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Contact-Impact Problems Using a Quasi-Linear Finite Element Appraoch

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Abstract: Contact-impact problems are a class of problems that exist in almost all machinery, automotive, aerospace, and structural applications. The contact-impact phenomena is inherently nonlinear and numerous approaches were used to solve such problems. In this paper, we exploit the linear Finite Element formulation within an incremental approach to solve Contact-impact problems under random excitations. The random excitations were modeled using Monte-Carlo simulation. We studied the input-output relationship for some of these problems to see the effect of random excitation on structural behavior.

Key words: Contact, Impact, Finite Element.

INTRODUCTION

Contact-impact problems are common class of problems that we face in most of engineering applications and especially in mechanical, automotive, aerospace, and civil engineering. The problem does not lend itself easily to solution due to its strong nonlinearity

Although the interest in contact problems is quite old since Hertz, the evolution of practical solution methods for dynamic contact problems is relatively new. The interest in dynamic contact systems is started as an interest in impact problems. Unfortunately, most of the researchers at that time were interested only in the wave propagation consequences of the impact problem, not in the contact phenomena.

Solution methods are classified mainly into two broad categories: analytical methods and Computational methods.

Analytical Methods:

Many analytical investigations have been employed to solve the dynamic contact problems. Most of the analytical model is limited to specific geometry, boundary conditions and loading schemes.

Nayak (1972) in his research work tried to reach, using a semi-heuristic approach, a general differential equation expressing the contact vibrations of point contact systems, through the analysis of three cases: undamped free vibration, forced damped vibrations with a sinusoidal input, and vibration with broad band random input. He approved the accuracy of his analysis by comparison with some experimental data.

Parton (1980) showed the use of integral equation approach to solve the dynamic wedge problem. Fomin (1984) investigated the problem of determining contact stresses under a periodic system of stamps located on the boundary of a homogenous elastic half-plane. The problem was reduced to solving Fredholm integral equation of the first kind.

Zhigalco *et al.,* (1989) offers a general formulation of the dynamic contact problem of an elastic shell of arbitrary shape with specified kinematics of motion of points belonging to a fixed contact region.

Glushkov *et al.,* (1992) proposed a model for solving dynamic contact problems in which an infinite system of linear algebraic equations is obtained that , taking into account the asymptotic bahaviour of the unknowns, reduces to an asymptotically equivalent finite system. The model yields simple relations between the unknowns corresponding to the tangential and normal components and the dimensions of the system is thereby halved. The model is applied to the case of a circular punch adheres to an elastic layer.

Belokon (1992) proposed a method that differs from the previous studies and is based on the introduction of special functional spaces. He establishes the discrete nature of the spectrum of the contact problem and the completeness of the system of eigen- functions.

Computational Methods:

Basically, the computational models of contact problems could be classified to one of the following categories:

a) Iterative or incremental approach.
b) Mathematical programming approach.

Iterative and Incremental Approaches:

Hughes *et al.,* (1976) were of the earliests to deal with the dynamic contact problem. They developed an algorithm in which they proposed interface conditions for cases of frictionless and perfect - friction contact.

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They described the spatial discretisation aspects of this formulation. Also considered how such formulation could be extended to problems of contact bodies of different dimensions. They demonstrated their approach by a number of interesting examples.

Hallquist investigated some explicit finite element codes (1976a) which were first implemented in HONDO, then in DYNA2D and later extended to three dimensions and implemented in DYNA3D (Hallquist 1976b). Constraints are imposed into the global equations of motion by a transformation of the nodal displacement components along the interface.

Osmont (1982, 1985) studied long time response of structures which possibly come into contact by means of a time-step algorithm. He suggested an implicit time integration algorithm to prevent numerical instabilities. To model the contact between two nodes of the discretization which come into contact, he inserted a non-tensile spring between those nodes at the very moment they come close.

He considered first the continuous contact problem, then discretised contact problem where the choice of contact springs is explained above. At least he considered the discretised dynamic contact problem with the determination of the instant of contact and the instant of separation.

Hallquist (1985a) described the two and three dimensional contact algorithms used in the finite element programs developed at the Lawrence Livermore National Laboratory. They were interested in both static and dynamic impact problems and consequently, had pursued the development of two different algorithms. The first based on the hydrocode (Viscosity method) technology of the sixties is implemented in the two and three dimensional explicit finite element codes. The second, a symmetric penalty treatment, is used in his implicit codes and is optional in the explicit code. The second method is considered an optimization method.

Hallquist (1985b) have overcome the above the above difficulties in his new approach for handling sliding interfaces and impact in large deformation explicit finite element and finite difference codes that use four node constant stress quadrilateral zones in the spatial discretisation. The method is applicable to problems in solid mechanics and hydrodynamics.

Ascione *et al.,* (1985) solve the dynamic problem of elastic plate in unilateral contact with a winkler subgrade. The plate is modeled according to Mindlin's theory in which the effects of the shear stresses and the rotary inertia on the motion can be taken into account, for the elastic foundation, it is assumed as a continuous distribution of massless and tensionless elastic springs. Ascione et al expressed the plate dynamical equilibrium equations by means of virtual work principle. They gave a numerical treatment of the variational problem, discretising the region Ω by means of a family of isoparametric rectangular finite elements.

Ostachowicz and Szwedowics (1986) presented a method of analysis of longitudinal and transverse vibrations of beams, taking into account Coulomb friction forces at the nodes. The beams have been modeled by finite elements. An iterative procedure is then applied to solve this equation assuming in the first iterative step that there is no sliding in the faying surface; that is vector {p} has zero elements.

Hassan (1988) proposed an incremental computational model for the dynamic behaviour of elastic bodies in contact. The model is evolved from an elasto-static incremental algorithm proposed earlier by Mahmoud et al (1982) for elastic bodies in static contact. This model proved to be simple and has a clear physical justification. Ayari and Saouma (1991) developed a new finite element model for the contact -impact problem for both static and dynamic cases. In this geometrical nonlinear formulation, contact (from node to surface) is simulated through fictitious equivalent pressure along the boundaries. Contrary to most existing models, this formulation entails relatively few matrix decompositions and thus is computationally inexpensive. The model is first assessed through some classical contact problems, and is subsequently applied to the analysis of a cracked dam under seismic excitation.

Mathematical Programming Methods:

Talaslidis and Panagiotopoulos (1982) investigated the theoretical and numerical aspects of dynamic unilateral contact problems. They formulated the governing equations as an equivalent variational inequality expressing D'Alembert's principle in the inequality form. Two cases of unilateral support are considered: rigid foundation laying at a distance from the body and unilateral elastic support. The discretisation with respect to time and space leads to a static nonlinear programming problem which is solved by a linear analysis approach. The nonlinear minimization problem is treated by a method which combines the advantages or trial and error methods.

Mitsopoulou (1983) considered an elastic beam oscillating in proximity to a rigid profile or a Winkler ground, in the range of small deformation. Numerical method rests on a familiar finite element method and on an implicit, unconditionally stable time integration scheme is developed. The engineering motivation of this study was provided primarily by off-shore pipelines for transportation of oil or gas.

Kanto *et al.,* (1983) developed a finite element method to solve two dimensional and axisymmetric dynamic contact problems using th penalty function method. Their computer code is capable to solve problems with large deformation and finite rotations including the plasticity model of isotropic strain- hardening flow theory and the viscosity model of the proportional damping for the material. Due to the inherent irreversibility of the contact phenomenon, they treated the problem with an incremental technique in which the load is divided into some incremental steps and the contact equations are solved by the respective method in each incremental step. In order to verify the method, twp problems were analyzed. On is the normal impact of two bars with the same cross section, and the other is the dynamic buckling of an arch in contact dynamically with a rigid plate moving at a constant velocity.

Zhi- Hua (1988) reviewed and formulate general contact-impact problems. The formulations are based on the updated Lagrangian formulation for incremental nonlinear analysis with the principle of virtual work employed. To treat the contact-impact interface between the contact boundaries, a new interface algorithm based on the finite element method is proposed. This interface algorithm transforms the contact-impact interface between the hitting node and a defense node. The algorithm is implemented in the finite element code DYNA3D. The implementations are examines via numerical examples.

Taylor & Papadopoulos (1991) formulated the dynamic contact initial value problem and discretise it spatially using Lagrange multiplier formulation. The continuous problem is presented in the context of nonlinear kinematics and the standard Newmark integration method is used to integrate the resulting equations. They show that these integration methods are unsuccessful in modeling the kinematical constraints imposed on the contacting bodies during persistent contact and a modified version is developed to get better results. The numerical work carried out within the environment of the fully nonlinear general purpose Finite Element Analysis Program (FEAP).

Simulation Scheme:

The mathematical model can be found in Mahmoud (1990).

The simulation scheme is as follows:

1) Input the preliminary data, which describe the onset of the problem (e.g initial conditions, boundary conditions, system characteristics…. etc).
2) Setting the equations in its incremental form.
3) Solving the system of equations by one of the direct integral methods.
4) Checking the contact or separation condition by an appropriate criterion i.e. according to displacement or force.
5) Correcting the system of equations according to the new contact conditions.
6) Changing the forcing function value according to a chosen random number generator.
7) Going to step (3) through (6) again, until the chosen time domain of the problem satisfied.
8) Stop.

RESULTS AND DISCUSSION

A simple problem which consists of a uniform cantilever beam which comes into contact with a massless spring is shown in Fig. (1). the boundary constraints of the problem are the convention one at the fixed end and that due to contact at the spring beam interface.

The beam will be analyzed for transverse vibrations, due to an input random excitation force applied at the tip point, whose force-time history is of Gaussian distribution.

In analyzing transverse vibration of a beam we need three degrees of freedom per node to simulate the real situation; also a consistent mass matrix is used because the lumped mass approach will lead to considerable errors (Smith & Griffiths 1988). Therefore, the beam is modeled by a uniform mesh of 10 two-node cubic elements.

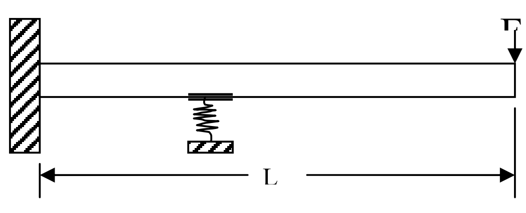

Fig. 1: A cantilever with massless spring.

The deflection of the tip point of the beam with time is shown in fig. 2, also the velocity of the same point with time is shown in fig. (3).

To illustrate the nonlinear nature of the problem, we calculated the correlation ration between the input force and the output velocity and it is found to be 0.04. This value indicates two main conclusions: firstly, the positive sign indicates that the output increases as the input increases. Secondly, the smallness of the ratio value indicates that the relation between the input and the output is *linearly independent.* This letter indication yields that the variables have some *nonlinear* relation between them.

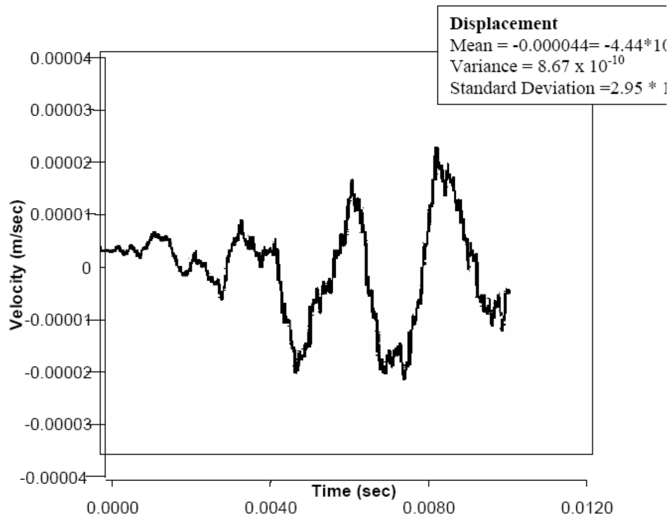

Fig. 2: Tip displacement of the cantilever.

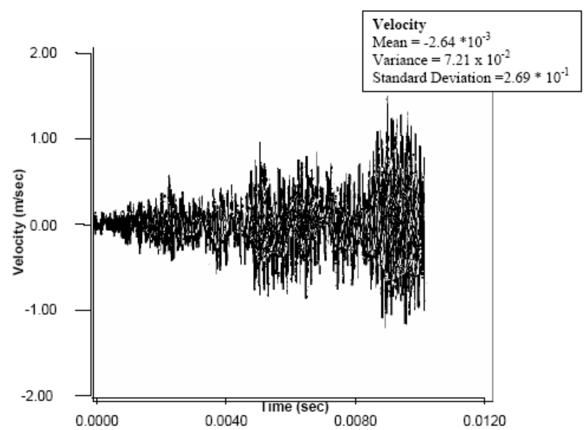

Fig. 3: Velocity of the tip of a contilever subjected to random excitatoin.

Conclusions:

In this paper, we applied a quasi-linear finite element approach to solve the problem of contact-impact under random vibration. The method was applied to a typical problem of a cantilever beam on a spring with a gap between them. The results were shown, and an input-output analysis was carried out to prove the nonlinear nature of such problems.

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