A Review of Relations for Physical Properties of Nanofluids.

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Abstract: So far a lot of researchers have attempted to find a suitable relation to predict physical properties of nanofluids. But most of traditional formulae which belong to big size particles were not useful for nanofluids and empirical correlations also have their working condition limitations. There are several factors such as particle shape, size and material, interaction between particles, temperature, velocity, type of fluid flow, physical properties of both particle and base fluid and which affect on determination of physical properties of nano fluids. In this paper has been reviewed and classified all existing formulae and correlations about physical properties of nanofluids including thermal conductivity, viscosity, density and specific heat.

Key words: Physical property; Thermal conductivity; Viscosity; Density; Specific heat; nanofluid.

INTRODUCTION

Thermal fluids provide a situation for changing the energy of system. Heat transfer efficiency of these fluids depends on their physical properties such as thermal conductivity, viscosity, density and specific heat (see figure 1). Duo to low thermal conductivity of traditional thermal fluid like water, ethylene glycol, and oils scientists try to find a new coolant with higher thermal capacity. Choi created a novel thermal fluid which is called nanofluid in 1995 (S.U.S. Choi, *et al*., 1998). Nanofluid is a suspension of solid particles in size of nanometer in base fluid to increase its thermal capacity. Before appearance of nanofluid were used micro or millimeter size particles. But their big size , made some problems like corrosion and rapid sedimentation. Nanoparticles duo to their ultra fine scale, have a greater effective surface and better stability in base fluid as a suspension. These advantages of nanofluids attracted researchers to focus and use them as a new generation of coolants. The first step in this way is knowledge about their physical properties. Most researches had tried to obtain a comprehensive formula or correlation for calculating these properties. But as we will bring in following, there are a lot of different relations which in each of them was considered some factors and ignore others.

Thermal conductivity is more important in nanofluid. In fact it's enhancement than traditional fluid, is the benefit aspect of nanofluids. Duo to thermal conductivity play a key role in laminar fluid flow than turbulent one, most researches focus on stationary state or very low velocity of nanofluids flow. In using suspensions we must pay special attention on particle motions.

For example importance level of viscosity in laminar natural convection flow is very lower than random motion of particle. However role of viscosity will be more important in forced convection (N. Putra, *et al*., 2003). Roles of density and specific heat of nanofluids because they have an important position in determination of non dimensional thermal numbers cab be ignored. In continuing, we classified all exist formulae and correlations about each of these thermal properties of nanofluid.

Thermal Conductivity:

In physical aspect, thermal conductivity, *k*, is the property of a substance which shows its ability to conduct heat. As we know thermal conductivity of metals are very higher than liquids. In nanofluid approach, scientist attempt to use this feature to enhance the thermal conductivity of traditional thermal fluids by suspending nano metallic particles and most researchers create new formulae, novel correlations or try to adopt last formulae for calculating thermal conductivity of nanofluids. In continuing, we review them. The first formula about thermal conductivity of suspension with spherical particle shape was given by Maxwell (J.C.Maxwell, *et al*., 1904) as:

$$\frac{k_{eff}}{k_f} = 1 + \left(\frac{3(\alpha-1)\varphi}{(\alpha+2)-(\alpha-1)\varphi} \right) \qquad (1)$$

The model of Maxwell and Garnett (J.C. Maxwell, 1904) also was presented in the same year and was called M-G model. We can obtain the
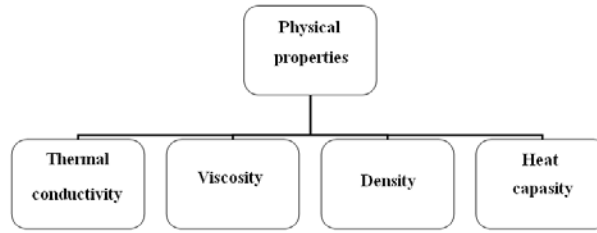M-G model for thermal conductivity of nanofluid by solving

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Fig. 1: Element of physical properties.

$\varphi\left(\dfrac{k_p - k_{eff}}{k_p + 2k_{eff}}\right) + (1-\varphi)\left(\dfrac{k_f - k_{eff}}{k_f + 2k_{eff}}\right) = 0$ equation as follow:

$$\frac{k_{eff}}{k_f} = \frac{(1-\varphi)(k_p + 2k_f) + 3\varphi k_p}{(1-\varphi)(k_p + 2k_f) + 3\varphi k_f} \tag{2}$$

Bruggeman (D.A.G. Bruggeman, *et al.,* 1935) gave a model for solid-liquid mixture as below:

$$k_{eff} = \frac{1}{4}\left[(3\varphi-1)k_p + (2-3\varphi)k_f\right] + \frac{k_f}{4}\sqrt{\Delta} \tag{3}$$

Where $\Delta = \left[(3\varphi-1)^2\left(\dfrac{k_p}{k_f}\right)^2 + (2-3\varphi)^2 + 2(2+9\varphi-9\varphi^2)\left(\dfrac{k_p}{k_f}\right)\right]$.

Hamilton and Crosser (R.L. Hamilton and O.K.Crosser, 1962) model was one of basic formula for two phase suspension which a lot of researcher's work was lied on this.

$$\frac{k_{eff}}{k_f} = \frac{k_p + (n-1)k_f - \varphi(n-1)(k_f - k_p)}{k_p + (n-1)k_f + \varphi(k_f - k_p)} \tag{4}$$

In this formula $n = \dfrac{3}{\psi}$ is called particle shape factor and ψ sphericity. Sphericity is defined as ratio of the surface area of a sphere with a volume equal to that of the particle to the surface area of the particle. From this model we can derive that sphericity has inverse relation to thermal conductivity. Consideration of ψ range is different among researchers. For example Xuan and Li (Y. Xuan and Q.Li, 2000) assumed that the range of ψ is between 0.3-1. However, Trisaksri and Wongwises (V. Trisaksri and S.Wongwises, 2007) mentioned range of $\psi = 0.5-1$, but most of them get the best answer in $\psi = 0.7$ (Xuan and Roetzel, 2000; Trisaksri and Wongwises, 2007) Beside that some researchers (Murshed, *et al*., 2005; A. Akbarinia and A.Behzadmehr, 2007) recommended $\psi = 1$ is suitable for spherical particles and $\psi = 0.5$ for cylindrical one and Zhang *et al.,* (2007) suggest a correlation to calculate φ as $\varphi = \dfrac{\varphi_w \rho_f}{\rho_p + \varphi_w \rho_f - \varphi_w \rho_p}$. Here φ_w is mass fraction of dispersed particles in nanofluid. Tsai and Chein (2007) used the H-C correlation as k_{static} and moreover they believed that k_{motion} also is necessary. Jana *et al.* assumed this limitation for using the H-C formula. $\varphi <= 1$ and $|\alpha - 1| <= 1$.

Considered $k = \dfrac{k_{eff} - k_f}{k_{H-C} - k_f}$ (P. Keblinski, *et al.,* 2002; S. Zeinali Heris, *et al.,* 2006). They said that

for $k_p >= k_f$, we have $k_{H-C} = k_f\left(\dfrac{1+2\varphi}{1-2\varphi}\right)$. Jeffrey (1973) suggested next model.

$$\frac{k_{eff}}{k_f} = 1 + 3\beta\varphi + \left(3\beta^2 + \frac{3}{4}\beta^2 + \frac{9}{16}\beta^3\frac{\alpha+2}{2\alpha+3} + ...\right)\varphi^2 \tag{5}$$

Where $\beta = \dfrac{(\alpha-1)}{(\alpha+2)}$ and high-order term represents pair interaction of randomly dispersed spheres. Jana

et al., assume this condition when used the model: $\varphi <= 1$ and $|\alpha-1| <= 1$. (Tinga also 2003) gave another correlation He assumed that there is an interfacial layer around each particle (K.C. Leong, *et al.*, 2006). His suggested model is:

$$\frac{k_{eff}}{kf} = 1 + \left(\frac{3\varphi_p\left[(\beta^3-1)(2k_{lr}+k_p)(k_{lr}-k_f)-(k_{lr}-k_p)(2k_{lr}+k_f)\right]}{(2k_f+k_{lr})(2k_{lr}+k_p)-\dfrac{2}{\beta^3-1}(k_{lr}-k_f)(k_{lr}-k_p)-3\varphi_p k_{lr}(k_p-k_f)}\right) \tag{6}$$

In this correlation β is equal to $1 + \dfrac{h}{r}$. Wasp *et al.*, 1997) simplified H-C correlation for spherical particles by assuming $\psi = 1$.

$$\frac{k_{eff}}{k_f} = \frac{k_p + 2k_f - 2\varphi(k_f - k_p)}{k_p + 2k_f + \varphi(k_f - k_p)} \tag{7}$$

Where $\varphi = \dfrac{V_p}{V_p + V_f} = m\dfrac{\pi}{6}\overline{d}_p^{-3}$. Yamada and Ota, (1980) gave a formula to calculate thermal conductivity of nanoparticles as:

$$\frac{k_{eff}}{k_f} = \frac{\dfrac{k_p}{k_f} + n - n\varphi\left(1 - \dfrac{k_p}{k_f}\right)}{\dfrac{k_p}{k_f} + n + \varphi\left(1 - \dfrac{k_p}{k_f}\right)} \tag{8}$$

But they consider new correlation for cylindrical particle's shape factor as $n = 2\varphi^{0.2}\left(\dfrac{l_p}{d_p}\right)$.

Formula (1) did not satisfy Davis, *et al.*, (1986) and he added other parts. His suggestion was:

$$\frac{k_{eff}}{k_f} = \left(1 + \frac{3(\alpha-1)\varphi}{(\alpha+2)-(\alpha-1)\varphi}\right)\left(\varphi + f(\alpha)\varphi^2 + O(\varphi)^3\right) \tag{9}$$

High-order terms represented pair interaction of randomly dispersed spheres and $f(\alpha)$ also had measured in two states: $f(\alpha) = 2.5$ for $\alpha = 10$ and $f(\alpha) = 0.5$ for $\alpha = \infty$. Bonnecaze and Brady, (1990, 1991) carried out numerical simulation about near and far field interactions among two or more particles which are mentioned by some researchers (Q.Z. Xue, 2003; C. Kim, Mathematical, 2004; Q. Xue and W.M.Xu, 2005). All formulae have designed for stationary fluids yet, but random contact effect of nanoparticles in fluid. This effect can involve Brownian motion, and interaction between molecules and particles must not be ignored. Amiri and Vafai (A. Amiri and K.Vafai, 1994) represented their formula for mixture by adding effect of particle's movement in this way:

$$k_{eff} = k_{eff(stagnant)} + k_d \tag{10a}$$

Which H-C correlation was replaced in $k_{eff(stagnant)}$

$$\frac{k_{eff\,(stagnant)}}{k_f} = \frac{k_p + (n-1)k_f - \varphi(n-1)(k_f - k_p)}{k_p + (n-1)k_f + \varphi(k_f - k_p)} \tag{10b}$$

And they defined random contact effect by:

$$k_d = C(\rho C_p)_{nf} \left| \overline{V} \right| \varphi d_p \tag{10c}$$

Where C is unknown constant that can determine by matching empirical data,

$\left| \overline{V} \right| = \sqrt{u^2 + v^2}$ and $(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_p$. Tsai and Chein (T.H. Tsai and R.Chein, 2007) considered 0<C<0.4 and C=0 also means there is not any thermal dispersion effect. (Choi, 2003) invented the nano particles and after that, attempts of researchers are focused on it. Lu and Lin were the first group who gave a model for nanofluids (Q.Z. Xue, 2003). They believed that their model can satisfy all particle's shape involve spherical or non spherical. Their model is:

$$\frac{k_{eff}}{k_f} = 1 + a\varphi + b\varphi^2 \tag{11}$$

Jana *et al.*, (2007) obtained these numbers for spherical particles. For $\alpha=10$, $a=2.25$, $b=2.27$ and for $\alpha=\infty$, $a=3$, $b=4.51$. (Model of Xuan *et al.*, 2003) tried to amend (Wasp *et al.*, 1977) model. They believed that some effective factors such as temperature, viscosity, density, thermal capacity etc are very important and play key roles for calculating thermal conductivity of nanofluids. Hence they added them as below:

$$\frac{k_{eff}}{k_f} = \frac{k_p + 2k_f - 2\varphi(k_f - k_p)}{k_p + 2k_f + \varphi(k_f - k_p)} + \frac{\rho_p \varphi C_p}{2k_f} \sqrt{\frac{k_B T}{3\pi r_c \eta}} \tag{12}$$

Where r_c depends on the fractal dimension of the cluster structure. In the same year, a theory was explained which assumed that there is a very thin fluid layer around nanoparticles. Physical properties of this layer are between liquid and solid and they act as a connection bridge to transfer the thermal energy between liquid and solid particles. Base of this theory, (Yu and Choi, 2003) established a emendation for Maxwell model and called this new finding as Modified Maxwell equation:

$$\frac{k_{eff}}{k_f} = \frac{k_{pe} + 2k_f + 2(k_{pe} - k_f)(1+\beta)^3 \varphi}{k_{pe} + 2k_f - (k_{pe} - k_f)(1+\beta)^3 \varphi} \tag{13}$$

Where $k_{pe} = \dfrac{\left[2(1-\gamma) + (1+\beta)^3(1+2\gamma) \right]\gamma}{-(1-\gamma) + (1+\beta)^3(1+2\gamma)} k_p$ and $\beta = \dfrac{h}{r}$. They also assumed h<10nm. (Mansour *et al.*, 2007) considered β=0.1 in their empirical work. Wang *et al.* (2003) considered effective thermal conductivity of clusters as follow:

$$\frac{k_{eff}}{k_f} = \frac{(1-\varphi) + 3\varphi \int_0^\infty k_{cl}(r)n(r)/[k_{cl}(r) + 2k_f]dr}{(1-\varphi) + 3\varphi \int_0^\infty k_f(r)n(r)/[k_{cl}(r) + 2k_f]dr} \tag{14}$$

Where $k_{cl}(r)$ is effective thermal conductivity of clusters and n(r) is the radius distribution function. Based on average polarization theory, (Xue *et al.*, 2003) presented an equation which can observed the effect of interface between the solid particles and base fluids:

$$9\left(1-\frac{\varphi}{\lambda}\right)\frac{k_{eff} - k_f}{2k_{eff} + k_f} + \frac{\varphi}{\lambda}\left[\left(\frac{k_{eff} - k_{c,x}}{k_{eff} + B_{2,x}\left(k_{c,x} - k_{eff}\right)}\right) + 4\frac{k_{eff} - k_{c,y}}{2k_{eff} + (1-B_{2,x})\left(k_{c,y} - k_{eff}\right)}\right] = 0 \tag{15}$$

Where $\lambda = \dfrac{abc}{(a+t)(b+t)(c+t)}$. Model of (Putnam *et al.* 2003.) also was presented in same year. Their formula is:

$$\frac{k_{eff}}{k_f} = 1 + 3\varphi \frac{\gamma - 1}{\gamma + 2} \tag{16}$$

Here, if we consider δ_{eq} as equivalent matrix thickness, therefore $\gamma = \dfrac{r_p}{\delta_{eq}}$. If $\delta eq \rightarrow 0$, we have $\gamma \rightarrow \infty$ and hence $\dfrac{k_{eff}}{k_f} = 1 + 3\varphi$. Kumar *et al.* (2004) found that velocity of particles in suspension and radius of molecules of fluid are essential dependent components of thermal conductivity of nanofluid. Their correlation is:

$$\frac{k_{eff}}{k_f} = 1 + CU_p \frac{r_f \varphi}{k_f r_p (1-\varphi)} \tag{17}$$

Where r_f is radius of the base fluid's molecules. The average particle velocity which was derived from the Stokes–Einstein equation is $U_p = \dfrac{8k_B T}{\pi \eta r_p^2}$. There are also some experimental findings which were presented as statistical regression correlation. For example, (Nguyen *et al.*, 2004) gave one of famous regression correlation as follows:

$$k = \frac{k_{nf}}{k_{bf}} = 4.97\varphi^2 + 2.72\varphi + 1 \text{ for Al}_2\text{O}_3 \tag{18a}$$

And

$$k = \frac{k_{nf}}{k_{bf}} = 28.905\varphi^2 + 2.827\varphi + 1 \text{ for CuO.} \tag{18b}$$

Model of (Bhattacharya *et al.*, 2004) seemed to be very simple.

$$k_{eff} = k_p \varphi + (1-\varphi)k_f \tag{19}$$

But they used a new formula for calculation of nanoparticles thermal conductivity as below:

$k_p = \dfrac{1}{K_B T^2 V} \displaystyle\sum_{j=0}^{n} \langle Q(0).Q(j\Delta t)\rangle \Delta t$. Where *n* is number of time steps used in simulation and $\langle Q(0).Q(j\Delta t)\rangle$ is the time- autocorrelation function of Q(t). *Nu* number is very important factor in heat transfer. It link to thermal conductivity directly. Two non dimensional numbers, *Re* and *Pr*, effect on *Nu* number. However, nobody pays attention to *Pr* and *Re* yet. The first persons who employed *Pr* and *Re* numbers in their correlation were (Jang and Choi 2004).

$$k_{eff} = k_f (1-\varphi) + k_p \varphi + 3C \frac{d_f}{d_p} k_f \text{Re}_{dp}^2 \text{Pr} \varphi \tag{20}$$

Where C is the proportional constant. Trisaksri and Wongwises, (2007) defined $\mathrm{Re}_{dp} = \dfrac{\overline{C_{RM}}d_p}{\eta}$, which

$\overline{C_{RM}}$ is random motion velocity of nanoparticles. Trisaksri and Wongwises, (2007) and Wang and Mujumdar (2007) also divided this formula in three parts.

1. collision between base fluid molecules $k_1(1-\varphi)$
2. thermal diffusion in nanoparticles in fluids $k_p\varphi$ and
3. thermal interaction of dynamic or "dancing" nanoparticles with the base fluids molecules $C\dfrac{d_f}{d_p}k_f\,\mathrm{Re}_{dp}^2\,\mathrm{Pr}\,\varphi$. Unfortunately collision among nanoparticles due to Brownian motion assumed neglected

by authors. Beck *et al.*, 2007) use $\mathrm{Re}_{dp} = \dfrac{\rho K_B T}{3\pi\eta^2 L_f}$, which L_f is mean free path of a liquid molecule. Koo and

Kleinstreuer 2004. also gave their model. They used Maxwell formula as k_{static}:

$$k = k_{static} + k_{brownian} \tag{21a}$$

$$k(static)_{maxwell} = \frac{k_{static}}{k_f} = \left[1 + \frac{3\left(\dfrac{k_p}{k_f}-1\right)\varphi}{\left(\dfrac{k_p}{k_f}+2\right)-\left(\dfrac{k_p}{k_f}-1\right)\varphi} \right] \tag{21b}$$

$$k_{brownian} = 5\times10^4\,\beta\varphi\rho_p C_p\sqrt{\frac{K_B T}{\rho_p d}}\cdot[f\,(T,\varphi)] \tag{21c}$$

To calculate $f(T,\varphi)$, they suggested an empirical correlation:

$f(T,\varphi) = (-6.04\varphi + 0.4705)T + (1722.3\varphi - 134.63)$. And β represents the fraction of the liquid volume V_f which travels with a particle. It can be achieved by $\beta = 0.0011(100\varphi)^{-0.7272}$ for CuO in $\varphi > 1\%$ and $\beta = 0.0017(100\varphi)^{-0.0841}$ for Al$_2$O$_3$ in $\varphi > 1\%$. Chakraborty and Roy (2008) considered this limitations for using the above formula as: $f(T,\varphi)$: $1\% \leq \varphi \leq 4\%$ and $300K \leq T \leq 400K$. Likewise some others (Chakraborty and S.Roy, 2008; Wang, D.W.Tang, *et al.*, 2007) also put this condition: In $\varphi < 1\%$ we can calculate $\beta = 0.137(100\varphi)^{-0.8229}$ for CuO. Yu and Choi (2003) gave a novel model for all elliptical particles. However, we can use this correlation by considering same value for all radiuses for sphere particles:

$$\frac{k_{eff}}{k_f} = \left(1 + \frac{n\varphi_{eff}A}{1-\varphi_{eff}A}\right) \tag{22}$$

Where A is defined as $A = \dfrac{1}{3}\displaystyle\sum_{j=a,b,c}\dfrac{\left(k_{pj}-k_f\right)}{\left(k_{pj}+(n-1)k_f\right)}$. For spherical particles we have

$\dfrac{k_{pj}}{k_s} = \left(1 + \dfrac{3\left(k_p - k_s\right)}{k_p\left(\varphi_{eff}-1\right) - k_s\left(\varphi_{eff}-1-3r\right)}\right)$ where k_s is thermal conductivity of surrounding layer of particles

and $\varphi_{eff} = \dfrac{\varphi\sqrt{\left(a^2+t\right)\left(b^2+t\right)\left(c^2+t\right)}}{\sqrt{abc}}$. Here $a,b,$ and c are radiuses of elliptical particle and t is the layer's

thickness. In the following year we also have several models. Interfacial shell is brought in equation by Xue and Xu 2005. as an important part.

$$\left(1-\frac{\varphi}{v}\right)\frac{k_{eff}+k_f}{2k_{eff}+k_f}+\frac{\varphi}{v}\frac{(k_{eff}-k_{is})(2k_{is}+k_p)-v(k_p-k_{is})(2k_{is}+k_{eff})}{(2k_{eff}+k_{is})(2k_{is}+k_p)+2v(k_p-k_{is})(k_{is}-k_{eff})}=0 \qquad (23)$$

v is the volume ratio of spherical nanoparticles to complex nanoparticle and is obtained by this formula:

$v=\left(\dfrac{r}{r+t}\right)^3$. Consequent of their last work, Khaled and Vafai (2005). used their last formula in channel as follows:

$$k=\frac{k_p+2k_f-2\varphi(k_f-k_p)}{k_p+2k_f+2\varphi(k_f-k_p)}k_f+C^*(\rho C_p)_{nf}\,\varphi h U \qquad (24)$$

Which $U=u_m\dfrac{3}{2}\left(1-\left(\dfrac{y}{H}\right)^2\right)$ and $(\rho C_p)_{nf}=(1-\varphi)(\rho C_p)_f+\varphi(\rho C_p)_p$. C^* is constant that depending on

the diameter of the nanoparticles and its surface geometry. Prasher *et al*., (2005). Brought a new factor which was effect of Kapitza resistance and called it R_b. They gave BMM model as follow:

$$\frac{k_{eff}}{k_f}=\left(1+A\,\mathrm{Re}^M\,\mathrm{Pr}^{0.33}\,\varphi\right)\frac{(1+2\lambda)+2\varphi(1-\lambda)}{(1+2\lambda)-\varphi(1-\lambda)} \qquad (25)$$

Where $\lambda=2R_b\dfrac{k_m}{d}$ and $\mathrm{Re}=\dfrac{1}{\mu}\sqrt{\dfrac{18K_BT}{\pi\rho_p d_p}}$. Kapitza resistance may arise from the phonon scattering at the

interface. Range of R_b was determined as low value 0.77×10^{-8} (km²w⁻¹) < R_b <20×10^{-8} (km²w⁻¹) high value. If R_b <low, we have λ=1. k_m is matrix conductivity that introduced as $k_m=k_f\left[1+\dfrac{\mathrm{Re.Pr}}{4}\right]$. For simplifying the

correlation, they assumed A=40000 and M is between 2 to 2.5. A similar model was presented by Xie *et al*., (2005). which name of NLBMM:

$$\frac{k_{eff}}{k_f}=\left(1+A\,\mathrm{Re}^M\,\mathrm{Pr}^{0.33}\,\varphi\right)\frac{1+3\theta}{1-\theta} \qquad (26)$$

Function θ is introduced as: $\theta=\beta_{nl}\dfrac{\left[(1+\gamma)^3-\dfrac{\beta_p}{\beta_{nf}}\right]}{\left[(1+\gamma)^3+\dfrac{\beta_{nl}}{\beta_p}\right]}$ where β wholly related to thermal conductivity as

$\beta_{nl}=\dfrac{k_{nl}-k_f}{k_{nl}+2k_f}$, $\beta_p=\dfrac{k_p-k_{nl}}{k_p+2k_{nl}}$, and $\beta_{nf}=\dfrac{k_f-k_{nl}}{k_f+2k_{nl}}$. In θ function $\gamma=\dfrac{h}{r}$. Here also can assume A=

40,000 and M=2-2.5. Another model which has been given by Xie *et al*., (2005).

$$\frac{k_{eff}}{k_f}=\left(1+3\Theta\varphi_T+\frac{3\Theta^2\varphi_T^2}{1-\Theta\varphi_T}\right)+f\left(r_p,\delta,\varphi,k_f,k_p,k_1\right) \qquad (27)$$

Where $\varphi_T=\varphi(1+\gamma)^3$ and $\Theta=\dfrac{\beta_{nl}\left[(1+\gamma)^3-\dfrac{\beta_p}{\beta_{nf}}\right]}{(1+\gamma)^3+2\beta_{nl}\beta_p}$. In this formula β and γ are same to last model. The

second part in right hand of equation 26 is the higher order pair interactions which is negligible for low particle concentration in suspension. Chon *et al*., (2005). found an experimental formula for measuring thermal conductivity of Al_2O_3 nanofluids. Their formula was a function of nanoparticle size (nominal diameters rang of nanoparticles were 11 nm to 150 nm and temperature range was 21 to 71 °C):

$$\frac{k_{eff}}{k_f} = 1 + 64.7 \varphi^{0.764} \left(\frac{d_f}{d_p}\right)^{0.369} \left(\frac{k_p}{k_f}\right)^{0.7476} Pr^{0.9955} Re^{1.2321} \tag{28}$$

He also defined *Pr* and *Re* numbers as: $Pr = \frac{\mu}{\rho_f \xi_f}$ and $Re = \frac{\rho_f K_B T}{3\pi\mu^2 L_f}$ where L_f is the mean free path

of water molecules. Viscosity was introduced from $\mu = 2.414 \times 10^{-5} \times 10^{\left(\frac{247}{T-140}\right)}$. Next model belongs to Patel *et al.*, (2005). which was for spherical particle

$$\frac{k_{eff}}{k_f} = 1 + \frac{k_p A_p}{k_f A_f} + C k_p Pe \frac{A_p}{k_f A_f} \tag{29}$$

Where $\frac{A_p}{A_f} = \frac{d_f \varphi}{d_p(1-\varphi)}$ and $Pe = \frac{U_p d_p}{\alpha_p}$. Here U_p is achieved from $U_p = \frac{2K_B T}{\pi\eta d_p^2}$. Only (A.K. Santra, *et al.,* 2008) mentioned this finding. By considering the effect of interfacial layer at the solid particles/liquid interface, Leong *et al.*, (2006) gave their model.

$$k_{eff} = \frac{\left(k_p - k_{lr}\right)\varphi_1 k_{lr}\left(2\beta_1^3 - \beta^3 + 1\right) + \left(k_p + 2k_{lr}\right)\beta_1^3 \left[\varphi_1 \beta^3 \left(k_{lr} - k_f\right) + k_f\right]}{\beta_1^3 \left(k_p + 2k_{lr}\right) - \left(k_p - k_{lr}\right)\varphi_1 \left(\beta_1^3 + \beta^3 - 1\right)} \tag{30}$$

Where $\beta_1 = 1 + \frac{h}{d}$ and $\beta = 1 + \frac{h}{r}$. In the same year, Prasher *et al.*, 2006 removed their last model's defaults and presented new formula which called it MSBM. This new model can use only for water base nanofluids.

$$\frac{k_{eff}}{k_f} = \left(1 + A Re^M Pr^{0.333} \varphi\right) \left[\frac{\left[k_p(1+2\lambda) + 2k_m\right] + 2\varphi\left[k_p(1+2\lambda) - k_m\right]}{\left[k_p(1+2\lambda) + 2k_m\right] - \varphi\left[k_p(1-\lambda) - k_m\right]}\right] \tag{31}$$

Here *A* dependent on fluid type and is assumed 40000. *M* is dependent of fluid and is considered 2.5±0.15 and R_b=0.77×10⁻⁸. As we said before some researchers made empirical correlation in especial laboratory conditions. Regarding particle concentration, thermal conductivity of base fluid and temperature, Li and Peterson 2006 explained their formula.

$$\frac{k_{eff} - k_f}{k_f} = 0.764\varphi + 0.0187(T - 273.15) - 0.462 \text{ for water-Al}_2\text{O}_3 \text{ and} \tag{32a}$$

$$\frac{k_{eff} - k_f}{k_f} = 3.761\varphi + 0.0179(T - 273.15) - 0.307 \text{ for CuO.} \tag{32b}$$

Buongiorno (2006) also gave an empirical correlation like this:

$$k_{nf} = k_f(1 + 7.47\varphi) \tag{33}$$

Some researchers' Models like Palm *et al.*, (2006) depended only on temperature and restricted by volume particle concentration for water-γAl₂O₃ nanofluid.

$$k_{nf} = 0.003352T - 0.3708 \text{ for } \varphi = 1\% \tag{34a}$$

And

$$k_{nf} = 0.004961T - 0.8078 \text{ for } \varphi = 4\%.\qquad(34b)$$

Wang *et al.*, (2007) tried to amend model of Koo and Kleinstreuer (2004) and consider the effect of Brownian motion by $k_{brownian}$:

$$k = k_{static} + k_{brownian}\qquad(35a)$$

$$k_{static}(Maxwell) = \frac{k_{static}}{k_f} = \left[1 + \frac{3\left(\dfrac{k_p}{k_f}-1\right)\varphi}{\left(\dfrac{k_p}{k_f}+2\right)-\left(\dfrac{k_p}{k_f}-1\right)\varphi}\right]\qquad(35b)$$

$$k_{brownian} = 5\times10^4 \beta\varphi\rho_p C_p \sqrt{\frac{K_B T}{\rho_p d}} [f(T,\varphi)]\left(\frac{T}{273}\right)^{2.5}\qquad(35c)$$

In this formula this limitation was considered: $291K \le T \le 338K$. Nie *et al.*, 2008. also explained a suggestion for $k_{brownian}$. Their opinion was:

$$k_{brownian} = \frac{85}{96\pi^2} \frac{\varphi K_B^2 T}{r^4 \eta}\qquad(36)$$

Next proposed model belong to Louge and Chen (2008). They suggested a new formula by studying nanofluid as two phase mixture entirely and assuming separate features for each part as follow:

$$k_{eff} = k_f + k_p\qquad(37)$$

Where $k_p = \rho_p \varphi C_p \xi_p$ and $\dfrac{k_{eff}}{k_f} = 1 + 3\varphi\left(\dfrac{\alpha-1}{\alpha+2}\right) + Pr_p + O(\varphi^2)$. Here $Pr_p = \dfrac{d\sqrt{\Theta}}{9\sqrt{\pi}\dfrac{k_f}{\rho_p C_p}}$. By

considering $\dfrac{k_p}{k_f} >= 1$, will have $\dfrac{k_{eff}}{k_f} = 3\varphi$. Finally, last group that we bring their model in this field is Velagapudi *et al.*, 2008. They considered that thermal conductivity of nanofluid is function of several factors like particles diameter, concentration, viscosity, temperature, density, and thermal conductivity of both particles and base fluid. They created a relation between them as $\dfrac{k_{nf}}{k_f} = f\left[Re_m, \varphi, \dfrac{k_p}{k_f}\right]$. They separated each of these factors and empirically found suitable coefficient for them.

$$\frac{k_{nf}}{k_f} = c\, Re_m^p\, \varphi^q \left(\frac{k_p}{k_f}\right)^r\qquad(38)$$

By non linear regression they obtained 0.175, 0.05, and 0.2324 for *p*, *q*, and *r* respectively. *c* is the unknown coefficient which must determined for each nanofluid separately. For example they obtained the values 1, 1.32, 1.298, 1.72, 0.74, 0.82, 1.5, and 1.98, for Al_2O_3-water, Al_2O_3-ethylene glycol, CuO-water, CuO- ethylene glycol, Cu-water, Cu- ethylene glycol, TiO_2-water, and TiO_2- ethylene glycol nanofluids respectively.

Viscosity:

Viscosity is a measure of the resistance of a fluid against deformation by either extensional stress or shear stress. In general terms it is the resistance of a liquid to flow. For nanofluids duo to its suspension nature, viscosity is very important in application design. For viscosity of solid-liquid mixture also are presented a lot of models and correlations. The first person who gave his idea about viscosity of suspension as a function of viscosity of base fluid and particle volume concentration was Einstein which is brought in paper of Roetzel and Xuan 2000.

$$\mu_{nf} = \mu_{bf}\left(1 + 2.5\varphi\right) \tag{39}$$

His formula was applicable for large particles size like micrometer or millimeter (A.G.A. Nnanna, *et al.*, 2008). In this formula, linearly viscous fluids which include a dilute suspension of small rigid spherical particles with $\varphi < 0.05$ is considered and particle-particle interactions are negligible. Most empirical researchers believe that results of above equation is very lower than real one (W.Q. Lu and Q.M.Fan, 2008; Y. He, *et al.*, 2007) and is not suitable for nanofluids (A.K. Santra, *et al.*, 2008). After him Vand 1948. suggested his formula to calculate solid-liquid mixture as below:

$$\mu_{nf} = \mu_{bf}\left(\frac{\exp\left(2.5\varphi + 2.7\varphi^2\right)}{1 - 0.609\varphi}\right) \tag{40}$$

Model of Mooney 1951. also only consider particle concentration too.

$$\mu_{nf} = \mu_{bf}\exp\left(2.5\varphi\frac{\varphi_m}{1-\varphi}\right) \tag{41}$$

φ_m is maximum packing of particle volume concentration. After them Krieger and Dougherty 1959 presented their formula. They assumed the power of this correlation parametrically and amended it in their next related work.

$$\mu_{nf} = \mu_{bf}\left(\frac{\varphi_m}{1-\varphi}\right)^{\alpha} \tag{42}$$

Regarding to maximum packing concentration Frankel and Acrivos (1976) gave their suggested formula as:

$$\mu_{nf} = \mu_{bf}\left(1.125\left[\frac{\left(\frac{\varphi}{\varphi_m}\right)^{\frac{1}{3}}}{1-\left(\frac{\varphi}{\varphi_m}\right)^{\frac{1}{3}}}\right]\right) \tag{43}$$

As studied earlier Krieger (1972) amended his last model by determination a suitable power as α. His new formula is:

$$\mu_{nf} = \mu_{bf}\left(\frac{\varphi_m}{1-\varphi}\right)^{1.82} \tag{44}$$

Yaws (1977) established his experiment on water and CuO particles. His result shows the logarithmic relation between viscosity and particle volume concentration

$$\log\mu_{nf} = A\,e^{-BT} \tag{45}$$

Here with $R^2 = 0.9873$ we have $A = 165.56 - 29.643\varphi + 1.8375\varphi^2$ and with $R^2 = 0.9881$ we can obtain $B = 0.0186 - 0.001\varphi + 4 \times 10^{-6}\varphi^2$. R^2 is correlation factor and temperature and viscosity units are Kelvin and centipoises respectively. A and B are the functions of volume concentration. This correlation is not applicable for nanofluids in the subzero temperature range (P.K. Namburu, *et al.*, 2007). By bringing Inter particle spacing (h) in calculation Graham 1981 gave his suggested formula as follow:

$$\mu_{nf} = \mu_{bf}\left(1 + 2.5\varphi + 4.5\left[\frac{1}{\left(\frac{2z}{d_s}\right)\left(2 + \frac{2z}{d_s}\right)\left(1 + \frac{2z}{d_s}\right)^2}\right]\right) \tag{46}$$

ds represented the solid particle's diameter. It believed that collision effect of particles play the key role to measuring the viscosity of suspension. Base on this theory Gidaspow *et al.*, (1991) gave their model. As we can see they separated the viscosity in two parts:

$$\mu = \mu_{collision} + \mu_{kinetic} \tag{47a}$$

$$\mu_{collision} = 0.8\varphi_s\rho_s d_s\left(1 + e_s\right)\left(\frac{\Theta_s}{\pi}\right)^{0.5} \tag{47b}$$

$$\mu_{kinetic} = \frac{10}{96}\frac{\rho_s d_s\sqrt{\Theta_s\pi}}{\varphi_s\left(1 + e_s\right)}\left[1 + 0.8\varphi_s\left(1 + e_s\right)\right]^2 \tag{47c}$$

In this formula value of *e* is issued 0.9 (Elgafy and Lafdi, 2006) In next year Miller and Gidaspow (1992) suggested their experimental result as a regression model:

$$\mu = \mu_{bf}\left(537.42\varphi - 0.188\right) \tag{48}$$

Last formula before invention of nanofluid belong to Chow (1993). This model is a theoretical analysis in base of a liquid lattice model. Without assuming dipole-dipole type interaction between particles, we can consider A as 4.67 for magnetic fluids (2004).

$$\mu_{nf} = \mu_{bf}\exp\left(2.5\frac{\varphi}{1 - \varphi}\right) + \frac{A\varphi^2}{1 - A\varphi^2\varphi_m} \tag{49}$$

In first years after invention of nanofluids base of researcher's experimental work had been on particles volume concentration and base fluid viscosity yet. For example Pak and Cho (1998) who worked on TiO_2 nanoparticles find correlation as follows:

$$\mu_{nf} = \mu_{bf}\left(108.2\varphi^2 + 5.45\varphi + 1\right) \tag{50}$$

Regard to nano particle volume concentration, Liu (1999). also gave his suggested formula as:

$$\mu_{nf} = \mu_{bf}\left(\left(\frac{1 - \varphi}{\varphi_m}\right)^2 + \left(\frac{k_1 - 2}{\varphi_m}\right)\varphi + \left(\frac{k_2 - 6}{\varphi_m^2}\right)\varphi^2\right) \tag{51}$$

There were an empirical working in same year by Wang *et al.*, (1999) which lead to two statistical correlation for viscosity of nano fluid. They tested γAl_2O_3 nanoparticle in two base fluid included water and ethylene glycol. Their correlations are

$$\mu_{nf} = \mu_{bf}\,(123\varphi^2 + 7.3\varphi + 1) \tag{52a}$$

And

$$\mu_{nf} = \mu_{bf}\,(306\varphi^2 - 0.19\varphi + 1) \tag{52b}$$

for water and ethylene glycol base fluids respectively. Orozco and Castillo (2002) also gave an experimental correlation as

$$\mu_{nf} = \mu_{bf}\,(6.17\varphi^2 + 2.5\varphi + 1) \tag{53}$$

As we can see, nobody had considered temperature effect on viscosity yet. All of them only focused on particle concentration. While temperature play a very important role for determination of suspension viscosity especially in higher compression. In fact temperature changes are effecting on Brownian motion and interaction of nanoparticles in nanofluid and this effect may be cause increase or decrease of mixture viscosity. The first person who studied the temperature factor effect on viscosity of nanofluid was Putra *et al.*, (2003). Their research was on γAl$_2$O$_3$ and water suspension in two particle concentration 1% and 4%. The report involves two empirical correlations for those particle percentages.

$$\mu_{nf} = 2.9\times10^{-7}\times T^2 - 2\times10^{-4}\times T + 3.4\times10^{-2} \text{ for 1vol\%} \tag{54a}$$

$$\mu_{nf} = 3.4\times10^{-7}\times T^2 - 2.3\times10^{-4}\times T + 3.9\times10^{-2} \text{ for 4vol\%} \tag{54b}$$

Model of Tang *et al.*, 2004 also related to temperature dependence of viscosity. Their logarithmic formula is

$$\mu_{nf} = 2.761\times10^{-6}\exp\left(\frac{1713}{T}\right) \tag{55}$$

He has another logarithmic correlation regarding the particle concentration dependence of viscosity as below:

$$\mu_{nf} = \mu_{bf}\,(0.904e^{0.1483\varphi}) \text{ for water-alumina in 47nm particle size and} \tag{56a}$$

$$\mu_{nf} = \mu_{bf}\,(0.015\varphi^2 + 0.025\varphi + 1) \text{ for water-alumina in 36nm particle size} \tag{56b}$$

They also suggested a novel formula to calculate μ_{bf} for above correlations. This formula is: $\mu_{bf} = 10^{-4}e^{\frac{1.12646 - 0.039638T}{1 - 0.00729769T}}$. Maiga *et al.*, 2004 tried to show the importance of viscosity in turbulent flow for nanofluids.

$$\mu_t = \frac{\rho C_\mu K^2}{\varepsilon} \tag{57}$$

Where μ_t is turbulent viscosity. C_μ also can assume 0.09 (W. Daungthongsuk and S.Wongwises, 2007). However attractive aspect of nanofluid belongs to its thermal conductivity and this property in more effective in laminar flow to turbulent one. To develop the Einstein formula, Brinkman 2004 presented his model

$$\mu_{nf} = \mu_{bf}\,\frac{1}{(1-\varphi)^{2.5}} \tag{58}$$

But this model also was very far in answer than experimental results. Ding and Wen (2005). also extend Einstein formula.

$$\mu_{nf} = \mu_{bf} \left(1 + k_1 \varphi + k_2 \varphi^2 + k_3 \varphi^3 + ...\right) \tag{59}$$

In their correlation k_1, k_2 and k_3 are constant. For dilute suspensions, $k_1 = 2.5$ like the Einstein constant. Batchelor suggested 6.2 for k_2. In fact k1 is the virial coefficient and k_2 is Huggins coefficient (P.K. Namburu, *et al.*, 2007). Ding and Wen 2005 brought a model for viscosity of nanofluid which belongs to Eiler. Their models also contain dependence only on particle concentration and base fluid viscosity.

$$\mu_{nf} = \mu_{bf} \left(1 + 0.5 \mu_{in} \varphi \left(\frac{1-\varphi}{\varphi_m}\right)\right)^2 \tag{60}$$

This equation was tested for double-layer covered Fe_3O_4 spherical nanoparticles. There are two correlations related to viscosity in year (2006) they are belong to Batchelor (2006) and Williams (2006) Batchelor gave his formula regarding to effect due to the Brownian motion of particles for an isotropic suspension of rigid and spherical particles as below:

$$\mu_{nf} = \mu_{bf} \left(6.5 \varphi^2 + 2.5 \varphi + 1\right) \tag{61}$$

Williams's model is logarithmic formula which considers the particle volume concentration and base fluid viscosity.

$$\mu_{nf} = \mu_{bf} \exp\left(\frac{4.91\varphi}{(0.2092 - \varphi)}\right) \tag{62}$$

Results of Chen *et al.*, (2007) examinations on TiO_2–ethylene glycol and Cu-water nanofluids are as follows respectively

$$\mu_{bf} \left(10.6 \varphi^2 + 10.6 \varphi + 1\right) \text{ and} \tag{63a}$$

$$\mu_{bf} \left(468.72 \varphi^2 + 3.645 \varphi + 0.995\right) \tag{63b}$$

Nguyen *et al.*, 2008 worked on aqueous nanofluids. They used Alumina and copper oxide as nanoparticles. They made statistical correlations for two concentrations of alumina regard to temperature dependence. There is non linear relation between temperature and nanofluid viscosity in their formula for 4% alumina.

$$\mu_{bf} \left(1.125 - 0.0007T\right) \text{ for 1\% and} \tag{64a}$$

$$\mu_{bf} \left(2.1275 - 0.0215T + 0.0002T^2\right) \text{ for 4\%} \tag{64b}$$

His correlation for copper oxide nanofluid is related to third power of particle volume concentration as:

$$\mu_{bf} \left(0.009 \varphi^3 + 0.051 \varphi^2 - 0.319 \varphi + 1.475\right) \tag{65}$$

The last viscosity model which we are going to mention here belong to Namburu *et al.*, (2008). They used Alumina-ethylene glycol and water and SiO_2-ethylene glycol and water as working fluids in particle concentration of 1 to 10 and 2 to 10 volume percents respectively. They generally recommended a logarithmic formula and mentioned two unknown parameters which must determine for each nanofluid separately. Their formula is

$$\log \mu_{nf} = A e^{-BT} \tag{66}$$

Here A and B are unknown and must be determined empirically. For example to calculating A and B for alumina nanofluid, they suggested with $R^2 = 0.9978$ we can use $A = 236.11 - 55.444\varphi + 6.7388\varphi^2 - 0.29956\varphi^3$ and with $R^2 = 0.9994$ also B can obtain from $B = (20341 - 1478.5\varphi + 140.03\varphi^2 - 6.4745\varphi^3)/10^6$ and also for SiO_2 nanofluid we can use $A = 167.17 - 2.245\varphi - 1.9289\varphi^2 0.1193\varphi^3$ with $R^2 = 0.981$ and $B = 0.0192 - 0.0004\varphi - 7 \times 10^{-6}\varphi^2$ with $R^2 = 0.99$ which R^2 is correlation factor.

Density:

The materials density is defined as their mass per unit volume. Density has very important role in heat transfer especially in natural convection. Base of natural convection is on buoyancy force and density gradient. Density has inverse effect on natural convection (G. Roy, *et al.*, 2004). However in spite of experimental results, Trisaksri and Wongwises (2007) believe that increasing the density cause increase the natural convection. All researchers used mixture rule to calculate density of nanofluids.

$$(\rho)_{nf} = (1 - \varphi)(\rho)_f + \varphi(\rho)_p \tag{67}$$

In using this formula if we assume a metallic nanoparticle with density n times more than water density, eventually, the density of mixture will be:

$$(\rho)_{nf} = (1 - \varphi)(\rho)_f + \varphi(\rho)_p = (1 - \varphi) + n\varphi = (n - 1)\varphi + 1$$

Increase the particle volume concentration lead to increase the nanofluid density. Riehl, carried out an experimental work by nickel-water nanofluid and observed an increasing the density near 3.2% and 4.7% for particle concentration rising from 3.5% to 5%. But the inverse effect of mass flow rate and temperature on suspension density. There is a hypothesis is called Boussinesq which is believed that density has a linearly relation with temperature (S. Mirmasoumi and A.Behzadmehr, 2008). In fact, duo to density of base fluid reduces by increasing temperature, whole mixture is affected by this decreasing and eventually was obtain lower density for nanofluid by temperature (D. Zhu, *et al.*, 2008). In following way, Yu *et al.*, (2006) gave an empirical correlation about effect of temperature on density as follows:

$$\rho_f = \rho_{f0}\left[1 - \beta(T_f - T_0)\right] \tag{68a}$$

$$\rho_s = \rho_{s0}\left[1 - \beta(T_s - T_0)\right] \tag{68b}$$

Where β is heat expansion coefficient. Yang and Liu 2006. limited using above correlation in the restricted β range as 1E-4/K < β < 1E-3/K. Density is effecting on particle inertia. Increase the density cause to increase the momentum and convection heat transfer (K. Khanafer, *et al.*, 2003). High density increases the pressure drop in microchannels (SW. Kang and SC.Tseng, 2006.) The particles like metallic one which has a large density different with fluid also are more susceptible to settle (K. Kwak and C.Kim, 2005).

The density gradient between particles and fluid temperature deference may cause fluid to mix (D.D.Doug Cobos, 1998). This swing of local density make a large hole next to the molecules of liquid and cause to move the molecules (R.L. Laurence, 2006). Inhomogeneous density of fluid leads to instability in fluid flow type and may cause change from laminar to turbulent when we use large scale particles (L.A. Pozhar and K.E.Gubbins, 1997).

Uniformity of density depend on several factors like particle material, time and position of particles (X.B. Mi and A.T.Chwang, 2003) for example lower density particles prefer to swim in higher altitudes within denser liquids (A.R.A. Khaled and K.Vafai, 2005). It is also valuable to note that irregular distribution of density may effect on other physical properties and can change the dynamic behavior of nanofluid (X.B. Mi and A.T.Chwang, 2003).

Specific Heat:

Specific heat capacity which known simply as specific heat, is the measure of the heat energy required to increase the temperature of a unit quantity of a substance by a certain temperature interval. It is another thermo

physical property which is important as well as other ones. Nobody can deny its effect on Prandtl number, heat capacity per unit mass and in general heat transfer. Specific heat of metals is less than liquids. Therefore due to lower specific heat of nanofluid we expect that see a reduction in specific heat of nanofluids comparing pure fluids (R. Chein and J.Chuang, 2007). Specific heat of nanofluid decreases by increasing particle volume concentration. For example the specific heat of SiO_2 nanofluid with 10vol% reduces about 12% relative to base fluid (P.K. Namburu, 2007). Banerjee and Ponnappan (2007) reported that in their experiment with exfoliated graphite nano-particle fibers which were suspended in poly-alpha-olefin oil as base fluid, observed up to 33% decrease in specific heat than base fluid by only 0.6% and 0.3% particle concentration. However Akbarinia (2008) believed that increasing particles volume concentration augments specific heat. To calculate heat capacity of mixture Longo (2007) suggest a formula which follows mixture rule.

$$(cp)_{nf} = \frac{(1-\varphi)(\rho.cp)_f + \varphi(\rho.cp)_p}{(1-\varphi)(\rho)_f + \varphi(\rho)_p} \qquad (69)$$

Specific heat also affected by temperature as direct relation (D.P. Kulkarni, 2007). It means that increasing the temperature lead to specific heat rising. Generally most researchers try to calculate density and specific heat simultaneously as:

$$(\rho.cp)_{nf} = (1-\varphi)(\rho.cp)_f + \varphi(\rho.cp)_p \qquad (70)$$

Nomenclature	
$B_{2,x}$	The depolarization factor along x-symmetrical axis which is derived from the average polarization theory
	Dispersive coefficient
C^*	Diameter
d	Coefficient of restitution for particle collisions
$\overline{d}$	average diameter
	Interfacial layer thickness
e	Half channel height
h	Thermal conductivity
H	Boltzman constant = 1.38×10^{-23} J/k
k	Modified thermal conductivity of particles
K_B	Effective dielectric constant
k_{pe}	Length
$k_{c,j}$	number of particles per unit volume
l	Particle shape factor
m	Particle radius
n	apparent radius of the clusters
r	System Temperature (k)
r_c	Velocity field in the channel
T	Brownian motion velocity
U	mean flow speed
U_B	interstitial velocity components
u_m	Volume
u,v	distance from center of pipe
V	Inter particle spacing
y	
z	
A	The ratio of thermal conductivities of particle to base fluid
ψ	Sphericity
φ	Particle volume fraction
η	Dynamic Viscosity of liquid
γ	Ratio of nanolayer thermal conductivity to particle thermal conductivity
υ	The ratio of volume of spherical nano particle to complex nano particle
Δt	Time step
ζ	Thermal diffusivity
μ	Viscosity
ε	Turbulent dissipation rate
Θ	Granular temperature
Subscripts	
eff	Effective
f	Base fluid
in	intrinsic
is	Interfacial shell
lr	Interfacial layer
nf	Nanofluid
nl	Nanolayer
p	Nanoparticles

Conclusions:

There are a lot of correlations and formulae to determined nanofluids physical properties. Because of inability of most formulae, researchers preferred to create correlations by empirical work. However, these correlations also are restricted by their experimental conditions. Several factors effect on physical properties of nanofluids. Among them, it seems that particle concentration, physical properties of both particle and base fluid, and temperature are more important. Of course all correlation depended to particle size and material. Effectiveness of viscosity, density and specific heat is more obvious in higher particle volume concentration. There is no considerable change for them in low compression of particles in nanofluids therefore researchers assumed the viscosity, density and specific heat are constant in this situation. It is valuable to mention here, temperature difference in fluid layers make gradient in fluid density and this created gradient cause the convection heat transfer.

Despite of some undesirable changes like rising viscosity or decreasing specific heat, we can consider nanofluids as good thermal fluids, even in equipment with high energy density. However, researches to obtain a comprehensive formula to determine physical properties of nanofluid must be continue.

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