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Perspectivity in Linear Programming Polygon

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Abstract: Thinking about the vertices of a linear programming polygon from a perspective and reaching each of them in the polynomial time steps is in question here. On the contrary, Classical methods either have been traveling from edge to edge and vertex to vertex for optimality or trapping the optimal using ellipsoids or are mere improvisations to George B. Dantzig.

Key words: central perspectivity.

INTRODUCTION

Polytope is the general term of the sequence point, segment, polygon, polyhedron… in which point is a zero dimensional entity, segment is one dimensional entity, polygon is two dimensional entities, polyhedron is three dimensional entit. In n dimensions, the set of points whose coordinates satisfy a linear equation $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$ is called a hyper-plane, set of points whose coordinates satisfy a linear inequality $a_1x_1 + a_2x_2 + \ldots + a_nx_n \leq b$ is called a half-space, and closed half-space if boundary is also included. In two dimensions it is called half-plane. Half-space (half-plane) defined by a linear inequality is convex. A convex polygon is the intersection P of a finite number of closed half-planes in R^2. P is 2-dimensional polyhedron if the points in P affinely span R^2 i.e.,

$$\lambda_1 p_1 + \lambda_2 p_2 + \ldots + \lambda_n p_n = \sum \lambda_i p_i \text{ and } \sum \lambda_i = 1$$

Theorem 1:

The set of points corresponding to feasible solutions of the general linear programming problem constitutes a convex set.

Linear programming is the problem of maximizing a linear objective function ϕ subject to a finite set of linear inequalities. The relevance of convex polygon to linear programming is clear. The set of feasible solutions for a linear programming problem is a polygon which also implies the convex hull P.

The linear objective function ϕ attains different values on different vertices of P and finding the optimal in polynomial time has been in question since the beginning of linear programming. P, a *d*-polytope i.e., a *d*-dimensional convex polytope has $f_0, f_1, f_2 \ldots$ *i*-dimensional faces. Thus f_0 is the number of vertices of P and f_1 is the number of edges of P called the *f*-vector of P.

Conventionally, Simplex Algorithm, developed in 1947, is the method widly used to solve the linear programming problem by repeatedly moving from one vertex to an adjacent vertex of the feasible polytope P so that in each step the value of the objective function is increased. Generally, simplex method requires $2m$ to $3m$ pivot steps to attain optimality and this fact was also know to Dantzig and he stated in his book on page 160. It has been established that various pivot rules may require exponentially many pivot steps in the worst case. In 1979 Khachiyan used the ellipsoid method and was first to show that linear programming was in class P of polynomial-time solvable problems. Despite a number of refinements the method is not competitive for linear programming. The idea of moving through the interior of the feasible region was developed by N. Karmarker in 1984.

Elements:

The raw material for a perspective look consists of points, lines and planes that is to say i.e., the face numbers of the polytope. These are related to each other according to certain axioms which are called the propositions of incidence. According to the propositions of incidence, if the line determined by the points A and B contains points X and Y, it follows that the line determined by X and Y is the same line and contains A and B. Consequently, three or more points which belong to a line are said to be collinear. Lines through a point are called concurrent. A plane is assumed to consist of an infinite set of points and to be completely

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determined by any three distinct non-collinear points upon it. If the homogeneous coordinates (x_1, x_2, x_3) are given for its non-homogeneous coordinates (x, y) then they are related as: $x = \frac{x_1}{x_3} = \frac{kx_1}{kx_3}$, $y = \frac{x2}{x3} = \frac{kx_2}{kx_3}$ and consequently equation $lx_1 + l_2 y + l = 0$ I rectangular coordinates becomes in homogenous coordinates $l_1 \frac{x_1}{x_3} + l_2 \frac{x_2}{x_3} + l_3 = 0$ or $l_1 x_1 + l_2 x_2 + l_3 x_3 = 0$ where $[l_1, l_2, l_3]$ are the coordinates of a line and (x_1, x_2, x_3) are the homogeneous coordinates of a point. The relation, usually called incidence condition, between a point and a line exists if and only if $l_1 x_1 + l_2 x_2 + l_3 x_3 = 0$. Three points $(a_1, a_2, a_3), (b_1, b_2, b_3), (c_1, c_2, c_3)$ are collinear when

$$a_1 l_1 + a_2 l_2 + a_3 l_3 = 0$$
$$b_1 l_1 + b_2 l_2 + b_3 l_3 = 0$$
$$c_1 l_1 + c_2 l_2 + c_3 l_3 = 0$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

that is to say determinant whose rows consists of coordinates of the points is equal to zero. Similarly, three lies will be concurrent when

$$l_1 a_1 + l_2 a_2 + l_3 a_3 = 0$$
$$m_1 l_1 + m_2 l_2 + m_3 l_3 = 0$$
$$n_1 l_1 + n_2 l_2 + n_3 l_3 = 0$$

$$\begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{vmatrix} = 0$$

that is to say three lies are concurrent if and only if the determinant whose rows consists of the coordinates of the lines is equal to zero. A point $C(c_1, c_2, c_3)$ is collinear with distinct points $A(a_1, a_2, a_3)$ and $B(b_1, b_2, b_3)$ if and only if $C = \alpha A + \beta B$ and similarly a line n is concurrent with the lines l and m if and only if $n = \alpha l + \beta m$. By the analogy of the arguments, the point D is linearly dependent on upon the three non-collinear points A, B, C when $D = \alpha A + \beta B + \gamma C$. Any line p is linearly dependent upon three non-concurrent lines l, m, n if it can be expressed as $p = \alpha l + \beta m + \gamma n$.

Gérard Desargues(1593-1663):

We have seen the convention that how points and lines in homogeneous coordinates are incident. A theorem which signifies the properties of incidence is due to Desargues which is stated without proof and subsequently applied.

Theorem 2:

If two triangles correspond to each other in such a way that the joins of their corresponding vertices are concurrent, then the intersections of their corresponding sides are collinear, and conversely.

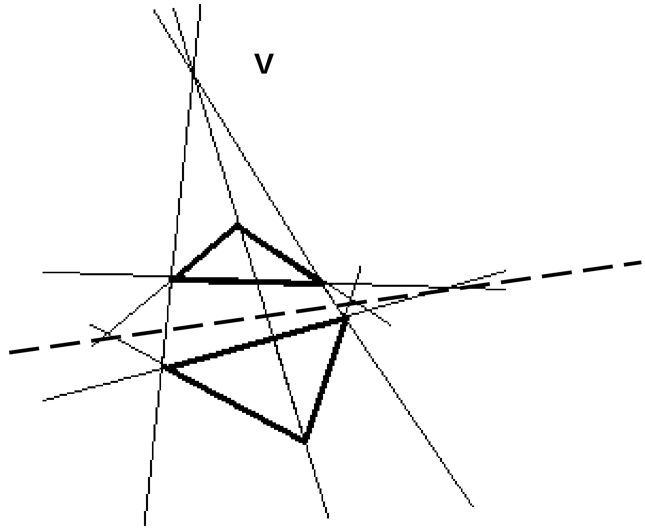

Fig. 1: Desargues' Theorem Illustration.

Perspectivities:

Correspondence between two forms in perspective is Perspectivity, that is to say, viewing from a fixed point V , the line joining the view point V to any point in the object *A* and its image $A^{'}$ are collinear.

***Theorem 3*:**

If V is the central perspective point of LP polygon, then every point of the LP polygon is in linear combination with V .

Proof:

let *A, B, C* and *A', B', C'* be the vertices of the two triangulated LP polygon. Let the lines *AA', BB', CC'* be concurrent in a point *V*. As *A = A', C = C'* then *V* can only be expressible as linear combination of and B' that is $V = \lambda B + \mu B'$. Hence two sets of points *A, B, C* and *A', B', C'* are centrally perspective from the center V.■
Equivalently, let $A(a_1, a_2, a_3)$ and $B(b_1, b_2, b_3)$ be points on line l with parameters λ and μ . The coordinates of an arbitrary point $X(x_1, x_2, x_3)$ on l can be written as $x_i = \lambda a_i + \mu b_i, \;\; i = 1, 2, 3$. Similarly on the second line l', $A'(a'_1, a'_2, a'_3)$ and $B'(b'_1, b'_2, b'_3)$ with λ' and μ' also with an arbitrary point $X'(x'_1, x'_2, x'_3)$ which has the coordinates $x'_i = \lambda' a'_i + \mu' b'_i, \;\; i = 1, 2, 3$. Let $V(v_1, v_2, v_3)$ be the center of perspectivity between l and l' . Now if $X(x_1, x_2, x_3)$ and $X'(x'_1, x'_2, x'_3)$ are collinear with $V(v_1, v_2, v_3)$ then .

$$\begin{vmatrix} v_1 & v_2 & v_3 \\ x_1 & x_2 & x_3 \\ x_1^{'} & x_2^{'} & x_3^{'} \end{vmatrix} = 0$$

or

$$\begin{vmatrix} v_1 & v_2 & v_3 \\ \lambda a_1 + \mu b_1 & \lambda a_2 + \mu b_2 & \lambda a_3 + \mu b_3 \\ \lambda' a'_1 + \mu' b'_1 & \lambda' a'_2 + \mu' b'_2 & \lambda' a'_3 + \mu' b'_3 \end{vmatrix} = 0$$

or

$$\lambda\lambda' \begin{vmatrix} v_1 & v_2 & v_3 \\ a_1 & a_2 & a_3 \\ a_1' & a_2' & a_3' \end{vmatrix} + \lambda\mu' \begin{vmatrix} v_1 & v_2 & v_3 \\ a_1 & a_2 & a_3 \\ b_1' & b_2' & b_3' \end{vmatrix} + \mu\lambda' \begin{vmatrix} v_1 & v_2 & v_3 \\ b_1 & b_2 & b_3 \\ a_1' & a_2' & a_3' \end{vmatrix} + \mu\mu' \begin{vmatrix} v_1 & v_2 & v_3 \\ b_1 & b_2 & b_3 \\ b_1' & b_2' & b_3' \end{vmatrix} = 0$$

or

$$\lambda\lambda'\alpha + \lambda\mu'\beta + \mu\lambda'\gamma + \mu\mu'\delta = 0$$

Thus, if there is a perspectivity between the line l and the line l' with parameters (λ, μ) and (λ', μ'), then there exits numbers $\alpha, \beta, \gamma, \delta$ such that $\lambda\lambda'\alpha + \lambda\mu'\beta + \mu\lambda'\gamma + \mu\mu'\delta = 0$, which is also said to be the equation of perspectivity.

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