

## HMT-Contourlet Image Segmentation Based On Majority Vote

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**Abstract:** Contourlet transform is a new multiscale and multidirectional image representation which effectively captures the edges and contours of images. Hidden Markov Tree (HMT) model can capture all inter-scale, inter-direction and inter-location dependencies. Also, as HMT can capture the statistical properties of the contourlet coefficients it maybe used to detect the image singularities (edges and ridges). In this paper, we have proposed texture segmentation methods, based on the HMT contourlet model. At first, contourlet coefficient is computed and then, for each texture an HMT Contourlet model is trained. For the test phase, a set of decisions are made for each block of input image based on likelihood criterion. Final decision will be based on the majority vote criterion. The proposed method has been examined on the test images and promising results in terms of low segmentation errors has been obtained.

**Key words:** Contourlet transform; HMT model; texture segmentation.

### INTRODUCTION

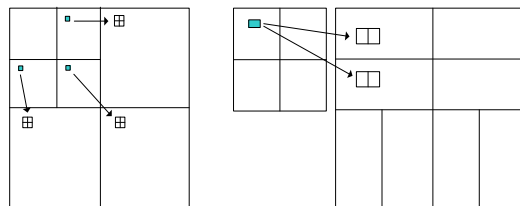
In the last decades capturing both global and local statistical properties of texture images has been of interest in statistical texture segmentation. Also, much attention has been paid to texture segmentation because of its applications in document images, aerial photos, synthetic aperture radar (SAR), medical images and so on.

The HMT-wavelet (Crouse, *et al.*, 1998) captures the inter scale dependencies of the coefficients across the scales and includes the statistical information for images. There are many algorithms for texture segmentation based on the HMT in the wavelet domain (Choi and baraniuk, 2001; Romberg, *et al.*, 2001; Arivazhagan and Ganesan, 2003). The contourlet transform can represent natural images in varying directions in a multiple scale, while the HMT-contourlet model characterizes more anisotropic information than the HMT-wavelet model. Unlike the HMT-wavelet, the HMT-contourlet model has inter-direction dependencies. This model has a parent-children relationship in the same directional subbands in different scales (same as the HMT-wavelet model) and also, unlike the HMT-wavelet, as shown in FIG.1, each parent can have its children over two directional subbands.

In this paper we have proposed three methods for HMT segmentation, based on the contourlet transform and majority vote rule. In all methods, the block-based HMT contourlet model is implemented for each texture based on the training data. There are different decisions for each block for different methods. Each block is assigned to the texture that yield majority vote.

As a first proposed method, we compute the average of the children and descendants of each coefficient in the coarsest scale, and consider it as an observation. We then apply texture segmentation based on HMT contourlet model.

As a second method, we compute the mean of four children of each parent (quad-tree structure), and consider them as set of observations. The texture segmentation is then performed based on contourlet hidden markov tree model. There are different decisions for each block. We assign block to the texture which obtain majority vote.



**Fig.1:** Parent-children relationship for (a) Wavelets and (b) Contourlet transform.

In the third method, we implement contourlet HMT texture segmentation by considering the coefficients as set of observations. In this approach, we have different decisions for different block sizes. The block is assigned to the texture which have majority vote.

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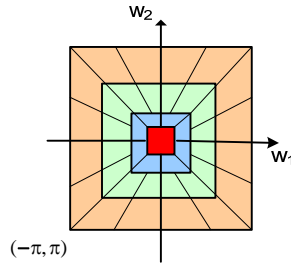
In this paper in section 2 we explain contourlet transform as a new two-dimensional extension of wavelet transform. Then in section 3 we develop a hidden Markov tree (HMT) model for contourlet transform which uses a two state mixture Gaussian model. Three methods are proposed for Texture segmentation in section 4, and finally, in sections 5 and 6 we demonstrate the experimental results and conclusion respectively.

### **Contourlet transform:**

The contourlet transform is an extension of wavelet transform in two dimensions, which has been introduced by Minh Do and Martin Vetterli (Duncan, *et al.*, 2006) the contourlet transform combines Laplacian Pyramid (LP) with a directional filter bank (DFB). The Laplacian pyramid is first used to capture the point discontinuities, and is then followed by a directional filter bank to link point discontinuities into linear structures.

The Laplacian pyramid (LP) is used to decompose an image into a number of radial subbands and the directional filter banks (DFB) decompose each LP detail subband into any power of two's number of directional subbands. FIG. 2 shows an example frequency partition of the contourlet transform where the three scales are divided into four, eight and eight directional subbands from coarse to fine scale, respectively. FIG. 3 shows an example of the contourlet transform on the Barbara image.

The contourlet coefficient can be represented in a quad-tree structure. Each coefficient in the coarsest scale has four children in the next higher subband and each of the children has four children in the next higher subband and a quad -tree will emerge.



**Fig. 2:** An example frequency partition by contourlet transform.



**Fig. 3:** (a) Barbara image. (b) Contourlet transform of Barbara image.

### **Contourlet HMT model:**

#### **3.1 Two State Model**

We want to model the joint pdf of contourlet coefficient by using HMT model (Rabiner, 1989). Each contourlet coefficient can be accurately modeled by a mixture of Gaussian distributions. Contourlet transform

consist of a small number of large coefficients and a large number of small coefficients. Most of the contourlet coefficients have small values that represent very little signal information.

A few contourlet coefficients have large values therefore, contain significant signal information. We model each contourlet coefficient as being in one of two states, high and low.

Each coefficient  $C_i$  is associated with a set of discrete hidden states  $S_i=0,1,\dots,M-1$ (for  $M$  state model) which have probability mass function (pmf)  $ps_i(m)$ .

Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$  are given by:

$$g(x; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\} \quad (1)$$

The parameter vector of a  $M$  state Gaussian Mixture Model is represented by:

$$\pi = \{p_{s_i}(m), \mu_m, \sigma_m^2 \mid m = 0, 1, \dots, M-1\} \quad (2)$$

And the general pdf of  $C$  is obtained by the sum:

$$f_c(c) = \sum_{m=0}^{M-1} p_{s_i}(m) f_{c|s_i}(c \mid s_i = m) \quad (3)$$

$$f_{c|s_i}(c \mid s_i = m) = g(c; \mu_m, \sigma_m^2) \quad (4)$$

A two state Gaussian Mixture Model are considered, thus each contourlet coefficient  $C_i$  is associated with a hidden state  $S_i$  taking value 0 and 1(low and high).

State high ( $S_i=1$ ) corresponds to a zero-mean, high variance Gaussian with:

$$f(c_i \mid s_i = 1) = g(c_i; \mu_i, \sigma_{L_i}^2) \quad (5)$$

State low ( $S_i=0$ ) corresponds to a zero-mean, low variance Gaussian with:

$$f(c_i \mid s_i = 0) = g(c_i; \mu_i, \sigma_{S_i}^2) \quad (6)$$

Where  $\sigma_L^2 > \sigma_S^2$ . The marginal pdf is yielded by:

$$f(c_i) = p_i^S g(c_i; \mu_i, \sigma_{S_i}^2) + p_i^L g(c_i; \mu_i, \sigma_{L_i}^2) \quad (7)$$

$$p_i^S + p_i^L = 1 \quad (8)$$

The state value Pmfs for  $S_i=\{0,1\}$  are represented by  $p_i^S$  and  $p_i^L$  respectively and defined as the probability that  $C_i$  is small or large.

We define the parameter  $\varepsilon_{j,j-1}^{m,n} = p(s_i = n \mid s_{p(i)} = m)$  as the probability that the coefficient is in a hidden state  $n$  when it's parent is in state  $m$ , where  $m,n=0,1$  and  $p(i)$  is the parent of node  $i$  and scale  $j=1,\dots,J-1$  ( $J$  is the coarsest scale) (Rabiner, 1989).

Each parent to child state-to-state link (transition probabilities) has a corresponding state transition matrix as follow:

$$A_i = \begin{bmatrix} p_i^{0 \rightarrow 0} & p_i^{0 \rightarrow 1} \\ p_i^{1 \rightarrow 0} & p_i^{1 \rightarrow 1} \end{bmatrix} \quad (9)$$

With  $p_i^{0 \rightarrow 1} = 1 - p_i^{0 \rightarrow 0}$  and  $p_i^{1 \rightarrow 0} = 1 - p_i^{1 \rightarrow 1}$ . The parameters  $p_i^{0 \rightarrow 0}(p_i^{1 \rightarrow 1})$  are defined as the probabilities that the contourlet coefficient is small(large) given that its parent is small(large). The parameters

$p_i^{0 \rightarrow 1}(p_i^{1 \rightarrow 0})$  are defined as the probabilities that the state values will change from one scale to the next. To propagate the large and small coefficient values down the quad-tree it is required that  $p_i^{0 \rightarrow 0} > p_i^{0 \rightarrow 1}$  and  $p_i^{1 \rightarrow 1} > p_i^{1 \rightarrow 0}$ .

### 3.2 HMT -contourlet parameter

An HMT model is defined by:

- 1) The Gaussian mixture variance  $\sigma_{i,m}^2$  and means  $\mu_{im}$  for each state.
- 2) The transition probabilities  $p(s_i | s_{p(i)}) = \varepsilon_{j,j-1}^{m,n}$ .
- 3) The pmf for the hidden state of the root node  $p_0(m)$  in the coarsest scale.

The parameters vector of HMT model is represented by  $\theta = \{p_{s_0}(m), \varepsilon_{j,j-1}^{m,n}, \mu_{i,m}, \sigma_{i,m}^2 | m, n = 0,1\}$ . In order to capture the contourlet characteristics from the image of interest, the HMT model is trained by using the Iterative Expectation Maximization (EM) algorithm.

#### Texture Segmentation:

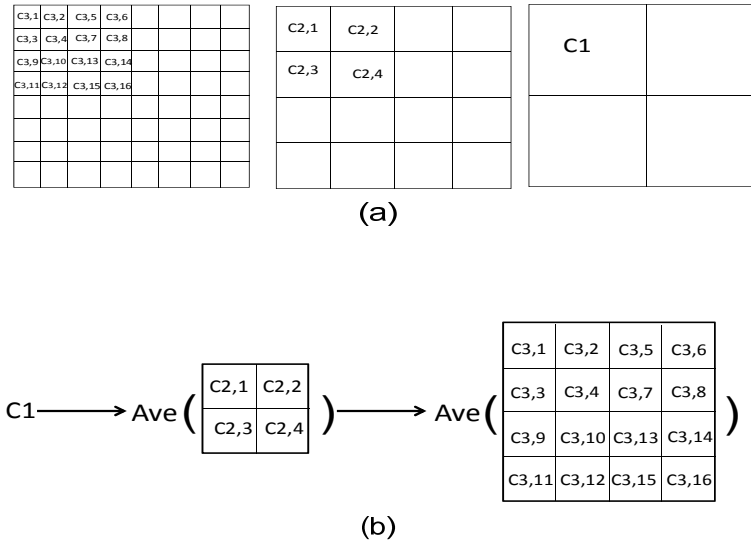
In this section we propose three methods for texture segmentation. Consider a texture image and divide it into blocks of size  $2M \times 2M$ , where  $M$  is the level of decomposition.

In the first method, contourlet coefficients have quad-tree structure we compute mean of children and descendants of each coefficient in the coarsest scale. For simplicity three level decomposition is considered. Each coefficient in coarsest scale has 4 children in next higher subband and 16 descendants in third scale, we compute their average and consider it as a coefficient in this scale (FIG.4). Then we compute  $p(S/C)$ .

$$p(S|C) = \frac{p(C|S) \times p(S)}{p(C)} \quad (10)$$

First, based on following relation, we obtain  $p(C|S) \times p(S)$ .

$$p(S) = p(S_{c_m}, S_{c_{m-1}}, \dots, S_{c_1}) \quad (11)$$



**Fig. 4:** (a) Contourlet coefficient. (b) HMT relationship of contourlet coefficients in the first method.

$$p(S_{c_m}, S_{c_{m-1}}, \dots, S_{c_1}) = p(S_{c_m} | S_{c_{m-1}}) \times p(S_{c_{m-1}} | S_{c_{m-2}}) \times \dots \times p(S_{c_1}) \quad (12)$$

Where,  $S_{c_m}$  is the state of finest scale and  $S_{c_1}$  is the state of the coarsest scale.

It must be considered, that in all methods we compute  $p(S_m/S_{m-1})$  from the transition matrix and  $p(S_i)$  from  $p_0(m)$  for all directions.

We consider two situations for each coefficient; the first one has low variance and the second, high variance situations.

$p(C|S)$  is computed by the use of Gaussian distribution (we assume contourlet coefficients have zero-mean). Then by computing  $p(C|S) \times p(S)$ ,  $2^m$  cases are obtained, the maximum value of this cases over  $s$  is obtained ( $\max_s(p(C|S) \times p(S))$ ) and related states are retained. This procedure is followed for each texture.

We obtain  $p(C, S|T_i)$  based on training model for each texture of interest.

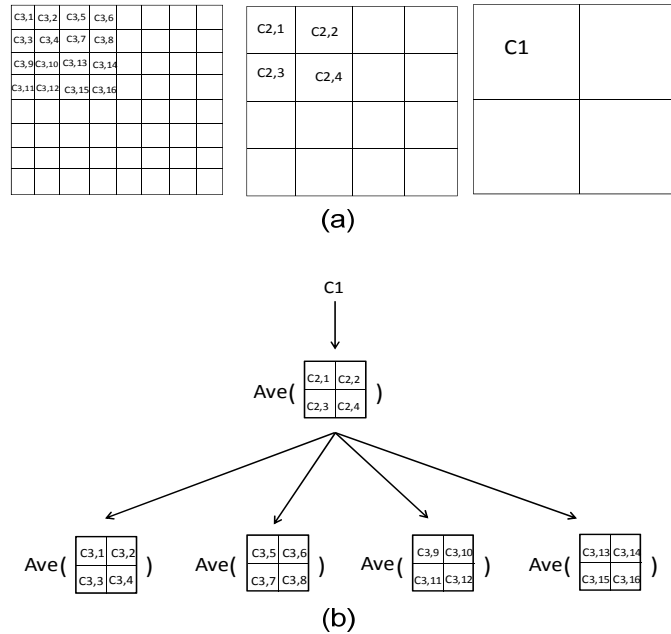
$$p(C, S|T_i) = p(C|S, T_i) \times p(S|T_i) \quad (13)$$

Then, the block is assigned to the texture for which we have

$$T_j = \arg(\max_{T_i} (p(C, S|T_i))) \quad (14)$$

In the second method, contourlet coefficients are represented in quad-tree structure we compute the mean of four children of each parent then consider this average as a coefficient. Next, we consider these coefficients as a set of observations that each one consist of  $m$  coefficients. For example, in three level decomposition, each block has one coefficient in the coarsest scale, one coefficient in the second scale and four coefficients in the third scale (FIG.5). Therefore, we have four observations that each of them consists of three coefficients

$$((C_1, C_2, C_{3,j})_{j=1,2,\dots,4}).$$



**Fig. 5:** (a) Contourlet coefficient. (b) HMT relationship of contourlet coefficients in the second method.

$$p(O|S) \times p(S) = p(o_1, o_2, \dots, o_m | s_1, s_2, \dots, s_m) \times p(S) \quad (15)$$

$$p(O|S) \times p(S) = p(o_1 | s_1) \times p(o_2 | s_2) \times \dots \times p(o_m | s_m) \times p(S) \quad (16)$$

Where,  $O_m$  is  $m$ 'th observation. We consider two situations for each observation; the first one has low variance and the second, high variance situations.  $p(o_m | s)$  is computed by use of Gaussian distribution (we assume contourlet coefficients have zero-mean). And  $p(S)$  is computed same as last methods. Then by

computing  $p(C|S) \times p(S)$ ,  $2^m$  cases are obtained, the maximum value of this cases over  $s$  is obtained ( $\max_s(p(C|S) \times p(S))$ ) and related states are retained. This procedure is followed for each texture.

$p(O, S | T_i)$  is obtained for each texture  $T_i$  by the following equation:

$$p(O, S | T_i) = p(O | S, T_i) \times p(S | T_i) \quad (17)$$

And the texture  $T_j$  is assigned to the input texture using the following criterion:

$$T_j = \arg(\max_{T_i} (p(O, S | T_i))) \quad (18)$$

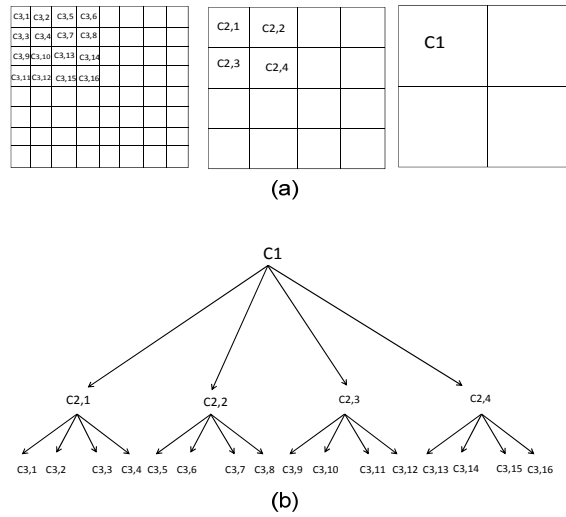
There are four decisions for three level decomposition in this method.

The block is assigned to the texture which has the majority vote.

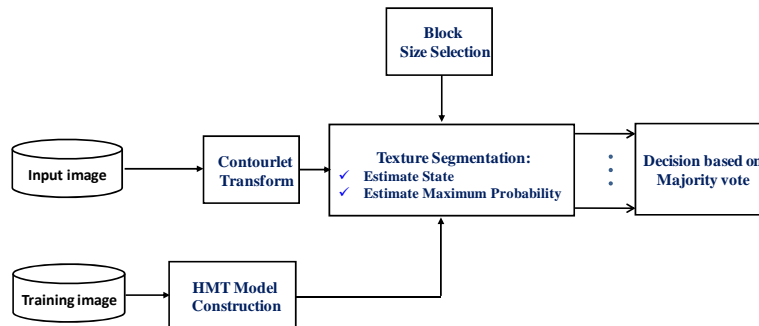
In the third method, coefficients are considered as a set of observations. In three level decomposition, each block has one coefficient in the coarsest scale, four coefficients in the second scale and 16 coefficients in the finest scale (FIG.6). Therefore, 16 sets of observations assigned to each block and each observation has three coefficients (( $C_1, C_{2,i}, C_{3,j}$ ),  $i=1,2,\dots,4$ ,  $j=(4i-3, 4i-2, 4i-1, 4i)$ ).

$$p(O|S) \times p(S) = p(o_1, o_2 \dots o_m | s_1, s_2, \dots s_m) \times p(S) \quad (19)$$

$$p(O|S) \times p(S) = p(o_1 | s_1) \times p(o_2 | s_2) \times \dots \times p(o_m | s_m) \times p(S) \quad (20)$$



**Fig. 6:** (a) Contourlet coefficient. (b) HMT relationship of contourlet coefficient in the third method.



**Fig. 7:**Block diagram of proposed methods.

Where  $P(o_m/s)$  and  $p(S)$  are computed same as last methods,  $2^m$  cases are obtained, the maximum value of this cases over  $s$  is obtained ( $\max_s(p(C|S) \times p(S))$ ) and related states are retained. This procedure is followed for each texture.

$p(O, S | T_i)$  is obtained for each texture  $T_i$  by the following equation:

$$p(O, S | T_i) = p(O | S, T_i) \times p(S | T_i) \quad (21)$$

And the texture  $T_j$  is assigned to the input texture using the following relation.

$$T_j = \arg(\max_{T_i} (p(O, S | T_i))) \quad (22)$$

There are 16 decisions for three level decomposition in this method. The block is assigned to the texture which has the majority vote. The general block diagram of proposed method is illustrated in FIG.7.

## RESULTS AND DISCUSSION

In order to implement all methods, Haar filter is used for both directional and multiscale decompositions in contourlet transform. Images are selected from *USC SIPI* database (*USC-SIPI Image Database*). We have selected two images to montage a test image. One forth of each selected image is used for training and one half is used for the test image. In this paper, we use blocks of size  $4 \times 4$  and  $8 \times 8$ , since we have applied two level and three level contourlet decomposition respectively. The first method only uses averaging scheme to produce observations. The second one uses averaging and majority vote decision making and finally the third one applies majority vote on individual decisions. The three methods have been discussed in previous subsections.

For blocks of size  $4 \times 4$ , the first and the second methods have been yielded one decision and the last method, four decisions. For blocks of size  $8 \times 8$ , the first method includes one, the second method achieves four and the last method 16 decisions. Final decision will be made, based on the majority vote criterion.

FIG. 8 is illustrates the montage image and its ground truth. Results of the first method for block size  $4 \times 4$  and  $8 \times 8$  are shown in FIG. 9. FIG. 10 demonstrates the results of the second method for block of sizes  $4 \times 4$  and  $8 \times 8$  respectively. Results of the last method are illustrated in FIG .11 for different block sizes. The percentage error in HMT contourlet segmentation, in each method, based on different block sizes, has been computed and reported in Table 1 and confusion matrix for different classes is demonstrated in Table 2.

As reported in Table 1, Table 2 and figures, while the size of blocks is increased the accuracy of classification is improved. The main reason is that more information is exploited in decision making process for larger block sizes. Also with the increase of block size, the textures can be represented more completely. Proposed method (3<sup>rd</sup> approach) is compared with HMM-contourlet HMT model, HMM-Real HMT and HMM-Complex HMT model in (Raghavendra and Subbanna Bhat, 2004) for blocks of size  $4 \times 4$  (Table3). These methods uses information of contiguous blocks and apply both HMT and HMM. Inter block statistics are modeled via HMM and intra-block statistics are modeled by using HMT. HMM-contourlet HMT utilizes contourlet transform, HMM-Real HMT make use of wavelet transform and HMM-Complex HMT exploits complex wavelet transform (Justin *et al.*, 2002).

Table3 shows our method (3<sup>rd</sup> approach) offers better results compared with other methods. The neighboring pixels information in the HMT contourlet approach maybe sufficient for texture segmentation as HMM modeling dose not causes HMT contourlet segmentation approach to give better results.

We have also examined the effect of different base filters on our proposed approach. Table 4 shows texture segmentation error rate by using different filters. The filters brought in this table are the most important ones used for image modeling with contourlet transform (Duncan, *et al.*, 2006). The approach has been based on our 3<sup>rd</sup> one. Compared with 9-7 and PKVA filters (Cohen and Daubechies, 1993; Phoong, *et al.*, 1995), Haar filters lead to better performance in texture segmentation applications.

**Table 1:** Percentage error in contourlet Hmt segmentation.

HMT Texture Segmentation methods		First method	Second method	Third method
Block size	4×4	4.5898	4.7485	1.9775
	8×8	2.4414	1.1719	0.90332

**Table 2:** Confusion Matrix for Different Classes.

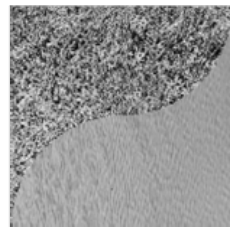
First proposed method			Second proposed method		Third proposed method	
<i>Block 4×4</i>	<i>Class 1</i>	<i>Class 2</i>	<i>Class 1</i>	<i>Class 2</i>	<i>Class 1</i>	<i>Class 2</i>
Class 1	7710	595	7672	633	8117	188
Class 2	157	7922	145	7934	138	7941
<i>Block 8×8</i>	<i>Class 1</i>	<i>Class 2</i>	<i>Class 1</i>	<i>Class 2</i>	<i>Class 1</i>	<i>Class 2</i>
Class 1	1725	381	16	2090	2085	21
Class 2	19	1971	1958	32	16	1974

**Table 3:** Average Percentage Error In Segmentation

Texture segmentation methods for block 4×4	Average Percentage Error(%)
HMM-Real HMT	3.20
HMM-complex HMT	3.59
HMM-contourlet HMT	4.16
Proposed method(3 <sup>rd</sup> approach)	1.9775

**Table 4:** Texture Segmentation Percent Error Rate Using Different Filters

Block 4×4			
Filter	First method	Second method	Third method
9-7	5.7251	5.1086	4.5959
PKVA	8.075	8.3801	3.5706
Haar	4.5898	4.7485	1.9775
Block 8×8			
Filter	First method	Second method	Third method
9-7	2.7722	3.4424	1.0806
PKVA	2.4719	3.0029	2.3926
Haar	2.4414	1.1719	0.9033



(a)



(b)

**Fig. 8:** (a) Montage image. (b) Ground truth.

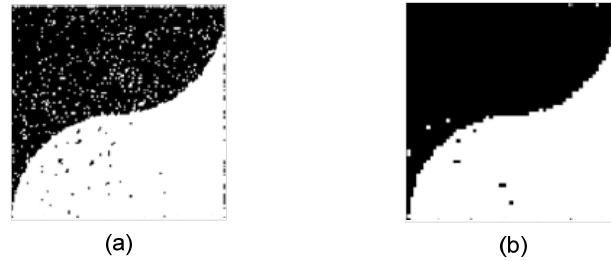
(a)



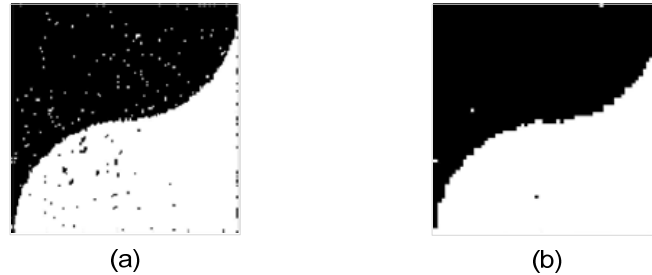
(b)

**Fig. 9:** Results of the first method.(a) 4×4 block (b) 8×8 block.





**Fig. 10:** Result of the second method. (a) 4×4 block (b) 8×8 block.



**Fig. 11:** Result of the third method. (a) 4×4 block (b) 8×8 block.

### **Conclusion:**

In this paper, we suggested three methods for image segmentation based on HMT contourlet model. The contourlet transform is applied on the image and contourlet coefficients are obtained. Then each texture is trained with HMT contourlet model. All methods have been employed the majority vote criterion. Different block sizes and/or methods provide different sets of decisions. Final decision will be made based on the majority vote criterion. The proposed methods have been examined on test images and promising results in terms of low segmentation errors has been obtained.

Future work may be conducted to provide some theoretical frame work for segmentation of image which include a considerable number of textures by using HMT contourlet segmentation.

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- USC-SIPI Image Database, available at <http://sipi.usc.edu/services/database/Database.html>.