A \prec B$.
5. For the fuzzy numbers A and B shown in Set 5 of Fig. 4, Cheng's method (Cheng, C. H., 1998), Chu and Tsao's method (Chu, T. and Tsao, C., 2002) cannot calculate the crisp-value fuzzy number, whereas Chen and Chen's method (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009) and $Mag-$method (Abbasbandy, S. and Hajjari, T., 2009) get the same ranking order: $A \prec B$.
6. For the fuzzy numbers A and B shown in Set 6 of Fig. 4, Cheng's method (Cheng, C. H., 1998), Chu and Tsao's method (Chu, T. and Tsao, C., 2002), Chen and Chen's method (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009) and $Mag-$method (Abbasbandy, S. and Hajjari, T., 2009) get the same ranking order: $A \prec B$.
7. For the fuzzy numbers A and B shown in Set 7 of Fig. 4, Cheng's method (Cheng, C. H., 1998), Chu and Tsao's method (Chu, T. and Tsao, C., 2002), Chen and Chen's method (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009) get the same ranking order: $B \prec A$, whereas the ranking order by $Mag-$method (Abbasbandy, S. and Hajjari, T., 2009) is $A \prec B$. By comparing the ranking result of $Mag-$method with other methods with respect to Set 7 of Fig. 4, we can see that $Mag-$method considers the fact that defuzzified value of a fuzzy number is more important than the spread of a fuzzy number.
8. For the fuzzy numbers A and B shown in Set 8 of Fig. 4, Cheng's method (Cheng, C. H., 1998), Chu and Tsao's method (Chu, T. and Tsao, C., 2002), Chen and Chen's method (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009) and $Mag-$method (Abbasbandy, S. and Hajjari, T., 2009) get the same ranking order: $A \prec B \prec C$, whereas the ranking order by Chen and Chen's method is $A \prec C \prec B$. By comparing the ranking result of mentioned method with other methods with respect to Set 8 of Fig. 4, we can see that Chen's method considers the fact that the spread of a fuzzy number is more important than defuzzified value of a fuzzy number.

4. Conclusion:

In spite of many ranking methods, no one can rank fuzzy numbers with human intuition consistently in all cases. Here, we have extended our previous method ($Mag-$method), which can rank all normal and non-normal fuzzy numbers. Moreover, to compare those fuzzy numbers that have same ranking order in defuzzification methods, we construct an operator so called promoter operator that can be applied for defuzzification method. From the presented operator the results are more accurate. It does not imply much computational effort and does not require a prior knowledge of the set of all alternatives. We also used comparative examples to illustrate the advantage of the proposed method.

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