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Application of Exp-Function Method to Some Nonlinear Equations

Shahrooz Abbasi nezhad, Bezhan shabani, Majid Soleimani

Department of Mechanical Engineering, Langroud branch, Islamic Azad University, Langroud, Iran.

Abstract: In this paper, the Exp-Function method is used to construct solitary wave solution for some nonlinear equations. The Khokhlov-Zabolotskaya, Newell-Whitehead and Buckmaster Equations are chosen to illustrate the effectiveness of this method. The method is straightforward and concise, and its applications are promising. It is shown that the Exp-function method, with the help of symbolic computation, provides a very effective and powerful new method for discrete nonlinear evolution equations in mathematical physics. In this paper, the results show that the Exp-Function method is a powerful mathematical tool for solving systems of nonlinear partial differential equations having wide applications in engineering.

Key words: Exp-Function method; Khokhlov-Zabolotskaya Equation; Newell-Whitehead Equation; Buckmaster Equation.

INTRODUCTION

Most scientific problems and phenomena in different fields of sciences and engineering occur nonlinearly. Except in a limited number of these problems are linear. This method has been effectively and accurately shown to solve a large class of nonlinear problems. In past several decades, many effective methods for obtaining the solutions of nonlinear evolution equations have been proposed; for example variational method (Ji-Huan He, 2006; Laila, 2008; Ji-Huan He and Xu-Hong, 2007), Iteration perturbation method (He, 2001; Liao, 2005; He, 2005; He, 2006) and Homotopy perturbation method (Ramos, 2008; He, 2008; Gangi *et al*., 2008; Ganji, 2006; Ganji and Rafie, 2006; Rafei *et al*., 2007; Ben-gong *et al*., 2008; Hosein *et al*., 2008) other methods (Ganji *et al*., 2008; Ranjbar and Hosein Nia, 2008).

All mentioned above methods have limitations in their applications. In this paper we suggest a novel method called Exp-Function method (Ben-gong *et al*., 2008; Sheng Zhang, 2008; Soliman, 2008; Ya-zhou *et al*., 2008; Turgut Öziş, 2008; Sheng Zhang, 2008; Ganji *et al*., 2008; Chaoqing Dai *et al*., 2008; Sheng Zhang, 2008; Sheng Zhang, 2008; Khan *et al*., 2008; Ji-Huan and Li-Na Zhang, 2008; El-Wakil *et al*., 2008; Zhang Mei, 2007; Sheng Zhang, 2007 Abdou, 2007; Ji-Huan and Abdou, 2007; JXu-Hong and Ji-Huan, 2008; Ji-Huan and Xu-Wong, 2006) to search for solitary of various nonlinear wave equation. The solution procedure of this method, by the help of maple, is of utter simplicity and this method can be easily extended to all kinds of nonlinear equations. In this paper, an application of Exp-Function method applied to solve the Khokhlov-Zabolotskaya, Newell-Whitehead and Buckmaster Equations. This method leads to both generalized solitonary solutions.

Basic Idea of Exp-Function Method (JXu-Hong and Ji-Huan, 2008; Ji-Huan and Xu-Wong, 2006):

We first consider nonlinear equation in form:

$$N(u,u_t,u_x,u_{xx},u_{tt},...)=0, \tag{1}$$

Introduction a complete variation defines as:

$$\eta = kx + \omega t , \qquad u=u(\eta), \tag{2}$$

And therefore, the Eq. (1) construct of ODE in form:

$$N(u,u',u'',u''',...)=0. \tag{3}$$

And then solution of $u(\eta)$ is in form:

Corresponding author: Shahrooz Abbasi nezhad, Department of Mechanical Engineering, Langroud branch, Islamic Azad University, Langroud, Iran.
E-mail: shahroozabasinejad@hotmial.com

$$u(\eta)=\frac{\sum_{n=-c}^{d}a_n\exp(n\eta)}{\sum_{m=-p}^{q}b_m\exp(m\eta)}=\frac{a_c\exp(c\eta)+...+a_{-d}\exp(-d\eta)}{a_p\exp(p\eta)+...+a_{-q}\exp(-q\eta)} \tag{4}$$

Where c, d, p and q are positive integers which are unknown to be further determined, a_n and b_n are unknown constants.

Application to Nonlinear Equations:

Khokhlov-Zabolotskaya Equation:

We consider the Khokhlov-Zabolotskaya Equation

$$u_{xt}-(uu_x)_x=u_{yy} \tag{5}$$

For simplicity, we consider one dimentional form of equation. By this assumption we have Eq. (5) in form:

$$u_{xt}-(u_x)^2-uu_{xx}=0 \tag{6}$$

Introduction a complete variation defines:

$$\eta = kx + \omega t, \qquad u=u(\eta), \tag{7}$$

We Have

$$\omega u''-k(u')^2-kuu''=0 \tag{8}$$

Where prime denotes the differential with respect to η.

The Exp-Function method is very simple and straight forward, it is based on the assumption of traveling wave solutions can be expressed in following form:

In order to determine values of c and p, we balance the linear term of the highest order of u'' with the highest order nonlinear term uu'' in Eq. (8), we have:

$$u''=\frac{c_1\exp[(3p+c)\eta]+...}{c_2\exp[4p\eta]+...}, \tag{9}$$

$$uu''=\frac{c_3\exp[(2p+2c)\eta]+...}{c_4\exp[4p\eta]+...} \tag{10}$$

Where c_i s are determined coefficients only for simplicity. Balancing highest order of Exp-Function in Eqs. (9) and (10), we have:

$$3p+c=2p+2c \tag{11}$$

which leads to the result:

$$p=c. \tag{12}$$

Similarly to determine values of *d* and *q*, we balance the linear term of lowest order in Eq.(8)

$$u'' = \frac{... + d_1 \exp[-(3q+d)\eta]}{... + d_2 \exp[-4q\eta]} \tag{13}$$

And

$$uu'' = \frac{... + d_3 \exp[-(2q+2d)\eta]}{... + d_4 \exp[-4q\eta]} \tag{14}$$

Where d_i s are determined coefficients only for simplicity. Balancing the lowest order of Exp-Function in Eqs. (13) and (14), we have:

$$-(3q+d) = -(2q+2d) \tag{15}$$

This leads to the result:

$$q = d\,. \tag{16}$$

Case 1: $p = c = 1, q = d = 1$

$$u(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1}\exp(-\eta)}{\exp(\eta) + b_0 + b_{-1}\exp(-\eta)} \tag{17}$$

We set $b_1 = 1$ for simplicity. Substituting Eq. (17) into Eq. (8), and using the Maple, equating to zero coefficients of all powers of Exp(nη) yield to a set of algebraic equations to solve $a_0, b_0, a_1, a_{-1}, b_{-1}, k$ and ω using Maple, we obtain coefficients:

$$\begin{array}{llll} a_0 = a_1 b_0, & k = k, & b_{-1} = b_{-1}, & a_1 = a_1, \\ b_0 = b_0 & \omega = \omega, & a_{-1} = a_1 b_{-1} & \end{array} \tag{18}$$

Where a_1, b_0, b_{-1}, k and ω are free parameters. We therefore obtain the following solution:

$$u(x,t) = \frac{a_1 b_{-1} e^{-kx-\omega t} + a_1 b_0 + a_1 e^{kx+\omega t}}{b_1 e^{-kx-\omega t} + b_0 + e^{kx+\omega t}} \tag{19}$$

Case 2: $p = c = 2, q = d = 2$

$$u(\eta) = \frac{a_2 \exp(2\eta) + a_1 \exp(\eta) + a_0 + a_{-1}\exp(-\eta) + a_{-2}\exp(-2\eta)}{b_2 \exp(-2\eta) + b_1 \exp(\eta) + b_0 + b_{-1}\exp(-\eta) + b_{-2}\exp(-2\eta)} \tag{20}$$

we set $b_1 = 1$ for simplicity, then the trial Function, Eq. (20) is simplified as follows:

$$u(\eta) = \frac{a_2 \exp(2\eta) + a_1 \exp(\eta) + a_0 + a_{-1}\exp(-\eta) + a_{-2}\exp(-2\eta)}{b_2 \exp(-2\eta) + \exp(\eta) + b_0 + b_{-1}\exp(-\eta) + b_{-2}\exp(-2\eta)} \tag{21}$$

As we explain above, we obtain:

$$a_1 = \frac{a_{-2}}{b_{-2}}, \qquad a_2 = -\frac{8}{7}\frac{a_{-2}}{b_{-2} b_0}, \qquad a_0 = \frac{a_{-2} b_0}{b_{-2}}, \qquad b_{-1} = 0, \tag{22}$$

$$b_2 = -\frac{8}{7b_0}, \quad a_{-1} = 0, \quad \omega = \omega, \quad a_{-2} = a_{-2}, \quad b_{-2} = b_{-2}, \; b_0 = b_0$$

Where a_{-2}, b_{-2}, k, ω and b_0 are free parameters. We therefore obtain the following solution:

$$u(x,t) = \frac{a_{-2}e^{-2kx-2\omega t} + \frac{a_{-2}b_0}{b_{-2}} + \frac{a_{-2}}{b_{-2}}e^{kx+\omega t} - \frac{8}{7}\frac{a_{-2}}{b_{-2}b_0}e^{2kx+2\omega t}}{b_{-2}e^{-2kx-2\omega t} + b_0 + b_1 e^{kx+\omega t} - \frac{8}{7b_0}e^{2kx+2\omega t}} \tag{23}$$

Newell-Whitehead Equation:

We consider the Newell-Whitehead Equation

$$u_t = u_{xx} + u - u^3. \tag{24}$$

Introducing a complete variation η defined as: $\eta = kx + \omega t$ and $u = u(\eta)$, we have:

$$\omega u' - k^2 u'' - u + u^3 = 0, \tag{25}$$

Where prime denotes the differential with respect to η. we suppose that the solution of Eq. (25), can be expressed as:

$$u(\eta) = \frac{a_c \exp[c\eta] + ... + a_{-d} \exp[-d\eta]}{a_p \exp[p\eta] + ... + a_{-q} \exp[-q\eta]} \tag{26}$$

By the same manipulation as illustrated in the previous section, we can determine values of c and p by using balancing u^3 and u'' in Eq. (25).

$$u'' = \frac{c_1 \exp[(3p + c)\eta] + ...}{c_2 \exp[4p\eta] + ...}, \tag{27}$$

$$u^3 = \frac{c_3 \exp[(3c + p)\eta] + ...}{c_4 \exp[4p\eta] + ...}, \tag{28}$$

Balancing the highest order of Exp-Function in Eqs. (27) and (28), we have:

$$3p + c = 3c + p, \tag{29}$$

Which leads to the result:

$$p = c. \tag{30}$$

By a similar derivation as illustrated in the previous section we obtain:

$$d = q. \tag{31}$$

Case 1: $p = c = 1, q = d = 1$

The trial Function Eq.(26) reduces:

$$U(\eta)=\frac{a_1\exp(\eta)+a_0+a_{-1}\exp(-\eta)}{\exp(\eta)+b_0+b_{-1}\exp(-\eta)}, \tag{32}$$

We set $b_1=1$ for simplicity. As we Explain above, we obtain:

$$\begin{aligned} &b_{-1}=b_{-1}, \quad a_0=a_0, \quad k=k, \quad b_0=-a_0, \\ &a_1=-1, \quad \omega=2-k^2, \quad a_{-1}=-b_{-1} \end{aligned} \tag{33}$$

Where a_0, b_{-1}, k are free parameters. We therefore obtain the following solution:

$$u(x,t)=\frac{-b_{-1}e^{-kx-(2-k^2)t}+a_0-e^{kx+(2-k^2)t}}{b_{-1}e^{-kx-(2-k^2)t}-a_0+e^{kx+(2-k^2)t}} \tag{34}$$

Case 2: $p=c=2, q=d=2$

$$u(\eta)=\frac{a_2\exp(2\eta)+a_1\exp(\eta)+a_0+a_{-1}\exp(-\eta)+a_{-2}\exp(-2\eta)}{\exp(2\eta)+b_1\exp(\eta)+b_0+b_{-1}\exp(-\eta)+b_{-2}\exp(-2\eta)} \tag{35}$$

There are some free parameters in Eq. (35), we set $b_1=1$, $a_{-2}=b_{-2}=b_2=0$ for simplicity, then the trial Function, Eq. (35) is rewritten in form:

$$u(\eta)=\frac{a_2\exp(2\eta)+a_1\exp(\eta)+a_0+a_{-1}\exp(-\eta)}{\exp(\eta)+b_0+b_{-1}\exp(-\eta)} \tag{36}$$

These set of solutions are obtained by the same way as we explained in the solution of case 1 of this equation:

$$\begin{aligned} &a_1=-1, \quad a_{-1}=0, \quad a_2=0, \quad b_0=-a_0, \\ &\omega=1+k^2, \quad k=k, \quad a_0=a_0, \quad b_{-1}=0 \end{aligned} \tag{37}$$

Where a_0, k are free parameters. Substituting Eq. (37) in to Eq. (36) yields the following solution:

$$u(x,t)=\frac{a_0-e^{kx+(1+k^2)t}}{-a_0+e^{kx+(1+k^2)t}} \tag{38}$$

Buckmaster Equation:

We consider the Buckmaster Equation

$$u_t=(u^4)_{xx}+(u^3)_x \tag{39}$$

Making the transformation (2), Eq. (39) becomes:

$$4k(u)^3u'+u^3-\omega u=0, \tag{40}$$

Where prime denotes the differential with respect to η. We suppose that the solution of Eq. (40), can be expressed as:

$$u(\eta)=\frac{a_c\exp[c\eta]+...+a_{-d}\exp[-d\eta]}{a_p\exp[p\eta]+...+a_{-q}\exp[-q\eta]} \tag{41}$$

By the same manipulation as illustrated in the previous section, we can determine values of c and p by using balancing u^3u' and u^3 in Eq. (40).

$$u = \frac{c_1 \exp[c\eta] + \ldots}{c_2 \exp[p\eta] + \ldots} = \frac{c_1 \exp[(4p + c)\eta] + \ldots}{c_2 \exp[5p\eta] + \ldots} \tag{42}$$

$$u'u^3 = \frac{c_3 \exp[(p + 4c)\eta] + \ldots}{c_4 \exp[5p\eta] + \ldots}, \tag{43}$$

Balancing the highest order of Exp-Function in Eq.(42) and (43), we have:

$$4p + c = p + 4c, \tag{44}$$

Which leads to the result:

$$p = c, \tag{45}$$

By a similar derivation as illustrated in the previous section we obtain:

$$d = q. \tag{46}$$

Case 1: $p = c = 1, q = d = 1$
The trial Function Eq. (42) reduces:

$$u(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{\exp(\eta) + b_0 + b_{-1} \exp(-\eta)}, \tag{47}$$

Substituting Eq. (47) into Eq. (40), and using the maple, equating to zero coefficients of all powers of exp(nη) yields a set of algebraic equations for $a_0, b_0, a_1, a_{-1}, b_{-1}, k$ (see Appendix). solving this system with the aid of Maple, we obtain coefficients:

$$k = \frac{1}{2}\frac{b_0}{a_0}, \qquad a_1 = \frac{a_0}{b_0}, \qquad a_{-1} = 0, \qquad b_{-1} = 0,$$
$$b_0 = b_0, \qquad \omega = \frac{a_0^{\,2}}{b_0^{\,2}}, \qquad a_0 = a_0 \tag{48}$$

where a_0, b_0 are free parameters. We therfore obtain the following solution:

$$u(x,t) = \frac{a_0 + \dfrac{a_0}{b_0} e^{\frac{1}{2}\frac{b_0}{a_0}x + \frac{a_0^{\,2}}{b_0^{\,2}}t}}{b_0 + e^{\frac{1}{2}\frac{b_0}{a_0}x + \frac{a_0^{\,2}}{b_0^{\,2}}t}} \tag{49}$$

Conclusions:

In this Letter, Exp-Function method is used for finding solitary solutions of Khokhlov-Zabolotskaya and Newell-Whitehead and Buckmaster Equations. It can be concluded that the Exp-Function method is a new promising and powerful method for nonlinear evolution equations arising in mathematical physics. Its applications are worth further studying. It is worth pointing out that the Exp-Function method presents a rapid convergence for solutions. The Exp-Function method has got more advantages in comparison with other methods. Calculations in Exp-Function method are simple and straightforward. The reliability of the method and the reduction in the size of computational domain give this method a wider applicability. The results show that the Exp-Function method is a powerful mathematical tool for solving nonlinear partial differential equation systems having wide applications in engineering.

Appendix

$a_1^3 - \omega a_1 = 0,$

$-4ka_1^3 a_0 + 4ka_1^4 b_0 - \omega a_0 - 4\omega a_1 b_0 + 3a_1^2 a_0 + 2a_1^3 b_0 = 0,$

$-\omega a_{-1} - 4\omega a_1 b_{-1} + 3a_{-1}a_1^2 + a_1^3 b_0^2 + 6a_1^2 a_0 b_0 - 8ka_{-1}a_1^3 + 2a_1^3 b_{-1} + 8ka_1^4 b_{-1}$
$-6\omega a_1 b_0^2 - 4\omega a_0 b_0 - 12ka_1^2 a_0^2 + 3a_1 a_0^2 + 12ka_1^3 a_0 b_0 = 0,$

$12ka_1^2 a_0^2 b_0 + 8ka_{-1}a_1^3 b_0 - 4\omega a_0 b_{-1} + 6a_{-1}a_1 a_0 - 4\omega a_{-1}b_0 - 12ka_1 a_0^3 + 2a_1^3 b_{-1}b_0 + 3a_1^2 a_0 b_0^2$
$-4\omega a_1 b_0^3 + 6a_1^2 a_0 b_{-1} + 6a_{-1}a_1^2 b_0 - 36ka_{-1}a_1^2 a_0 - 6\omega a_0 b_0^2 + 28ka_1^3 a_0 b_{-1} + a_0^3 + 6a_1 a_0^2 b_0$
$-12\omega a_1 b_{-1} b_0 = 0,$

$-6\omega a_{-1}b_0^2 + a_1^3 b_{-1} - 48ka_{-1}a_0^2 a_1 - 4ka_0^4 + 6a_1 a_0^2 b_{-1} - 4\omega a_{-1}b_{-1} - 12\omega a_0 b_{-1}b_0 + 3a_1 a_0^2 b_0^2$
$-6\omega a_1 b_{-1}^2 + 3a_{-1}a_1^2 b_0^2 + 6a_1^2 a_0 b_{-1}b_0 - 12\omega a_1 b_{-1}b_0^2 + 12ka_1^2 a_0 a_{-1}b_0 + 6a_{-1}a_1^2 b_{-1}$
$+12a_{-1}a_1 a_0 b_0 + 3a_{-1}a_0^2 + 2a_0^3 b_0 + 4ka_0^3 a_1 b_0 - 4\omega a_0 b_0^3 + 36ka_1^2 a_0^2 b_{-1} + 24ka_{-1}a_1^3 b_{-1} +$
$+3a_{-1}^2 a_1 - \omega a_1 b_0^4 - 24ka_{-1}^2 a_1^2 = 0,$

$-4\omega a_1 b_0^3 b_{-1} - 20ka_{-1}a_0^3 + 4\omega a_{-1}b_0^3 - 12\omega a_1 b_0 b_{-1}^2 - 6a^2{}_1 a_{-1}b_0 b_{-1}b_0 + 6a_{-1}a_0^2 b_0$
$-12\omega a_{-1}b_{-1}b_0 - 60ka_{-1}^2 a_0 a_1 - \omega a_0 b_0^4 - 6\omega a_0 b_{-1}^2 + 60ka_{-1}a_1^2 a_0 b_{-1} + 6a_{-1}^2 a_1 b_0$
$+2a_0^3 b_{-1} - 12\omega a_0 b_{-1}^2 + 6a_{-1}a_1 a_0 b_0^2 + 6a_1 a_0^2 b_{-1}b_0 + 20ka_1 a_0^3 b_{-1} + 3a_1^2 a_0 b_{-1}^2$
$+a_0^3 b_0^2 + +12a_{-1}a_1 a_0 b_{-1} + 3a_{-1}^2 a_0 = 0,$

$-4\omega a_1 b_{-1}^3 - 4\omega a_0 b_{-1}b_0^3 - 12ka_{-1}^2 a_0 a_1 b_0 + 3a_{-1}a_1^2 b_{-1}^2 + a_{-1}^3 - 24ka_{-1}^3 a_1 + 48ka_{-1}a_1 a_0^2 b_{-1}$
$-6\omega a_1 b_0^2 b_{-1}^2 - 36ka_{-1}^2 a_0^2 + 3a_1 a_0^2 b_{-1}^2 + 2a_0^2 b_{-1}b_0 + 6a_{-1}^2 b_0 a_0 + 24ka_1^2 a_{-1}^2 b_{-1} + 4ka_0^4 b_{-1}$
$+12a_{-1}a_1 a_0 b_{-1}b_0 - 12\omega a_{-1}b_{-1}b_0^2 + 6a_{-1}^2 a_1 b_{-1} + 3a_{-1}^2 a_1 b_0^2 + 3a_{-1}a_0^2 b_0^2 - \omega a_{-1}b_0^4 + 6a_{-1}a_0^2 b_{-1}$
$-12\omega a_0 b_0 b_{-1}^2 - 6\omega a_{-1}b_{-1}^2 - 4ka_0^3 a_{-1}b_0 = 0,$

$6a_{-1}^2 a_1 b_{-1}b_0 - 6\omega a_0 b_0^2 b_{-1}^2 + 2a_{-1}^3 b_0 + 6a_{-1}a_0^2 b_{-1}b_0 + a_0^3 b_{-1}^2 - 12\omega a_{-1}b_{-1}^2 b_0 - 28ka_0 a_{-1}^3$
$-4\omega a_{-1}b_{-1}b_0^3 + 6a_{-1}^2 a_0 b_{-1} - 4\omega a_1 b_0 b_{-1}^3 + 3a_0 a_{-1}^2 b_0^2 + 36ka_0 a_1 a_{-1}^2 b_{-1} + 12ka_{-1}a_0^3 b_{-1}$
$-12ka_{-1}^2 a_0^2 b_0 - 4\omega a_0 b_{-1}^3 + 6a_{-1}a_1 a_0 b_{-1}^2 - 8ka_{-1}^3 a_1 b_0 = 0,$

$a_{-1}^3 b_0^2 + 2a_{-1}^3 b_{-1} - 8ka_{-1}^4 - 4\omega a_0 b_0 b_{-1}^3 - 4\omega a_{-1}b_{-1}^3 + 3a_{-1}^2 a_1 b_{-1}^2 + 12ka_{-1}^2 a_0^2 b_{-1} - 12ka_{-1}^3 a_0 b_0$
$-6\omega a_{-1}b_{-1}^2 b_0^2 + 6a_{-1}^2 a_0 b_0 b_{-1} + 8ka_{-1}^3 a_1 b_{-1} - \omega a_1 b_{-1}^4 + 3a_{-1}a_0^2 b_{-1}^2 = 0,$

$4ka_{-1}^3 a_0 b_{-1} - 4\omega a_{-1}b_{-1}^3 b_0 + 2a_{-1}^3 b_{-1}b_0 - 4ka_{-1}^4 b_0 + 3a_{-1}^2 a_0 b_{-1}^2 - \omega a_0 b_{-1}^4 = 0,$

$$-\omega a_{-1}b_{-1}^4 + a_{-1}^3 b_{-1}^2 = 0,.$$

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