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Modified Goodness of Fit Tests for Exponentiated Pareto Distribution under Selective Ranked Set Sampling

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Abstract: This article deals with modified empirical distribution function (EDF) goodness of fit tests for exponentiated Pareto(EP) distribution based on two sampling techniques which are simple random sampling (SRS) and extreme ranked set sampling (ERSS). The performance of several modified goodness of fit tests are considered, such as, Kolmgorov-Simrnov; D, Kuiper; V, Cramer-von Mises, W^2 Anderson-Darling A^2, Watson U^2 and L test statistics. Tables of critical values for the proposed tests statistics under ERSS are obtained through simulation. The power of the modified test statistics under ERSS and SRS is investigated for a number of alternative distributions. A simulation study is conducted to compare the power functions of these tests under ERSS relative to SRS. The results of the power studies showed that the modified tests under ERSS are more efficient than their corresponding in SRS. In addition, the Watson test statistic is the most powerful goodness-of-fit test among the competitors.

Key words: Anderson-Darling test statistic, Cramer-von Mises test statistic, Exponentiated Pareto, Kolmogorov-Smirnov statistic, Watson statistic, Critical values, Ranked sample, Power test.

INTRODUCTION

The exponentiated Pareto (EP) distribution was introduced by Gupta *et al*. (1998) as a lifetime model. The cumulative distribution function (CDF) is expressed as

$$F(y;\theta,\lambda)=[1-(1+y)^{-\lambda}]^{\theta};\quad y>0,\lambda>0,\ \theta>0, \tag{1.1}$$

where, θ and λ are two shape parameters. The corresponding probability density function (PDF) is

$$f(y;\theta,\lambda)=\theta\,\lambda[1-(1+y)^{-\lambda}]^{\theta-1}(1+y)^{-(\lambda+1)};\quad y>0,\lambda>0,\theta>0, \tag{1.2}$$

when $\theta=1$, the EP distribution reduced to standard Pareto distribution of the second kind.

Ranked set sampling (RSS) is a method of sampling that can be advantageous when quantification of all sampling units is costly but a small set of units can be easily ranked, according to the character under investigation, without actual quantification. This sampling procedure was first introduced by McIntyre (1952) in his effort to find an efficient method to estimate the yield of pastures. From his study, he found that RSS was more efficient and cost effective method than the commonly used simple random sampling (SRS) technique. RSS scheme can be described as follows:

1. Randomly select m sets, each of size m elements from the population of interest.
2. Detect from the i^{th} sample, using a visual inspection, the i^{th} order statistic and choose it for actual quantification, say $Y_i, i=1,...,m$
3. RSS is the set of the order statistics $Y_1,Y_2,\ldots,Y_m$.
4. Repeat steps (1)-(3) r times to get more observations. These resulting measurements form an RSS of size (r m).

Takahasi and Wakitmoto (1968) provided the necessary mathematical theory of RSS. They showed that the RSS mean is an unbiased estimator for the population mean with smaller variance compared to the SRS mean. Dell and Clutter (1972) showed that the mean of RSS is still unbiased whether the ranking is perfect or not. Samawi *et al* (1996) investigated extreme ranked set sample (ERSS) i.e they quantify the smallest and the largest order statistics instead of detailed ranking.

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In fact, two factors affect the efficiency of an RSS, the set size and the ranking errors. The larger the set size, the larger efficiency of RSS. Thus the larger the set size, the more difficulty in visual ranking and ranking errors (Al-Saleh and Al-Omari, 2002). For this, several authors modified RSS to reduce the error in ranking and to make visual ranking tractable by an experimenter.

Muttlak (1997) introduced median ranked set sampling (MRSS) to estimate the population mean. Al-Saleh and Al-Kadiri (2000) considered double ranked set sample (DRSS), as a procedure that increases the efficiency of the RSS estimator without increasing the set size *m*. It was shown that the DRSS estimator of the mean is more efficient than that using RSS. Furthermore, ranking in the second stage is in some sense, easier than ranking in the first stage. Al-Saleh and Al-Omari (2002) generalized DRSS to multistage ranked set sample (MSRSS). Jemain and Al-Omari (2006) suggested multistage median ranked set sampling (MMRSS) method for estimating the population mean. Jemain *et al*. (2007) suggested multistage extreme ranked set sampling (MERSS) for estimating the population mean. For more about RSS see, Al-Omari and Jaber (2008), Islam *et al*. (2009), Mahdizadeh and Arghami (2009), Al-Omari *et al*. (2008), and Al-Omari *et al*.(2009)

Goodness-of-fit (GOF) tests have been applied in many areas of research. GOF tests measure the degree of agreement between the distribution of an observed data sample and the theoretical statistical distribution. One class of GOF tests can be used consists of tests based on the distance between the empirical and hypothesized distribution functions. Modified GOF tests under SRS was discussed by many authors (for example, Lilliefors (1967,1969), Stephness(1974,1979), Hassan (2005), and Abd-Elfattah *et al* (2010))

Stockes and Sager (1988) studied the characterization of RSS. Also, they gave an unbiased estimator for the population distribution function based on the empirical distribution function of RSS. Then, they proposed a Kolmogorov-Smirnov goodness of fit test based on the empirical distribution function. They derived the null distribution of their proposed test. Al-Subh *et al*. (2009) gave a comparison study for the power of a set of empirical distribution function goodness of fit tests for the logistic distribution under SRS and RSS. Ibrahim *et al* (2009) proposed a method to improve the power of empirical distribution function GOF tests for logistic distribution under ERSS. They also conduct a simulation study to compare the power of each test under ERSS and SRS. Shahabuddin *et al* (2009) investigate the power of several GOF tests under SRS and RSS.

In this article, extensive Tables of critical values for the EP distribution under ERSS are established. The power comparisons of a set of modified EDF tests are investigated for a number of alternative distributions under ERSS and SRS. Furthermore, the efficiency of test statistics under ERSS relative to SRS is calculated.

The layout of this article is as follows. Section 2 deals with maximum likelihood estimation of the unknown parameters from EP distribution. Section 3 presents the set of modified EDF goodness of fit tests under SRS and ERSS. In Section 4, a simulation study is conducted to obtain the critical values for the modified test statistics if the underlying distribution is EP based on ERSS. Section 5 gives the power efficiency for these test statistics under ERSS relative to SRS. Finally concluding remarks are presented in Section 6.

2. Maximum Likelihood Estimation:

In this Section the maximum likelihood estimators of the unknown parameters for the EP will be obtained. Let $Y_1, Y_2, \dots, Y_r$ be a random sample of size r selected via the ith order statistic from EP with unknown parameters θ and λ. Therefore, the PDF of the ith order statistics from EP distribution is given by using PDF and CDF defined in equations (1.1) and (1.2) as the following,

$$f(y_{i:r}) = \frac{m!}{(i-1)!(m-i)!}\theta\lambda[1-(1+y_{i:r})^{-\lambda}]^{\theta i-1}(1+y_{i:r})^{-(\lambda+1)}\{1-[1-(1+y_{i:r})^{-\lambda}]^{\theta}\}^{m-i} \tag{2.1}$$

The likelihood function of the sample $Y_1, Y_2, \dots, Y_r$ is given by

$$l = C\,\theta^r\,\lambda^r\prod_{i=1}^{r}[1-(1-y_{i:r})^{-\lambda}]^{\theta i-1}\prod_{i=1}^{r}(1+y_{i:r})^{-(\lambda+1)}\prod_{i=1}^{r}\{1-[1-(1+y_{i:r})^{-\lambda}]^{\theta}\}^{m-i}, \tag{2.2}$$

where, $c = \dfrac{m!}{(i-1)!(m-i)!}$. The log-likelihood function is

$$\ln l = r\ln C + r\ln\theta + r\ln\lambda + \sum_{i=1}^{r}(\theta i-1)\ln[1-(1+y_{i:r})^{-\lambda}] - (\lambda+1)\sum_{i=1}^{r}\ln(1+y_{i:r})$$
$$+\sum_{i=1}^{r}\,(m-i)\ln\{1-[1-(1+y_{i:r})^{-\lambda}]^{\theta}\} \tag{2.3}$$

Differentiate equation (2.3) with respect to θ and $\lambda,$ we obtain the following normal equations:

$$\frac{\partial \ln l}{\partial \theta} = \sum_{i=1}^{r} i \ln[1-(1+y_{i:r})^{-\hat{\lambda}}] + \frac{r}{\hat{\theta}} - \sum_{i=1}^{r} \frac{(m-i)\ln[1-(1+y_{i:r})^{-\hat{\lambda}}}{[1-(1+y_{i:r})^{-\hat{\lambda}}]^{-\hat{\theta}} - 1} = 0, \tag{2.4}$$

and,

$$\frac{\partial \ln l}{\partial \lambda} = \sum_{i=1}^{r} \frac{(\hat{\theta} i - 1)\ln(1+y_{i:r})}{(1+y_{i:r})^{\hat{\lambda}} - 1} + \frac{r}{\hat{\lambda}} - \sum_{i=1}^{r} \frac{(m-i)\hat{\theta}[1-(1+y_{i:r})^{-\hat{\lambda}}]^{\hat{\theta}-1}(1+y_{i:r})^{-\hat{\lambda}}\ln(1+y_{i:r})}{1-[1-(1+y_{i:r})^{-\hat{\lambda}}]^{\hat{\theta}}}$$
$$- \sum_{i=1}^{r} \ln(1+y_{i:r}) = 0.$$

Obviously, it is not easy to obtain a closed form solution for the two non-linear equations (2.4) and (2.5). Therefore, Newton Raphson method must be applied to solve these equations numerically.

3. Modified EDF Goodness of Fit Tests:

In goodness of fit test problem, the objective is to test hypotheses

$$H_0 : F(x) = F_0(x) \quad \forall x, H_1 : F(x) \neq F_0(x) \quad \text{for some } x \tag{3.1}$$

where, $F_0(x)$ is a known distribution function based on $X_1, X_2, \ldots, X_r$, a random sample from the distribution function $F(x)$. A goodness of fit test based on the EDF, where the parameters are estimated, is called modified goodness of fit test. The following set of a modified EDF goodness of fit tests under SRS defined as follows:

1. The Kolmgorov-Simrnov (K-S) test statistic D is
2.

$$D = \max\{\max_{1\le i\le r}[\frac{i}{r} - F_0(x_i), \hat{\theta}, \hat{\lambda})], \max_{r\le i\le 1}[F_0(x_{(i)}, \hat{\theta}, \hat{\lambda}) - \frac{i-1}{r}]\} \tag{3.2}$$

3. Kuiper statistic V is a modification of K-S test and takes the following form

$$V = \max_{1\le i\le r}[\frac{i}{r} - F_0(x_i), \hat{\theta}, \hat{\lambda})] + \max_{r\le i\le 1}[F_0(x_{(i)}, \hat{\theta}, \hat{\lambda}) - \frac{i-1}{r}]\} \tag{3.3}$$

4. The Cramer-von Mises (C-M) statistic W^2 is represented by the following formula

$$W^2 = \frac{1}{12r} + \sum_{i=1}^{r}[F_0(x_i), \hat{\theta}, \hat{\lambda}) - \frac{2i-1}{2r}]^2 \tag{3.4}$$

5. The Watson statistic U^2 is a modification of C-M statistic and takes the following form

$$U^2 = W^2 - r\sum_{i=1}^{r}[\frac{F_0(x_i), \hat{\theta}, \hat{\lambda})}{r} - \frac{1}{2}]^2 \tag{3.5}$$

6. The Anderson-Darling (A-D) statistic A^2 is

$$A^2 = -r - \frac{1}{r}\sum_{i=1}^{r}(2i-1)[\ln F_0(x_i), \hat{\theta}, \hat{\lambda}) + \ln[1 - F_0(x_{(r-i+1}, \hat{\theta}, \hat{\lambda})]$$

7. Liao and Shimokawa L , statistic is a modification of K-S test and takes the following form

$$L = \frac{1}{\sqrt{r}} \sum_{i=1}^{r} \{ \frac{\max[\frac{i}{r} - \mathrm{F}_0(\mathrm{x}_{(i)}, \hat{\theta}, \hat{\lambda}), \mathrm{F}_0(\mathrm{x}_{(i)}, \hat{\theta}, \hat{\lambda}) - \frac{i-1}{r}]}{\sqrt{\mathrm{F}_0(\mathrm{x}_{(i)}, \hat{\theta}, \hat{\lambda})[1 - \mathrm{F}_0(\mathrm{x}_{(i)}, \hat{\theta}, \hat{\lambda})]}} \}. \tag{3.6}$$

Let us denote the test statistics (3.2)-(3.6) by T, under SRS

To test the hypotheses based on RSS, let $Y_1, Y_2, \ldots, Y_r$ be a random sample of size r selected via ith order statistic. According to AL-Subh *et al* (2009), testing the hypotheses $H_0 : F(x) = F_0(x) \quad \forall x$ is equivalent to testing the hypotheses,

$$H_0^* : G_i(y) = G_{i0}(y) \qquad \forall y, H_1^* : G_i(y) \neq G_{i0}(y) \text{ for some } i, \tag{3.7}$$

where, $G_i(y), G_{i0}(y)$ are the CDF's of the ith of order statistics of random samples each of an odd size (m) chosen from $F(x)$ and $F_0(x)$, respectively. It can be noted that test of hypotheses (3.1) and (3.7) are equivalent tests.

According to Arnold *et al* (1992), $G_i(y)$ and $G_{i0}(y)$ have, respectively, the following representations:

$$G_i(y) = \sum_{j=i}^{m} \binom{m}{j} [F(y)]^j [1 - F(y)]^{m-j},$$

and

$$G_{i0}(y) = \sum_{j=i}^{m} \binom{m}{j} [F_0(y)]^j [1 - F_0(y)]^{m-j}.$$

For example, for m=3, and i=1,3, the CDF's , $G_i(y)'s$ and $G_{i0}(y)'s$ are given, respectively, by

$$G_1(y) = 1 - [1 - F(y]^3, \quad G_{10}(y) = 1 - [1 - F_0(y)]^3$$

and

$$G_3(y) = F^3(y), \quad G_{30}(y) = F^3(y),$$

where, $G_1(y)$, $G_{10}(y)$ are the CDF's from the smallest order statistics, and $G_3(y)$, $G_{30}(y)$ are CDF's from the largest order statistics.

It is clear that, $G_i(y) = G_{i0}(y)$ has the unique solution $F(x) = F_0(x)$.

Thus the goodness of fit test for the hypotheses (3.7), denoted by T^*, can be performed using the tests T as defined in the beginning Section, but using the data $Y_1, Y_2, \ldots, Y_r$.

4. Critical Values Based on ERSS:

The main aim in this Section is to obtain percentage points of $, T^*,$ for EP distribution under ERSS using Monte Carlo technique. The following steps are used in calculating critical values for the test statistics mentioned in Section 3:

Step(1) Generate ranked set sampling of sizes r=10(10),40 from EP distribution with $\theta = 3$ and $\lambda = 2$, for set sizes m, where, m=1,3,5,7 (m=1 means SRS case) using the ith order statistic. Then, $Y_1, Y_2, \ldots, Y_r$ be a random sample generated from $G_{i0}(x), i = 1,3$ (i.e the random sample of size r selected via smallest and largest order statistics).

Step (2): Based on a random sample obtained in step (1), estimate the unknown parameters of the EP using the maximum likelihood method by solving the two linear equations (2.4) and (2.5).

Step (3): The resulting maximum likelihood estimators of the unknown parameters were then used to determine the hypothesized cumulative distribution function for the EP distribution.

Step (4): Obtain the EDF, denoted by $\hat{F}_{ERSS}$, as follows:

$$\hat{F}_{ERSS} = \frac{1}{r}\sum_{j=1}^{r} I(Y_{(i)j} \le x), \quad I(Y_{(i)j} = \begin{cases} 1 & Y_{(i)j} \le x \\ 0 & otherwis \end{cases} \tag{4.1}$$

Step (5): Use steps(3) and (4) to calculate the statistics D, V, W^2, U^2, A^2, and L.

Step (6): Repeat steps (1-5) 5000 times, thus generating 5000 independent values of the appropriate test statistics. The 5000 test statistics for each type are sorted in ascending order and 75%, 80%, 85%, 90%, 95% and 99% percentiles are obtained. These percentiles approximate the critical values for significance levels α of 0.25, 0.20, 0.15, 0.1, 0.05 and 0.01 respectively..

Tables 1-2 list the critical values for the modified test statistics D, V, W^2, U^2, A^2, and L for EP distribution under ERSS using Monte Carlo method via MathCAD 14.

5. Power Efficiency:

The power of a test is useful in assessing the goodness of a test or in comparing competing tests. It is defined as the probability that a statistic will lead to the rejection of the null hypothesis, H_0, when it is false, i.e. when a sample is not from the hypothesized population but an alternative population (Mann *et al* (1974)). Let the complement of the null hypothesis be the alternative hypothesis H_1. The power of a goodness-of-fit test at the significance level α is denoted by $1-\beta$, where β is the probability of committing a type II error, failing to reject a false null hypothesis. A power comparison was made among K-S, Kuiper , C-M, A-D, Watson, and L test statistics for the EP distribution with two unknown shape parameters.

In this study the null hypothesis is that the random sample comes from EP and the alternative hypothesis H_1 is that the sample follows some other distributions. The considered alternative distributions are gamma, exponential, uniform, standard normal, lognormal, logistic. These distributions cover a wide range of various shapes, some of these shapes are close to the EP and some are far away from that distribution.

The procedure for calculating the power of $,T^*,$ under the alternative distributions are as follows

Step (1): Let $Y_1, Y_2, \ldots, Y_r$ be a random sample from the alternative distributions for smallest and largest order statistics.

Step (2): Obtain the EDF as defined in equation (4.1).

Step (3): Calculate the value of T^* as defined in tests from (3.2)-(3.6) but using data $Y_1, Y_2, \ldots, Y_r$. If the critical values of the test statistics for the alternative distribution exceed the corresponding critical values listed at Table (1), then the null hypothesis H_0 will be rejected at the significance level $\alpha = (0.05$ and $0.01)$

Step (4): Repeat the above steps from (1-3) 5000 times to generate 5000 independent sets of the test statistics.

Step (5): The power of each test is obtained by counting the number of rejections of the null hypothesis divided by 5000

Step (6): By the similar way the power of each test statistics is obtained under SRS but using data $X_1, X_2, \ldots, X_r$.

Step (7): The efficiency of test statistics $,T^*,$ under RSS relative to test statistics $,T,$ under SRS is calculated, where the relative efficiency is defined as the ratio of the powers $,T^*,$

$$eff(T^*, T) = \frac{Power\, of\, T^*}{Power\, of\, T}.$$

The efficiency values of tests at the significance level $\alpha = 0.01$ and 0.05 are presented in Tables (3-4) for smallest order statistics and in Tables (5-6) for largest order statistics.

From the simulation results given in Tables (1-6), the following remarks may be observed

1. For different significance levels and sample sizes, the change of critical values for all test statistics for smallest order statistic are greater than that the corresponding for largest order statistic. As the set size increases, the critical values for test statistics decrease monotonically, for all test statistics expect for few cases.
2. The efficiency of the modified tests are all greater than one, which mean that the power for test statistics under ERSS is larger than the corresponding under SRS.
3. As the set size increases from m =3 to m=5, the efficiencies for test statistics increase, but as the set size increase from m=5 to m=7 the efficiencies still the same in almost all cases expect for small cases.

4. It is found that the powers efficiency of modified EDF tests are broadly in the following order of descending power, $U^2 \rightarrow D \rightarrow V \rightarrow W^2 \rightarrow L \rightarrow A^2$.

Conclusion:

In order to investigate the behavior of the modified EDF tests of fit for EP distribution under ERSS and SRS, a power study is made using six alternatives families of distributions. This study shows the power of tests can be much improved if the sample collected via the ERSS. Furthermore, the modified EDF tests under ERSS are more powerful in comparison with their corresponding SRS. In general, the Watson statistic, K-S, and Kuiper tests appear to be the best EDF test statistics. The A-D tends to be least powerful among the six EDF considered here. So, Watson test statistic superior to other test statistic. The power of the modified EDF tests increases as the sample size increases in almost all cases.

Table 1: Critical Points of Test Statistics for ERSS (Using First Order Statistics)

Sample Size r	Set sizem	Test Statistics		Significance level α				
			0.01	0.025	0.05	0.1	0.15	0.2
		D	0.333	0.308	0.290	0.266	0.248	0.237
		V	0.571	0.530	0.495	0.454	0.427	0.408
	3	W^2	0.240	0.190	0.162	0.126	0.114	0.102
		U^2	0.213	0.167	0.143	0.117	0.103	0.092
		A^2	1.743	1.543	1.387	1.251	1.173	1.112
		L	1.360	1.148	0.954	0.793	0.697	0.626
		D	0.328	0.304	0.285	0.262	0.247	0.235
		V	0.572	0.543	0.496	0.457	0.430	0.412
10	5	W^2	0.235	0.199	0.163	0.134	0.116	0.103
		U^2	0.220	0.187	0.152	0.124	0.109	0.097
		A^2	1.942	1.666	1.481	1.306	1.211	1.144
		L	1.398	1.208	1.024	0.840	0.736	0.661
		D	0.314	0.293	0.276	0.256	0.243	0.233
		V	0.550	0.517	0.483	0.449	0.427	0.410
	7	W^2	0.217	0.178	0.155	0.128	0.112	0.101
		U^2	0.209	0.170	0.148	0.121	0.107	0.096
		A^2	1.956	1.678	1.501	1.308	1.219	1.140
		L	1.360	1.126	0.983	0.819	0.721	0.654
		D	0.236	0.220	0.206	0.190	0.178	0.170
		V	0.422	0. 394	0.367	0.367	0.317	0.301
	3	W^2	0.222	0.187	0.158	0.127	0.112	0.100
		U^2	0.199	0.165	0.140	0.115	0.101	0.091
		A^2	1.422	1.306	1.204	1.099	1.042	0.999
		L	1.292	1.105	0.956	0.795	0.694	0.631
		D	0.236	0.220	0.206	0.190	0.178	0.170
20	5	V	0.422	0.394	0.367	0.337	0.317	0.301
		W^2	0.222	0.187	0.158	0.127	0.112	0.100
		U^2	0.199	0.165	0.140	0.115	0.101	0.091
		A^2	1.422	1.306	1.204	1.099	1.042	0.999
		L	1.292	1.105	0.956	0.795	0.694	0.631
		D	0.235	0.218	0.204	0.188	0.176	0.169
		V	0.421	0.392	0.367	0.337	0.315	0.302
	7	W^2	0.226	0.188	0.158	0.130	0.112	0.101
		U^2	0.213	0.167	0.143	0.117	0.103	0.092
		A^2	1.743	1.543	1.387	1.251	1.173	1.112
		L	1.360	1.148	0.954	0.793	0.697	0.626

Table 1: (Continued)

Sample Size r	Set size m	Test Statistics		Significance level α				
			0.01	0.025	0.05	0.1	0.15	0.2
	3	D	0.225	0.205	0.187	0.172	0.162	0.155
		V	0.404	0.367	0.336	0.309	0.290	0.276
		W^2	0.221	0.187	0.157	0.129	0.112	0.101
		U^2	0.195	0.166	0.139	0.116	0.101	0.090
		A^2	1.396	1.265	1.176	1.070	1.010	0.967
		L	1.313	1.096	0.958	0.799	0.702	0.641
30	5	D	0.199	0.184	0.170	0.157	0.148	0.141
		V	0.366	0.334	0.309	0.284	0.267	0.255
		W^2	0.231	0.189	0.163	0.132	0.114	0.102
		U^2	0.211	0.178	0.150	0.122	0.106	0.096
		A^2	1.424	1.282	1.179	1.075	1.012	0.964
		L	1.390	1.177	1.011	0.838	0.740	0.670
	7	D	0.194	0.178	0.150	0.124	0.109	0.098
		V	0.356	0.328	0.304	0.281	0.266	0.253
		W^2	0.221	0.184	0.156	0.130	0.115	0.103
		U^2	0.212	0.174	0.150	0.124	0.109	0.098
		A^2	1.430	1.292	1.183	1.077	1.021	0.971
		L	1.406	1.167	1.010	0.842	0.746	0.668
	3	D	0.333	0.308	0.290	0.266	0.248	0.237
		V	0.571	0.530	0.495	0.454	0.427	0.408
		W^2	0.240	0.190	0.162	0.126	0.114	0.102
		U^2	0.213	0.167	0.143	0.117	0.103	0.092
		A^2	1.743	1.543	1.387	1.251	1.173	1.112
		L	1.360	1.148	0.954	0.793	0.697	0.626
40	5	D	0.236	0.220	0.206	0.190	0.178	0.170
		V	0.422	0.394	0.367	0.337	0.317	0.301
		W^2	0.222	0.187	0.158	0.127	0.112	0.100
		U^2	0.199	0.165	0.140	0.115	0.101	0.091
		A^2	1.422	1.306	1.204	1.099	1.042	0.999
		L	1.292	1.105	0.956	0.795	0.694	0.631
	7	D	0.235	0.218	0.204	0.188	0.176	0.169
		V	0.421	0.392	0.367	0.337	0.315	0.302
		W^2	0.226	0.188	0.158	0.130	0.112	0.101
		U^2	0.213	0.167	0.143	0.117	0.103	0.092
		A^2	1.743	1.543	1.387	1.251	1.173	1.112
		L	1.360	1.148	0.954	0.793	0.697	0.626

Table 2: Critical Points of Test Statistics for ERSS (Using Largest Order Statistics)

Sample Size r	Set size m	Test Statistics		Significance level α				
			0.01	0.025	0.05	0.1	0.15	0.2
	3	D	0.315	0.290	0.271	0.248	0.233	0.222
		V	0.529	0.485	0.457	0.415	0.393	0.376
		W^2	0.179	0.151	0.128	0.105	0.093	0.084
		U^2	0.157	0.135	0.115	0.096	0.084	0.076

		A^2	1.420	1.315	1.226	1.132	1.084	1.046
		L	1.003	0.860	0.737	0.621	0.555	0.507
10	5	D	0.307	0.287	0.268	0.246	0.232	0.222
		V	0.518	0.482	0.448	0.415	0.392	0.376
		W^2	0.169	0.147	0.127	0.104	0.092	0.082
		U^2	0.151	0.130	0.114	0.095	0.085	0.076
		A^2	1.467	1.320	1.220	1.136	1.081	1.045
		L	0.966	0.846	0.731	0.618	0.549	0.503
	7	D	0.311	0.286	0.269	0.247	0.233	0.221
		V	0.524	0.482	0.451	0.416	0.395	0.377
		W^2	0.173	0.146	0.125	0.105	0.1092	0.082
		U^2	0.155	0.131	0.115	0.096	0.085	0.077
		A^2	1.442	1.300	1.221	1.136	1.084	1.048
		L	0.986	0.836	0.732	0.623	0.552	0.506
	3	D	0.230	0.211	0.195	0.179	0.170	0.162
		V	0.409	0.374	0.343	0.313	0.295	0.282
		W^2	0.183	0.154	0.128	0.106	0.093	0.084
		U^2	0.155	0.135	0.115	0.095	0.084	0.075
		A^2	1.274	1.176	1.091	1.010	0.960	0.925
		L	1.027	0.868	0.747	0.626	0.558	0.511
20	5	D	0.234	0.212	0.196	0.180	0.168	0.160
		V	0.419	0.376	0.344	0.315	0.294	0.281
		W^2	0.187	0.156	0.130	0.105	0.091	0.082
		U^2	0.165	0.141	0.118	0.096	0.084	0.075
		A^2	1.287	1.201	1.106	1.012	0.962	0.924
		L	1.056	0.894	0.764	0.634	0.555	0.505
	7	D	0.228	0.209	0.194	0.178	0.168	0.161
		V	0.406	0.370	0.341	0.314	0.296	0.282
		W^2	0.178	0.152	0.129	0.106	0.092	0.082
		U^2	0.163	0.139	0.116	0.098	0.085	0.076
		A^2	1.288	1.182	1.095	1.016	0.964	0.928
		L	1.020	0.873	0.760	0.633	0.560	0.507

Table 2: (Continued)

Sample Size r	Set size m	Test Statistics		Significance level α				
			0.01	0.025	0.05	0.1	0.15	0.2
	3	D	0.188	0.174	0.161	0.148	0.140	0.134
		V	0.346	0.315	0.290	0.264	0.250	0.238
		W^2	0.185	0.154	0.132	0.108	0.093	0.083
		U^2	0.162	0.136	0.115	0.096	0.084	0.076
		A^2	1.200	1.114	1.039	0.961	0.910	0.872
		L	1.052	0.880	0.766	0.635	0.564	0.513
30	5	D	0.193	0.177	0.167	0.153	0.144	0.137
		V	0.350	0.317	0.294	0.265	0.251	0.239
		W^2	0.201	0.169	0.145	0.117	0.102	0.090
		U^2	0.166	0.136	0.119	0.096	0.084	0.077

		A^2	1.312	1.184	1.090	0.996	0.942	0.901
		L	1.185	0.998	0.864	0.715	0.633	0.564
		D	0.187	0.174	0.161	0.149	0.140	0.134
		V	0.341	0.316	0.290	0.267	0.251	0.239
	7	W^2	0.183	0.151	0.129	0.107	0.093	0.084
		U^2	0.165	0.135	0.117	0.098	0.086	0.077
		A^2	1.234	1.113	1.041	0.958	0.912	0.873
		L	1.045	0.880	0.758	0.645	0.568	0.518
		D	0.168	0.154	0.143	0.131	0.123	0.117
		V	0.312	0.283	0.261	0.238	0.224	0.212
	3	W^2	0.186	0.157	0.132	0.108	0.095	0.084
		U^2	0.166	0.137	0.118	0.096	0.085	0.076
		A^2	1.169	1.076	1.002	0.922	0.875	0.839
		L	1.055	0.884	0.767	0.643	0.570	0.516
		D	0.169	0.151	0.141	0.129	0.122	0.116
		V	0.312	0.278	0.257	0.235	0.221	0.210
40	5	W^2	0.187	0.155	0.127	0.104	0.090	0.082
		U^2	0.166	0.139	0.115	0.095	0.083	0.076
		A^2	1.172	1.080	0.993	0.915	0.865	0.833
		L	1.068	0.901	0.749	0.633	0.556	0.507
		D	0.164	0.150	0.140	0.129	0.121	0.116
		V	0.303	0.276	0.257	0.234	0.219	0.209
	7	W^2	0.177	0.147	0.125	0.105	0.092	0.082
		U^2	0.161	0.134	0.116	0.096	0.084	0.075
		A^2	1.145	1.055	0.986	0.915	0.870	0.838
		L	1.036	0.844	0.746	0.631	0.563	0.509

Table 3: Power Efficiency of Test Statistics for the EP using ERSS Relative to SRS (Using First Order Statistics) with Significance Level $\alpha = 0.01$

Sample Size r	Set sizem	Test Statistics		Alternatives				
			Gamma(3)	Exp.(2)	U(0,1)	N(0,1)	LN(0,1)	Logistic(2)
		D	1.411	1.406	1.393	1.321	1.396	1.445
		V	5.163	5.233	5.217	4.969	5.073	5.158
	3	W^2	3.661	3.725	3.682	3.654	3.607	3.702
		U^2	8.271	8.278	7.864	7.901	8.057	8.484
		A^2	3.084	3.149	3.143	3.099	3.051	3.155
		L	3.171	3.285	3.299	3.261	3.208	3.284
		D	5.476	5.637	5.580	5.258	5.429	5.459
		V	5.428	5.506	5.480	5.208	5.342	5.422
10	5	W^2	3.720	3.794	3.734	3.709	3.679	3.768
		U^2	5.888	5.849	5.452	5.836	5.781	6.131
		A^2	3.123	3.195	3.177	3.139	3.104	3.203
		L	3.284	3.401	3.408	3.357	3.331	3.404
		D	5.476	5.637	5.580	5.258	5.429	5.459
		V	5.429	5.507	5.482	5.208	5.342	5.423
	7	W^2	3.720	3.794	3.734	3.654	3.674	3.768
		U^2	13.253	13.454	12.581	12.935	13.055	13.661

		A^2 L	3.123 3.285	3.195 3.401	3.177 3.408	3.139 3.358	3.104 3.330	3.203 3.403
	3	D	7.133	6.411	7.081	6.702	7.238	6.668
		V	7.133	6.411	7.081	6.693	7.227	6.641
		W^2	4.324	4.072	4.505	4.401	4.448	4.317
		U^2	13.525	14.671	14.421	15.675	15.231	14.143
		A^2 L	3.759 4.234	3.546 4.044	3.808 3.280	3.762 4.324	3.774 3.364	3.660 4.461
20	5	D	7.143	6.418	7.092	6.711	7.238	6.668
		V	7.143	6.418	7.092	6.702	7.227	6.641
		W^2	4.325	4.072	4.505	4.401	4.448	4.317
		U^2	15.006	15.971	15.714	17.123	15.231	14.143
		A^2 L	3.759 4.234	3.546 3.045	3.808 4.281	3.762 4.325	3.774 4.364	3.660 4.161
	7	D	7.143	6.418	7.092	6.711	7.246	6.676
		V	7.143	6.418	7.092	6.702	7.236	6.649
		W^2	4.325	4.072	4.505	4.401	4.448	4.318
		U^2	15.015	15.974	15.974	17.123	16.722	15.528
		A^2 L	3.759 4.234	3.546 3.045	3.808 4.281	3.762 4.325	3.779 4.367	3.660 4.163

Entries are ratio of probabilities under ERSS relative to SRS for rejecting H_0 when the random sample is actually from the stated alternatives distribution.

Table 3: (Continued)

Sample Size r	Set sizem	Test Statistics		Alternatives				
			Gamma(3)	Exp.(2)	U(0,1)	N(0,1)	LN(0,1)	Logistic(2)
	3	D	6.964	7.022	6.964	7.496	6.585	6.578
		V	6.906	6.944	6.916	7.429	6.568	6.552
		W^2	4.942	4.136	3.971	4.344	4.000	3.925
		U^2	16.296	15.391	15.524	16.668	14.687	14.663
		A^2 L	3.663 3.934	3.679 3.876	3.531 3.820	3.882 4.149	3.579 3.885	3.531 3.900
30	5	D	6.964	7.022	6.964	7.496	6.585	6.578
		V	6.906	6.944	6.916	7.429	6.568	6.552
		W^2	4.042	4.136	3.971	4.344	4.000	3.925
		U^2	16.611	15.625	15.524	16.949	14.925	14.881
		A^2 L	3.663 3.934	3.679 3.876	3.531 3.820	3.882 4.149	3.579 3.885	3.531 3.900
	7	D	6.964	7.022	6.964	7.496	6.585	6.578
		V	6.906	6.944	6.916	7.429	6.568	6.552
		W^2	4.042	4.136	3.971	4.344	4.000	3.925
		U^2	16.611	15.625	15.773	16.949	14.925	14.743
		A^2 L	3.663 3.934	3.679 3.876	3.531 3.820	3.882 4.149	3.579 3.885	3.531 3.900
	3	D	6.775	7.092	6.964	7.257	6.510	7.013
		V	6.631	6.022	6.916	7.153	6.435	6.916
		W^2	4.517	4.355	3.971	4.480	4.413	4.566

		U^2	18.338	17.412	15.625	18.756	18.608	18.838
		A^2 L	3.834 3.855	3.628 3.768	3.531 3.820	3.704 3.931	3.712 3.931	3.748 3.879
40	5	D	6.775	7.092	6.964	7.257	6.510	7.013
		V	6.631	6.022	6.702	7.153	6.435	6.435
		W^2	4.517	4.355	3.971	4.480	4.413	4.566
		U^2	18.382	18.248	19.455	18.797	18.657	18.868
		A^2 L	3.834 3.855	3.628 3.768	3.531 3.820	3.704 3.931	3.712 3.931	3.748 3.879
	7	D	6.775	7.092	6.964	7.257	6.510	7.013
		V	6.631	6.022	6.702	7.153	6.435	6.435
		W^2	4.517	4.355	3.971	4.480	4.413	4.566
		U^2	18.382	17.921	19.455	19.455	18.657	18.868
		A^2 L	3.834 3.855	3.628 3.768	3.531 3.820	3.704 3.931	3.712 3.931	3.748 3.879

Entries are ratio of probabilities under ERSS relative to SRS for rejecting H_0 when the random sample is actually from the stated alternatives distribution.

Table 4: Power Efficiency of Test Statistics for the EP using ERSS Relative to SRS (Using First Order Statistics) with Significance Level $\alpha = 0.05$

Sample Size r	Set sizem	Test Statistics		Alternatives				
			Gamma(3)	Exp.(2)	U(0,1)	N(0,1)	LN(0,1)	Logistic(2)
	3	D	2.970	3.014	2.967	2.954	2.952	2.991
		V	2.938	3.039	2.918	2.967	2.924	2.951
		W^2	2.322	2.387	2.338	2.333	2.292	2.307
		U^2	4.151	4.321	4.172	4.160	4.124	4.117
		A^2 L	2.012 2.213	2.074 2.276	2.048 2.271	2.047 2.272	1.989 2.221	2.009 2.227
10	5	D	3.027	3.077	3.016	3.005	3.008	3.038
		V	2.992	3.106	3.034	3.021	2.982	3.008
		W^2	2.332	2.403	2.346	2.345	2.309	2.322
		U^2	3.254	3.361	3.180	3.244	3.272	3.263
		A^2 L	2.019 2.237	2.083 2.306	2.053 2.297	2.053 2.301	2.002 2.257	2.017 2.254
	7	D	3.027	3.077	3.016	3.005	3.008	3.038
		V	2.992	3.106	2.969	3.021	2.982	3.008
		W^2	2.332	2.403	2.346	2.345	2.309	2.322
		U^2	5.168	5.363	5.176	5.152	5.165	5.144
		A^2 L	2.019 2.237	2.083 2.306	2.053 2.297	2.053 2.301	2.002 2.257	2.017 2.254
	3	D	3.339	3.163	3.339	3.297	3.360	3.270
		V	3.324	3.151	3.297	3.269	3.340	3.255
		W^2	2.559	2.480	2.571	2.541	2.588	2.547
		U^2	5.865	5.464	5.641	5.830	5.663	5.447
		A^2 L	2.266 2.297	2.149 2.206	2.243 2.352	2.297 2.312	2.349 2.334	2.244 2.320

20	5	D	3.340	3.163	3.339	3.297	3.360	3.270
		V	3.324	3.151	3.297	3.270	3.340	3.255
		W^2	2.559	2.480	2.571	2.541	2.588	2.547
		U^2	6.031	5.593	5.807	5.959	5.814	5.593
		A^2	2.266	2.149	2.243	2.249	2.349	2.244
		L	2.297	2.206	2.352	2.312	2.334	2.320
	7	D	3.340	3.163	3.339	3.297	3.360	3.270
		V	3.324	3.151	3.297	3.270	3.340	3.255
		W^2	2.559	2.480	2.571	2.541	2.588	2.547
		U^2	6.031	5.593	5.807	5.959	5.814	5.593
		A^2	2.266	2.149	2.243	2.249	2.349	2.244
		L	2.297	2.206	2.352	2.312	2.334	2.320

Entries are ratio of probabilities under ERSS relative to SRS for rejecting H_0 when the random sample is actually from the stated alternatives distribution.

Table 4: Continued

Sample Size r	Set sizem	Test Statistics	Alternatives					
			Gamma(3)	Exp.(2)	U(0,1)	N(0,1)	LN(0,1)	Logistic(2)
	3	D	3.300	3.342	3.213	3.470	3.205	3.220
		V	3.347	3.362	3.243	3.506	3.232	3.232
		W^2	2.639	2.611	2.548	2.738	2.537	2.611
		U^2	6.084	5.862	5.959	6.527	5.650	5.891
		A^2	2.234	2.139	2.178	2.299	2.197	2.212
		L	2.328	2.335	2.269	2.403	2.271	2.312
30	5	D	3.300	3.342	3.213	3.470	3.205	3.220
		V	3.347	3.362	3.243	3.506	3.232	3.232
		W^2	2.639	2.611	2.548	2.738	2.537	2.611
		U^2	6.098	5.862	5.981	6.663	6.545	5.903
		A^2	2.234	2.139	2.178	2.299	2.197	2.212
		L	2.328	2.335	2.269	2.403	2.271	2.312
	7	D	3.300	3.342	3.213	3.470	3.205	3.220
		V	3.347	3.362	3.243	3.506	3.232	3.232
		W^2	2.639	2.611	2.548	2.738	2.537	2.611
		U^2	6.098	5.862	5.981	6.545	5.663	5.903
		A^2	2.234	2.139	2.178	2.299	2.197	2.212
		L	2.328	2.335	2.269	2.403	2.271	2.312
	3	D	3.439	3.413	3.344	3.365	3.324	3.477
		V	3.499	3.467	3.415	3.446	3.394	3.536
		W^2	2.706	2.614	2.610	2.660	2.621	2.637
		U^2	5.675	5.806	6.141	6.179	5.754	5.859
		A^2	2.264	2.234	2.217	2.311	2.229	2.262
		L	2.366	2.332	2.281	2.395	2.322	2.367
40	5	D	3.439	3.413	3.344	3.365	3.324	3.477
		V	3.499	3.467	3.415	3.415	3.394	3.536
		W^2	2.706	2.614	2.610	2.660	2.621	2.505
		U^2	5.675	5.806	6.143	6.180	6.180	5.862
		A^2	2.264	2.234	2.217	2.305	2.229	2.262

		L	2.366	2.332	2.281	2.395	2.322	2.367
	7	D	3.439	3.413	3.344	3.365	3.324	3.477
		V	3.499	3.467	3.415	3.446	3.394	3.394
		W^2	2.706	2.614	2.610	2.660	2.621	2.637
		U^2	5.675	5.807	6.141	6.180	5.754	5.862
		A^2	2.264	2.234	2.217	2.311	2.229	2.262
		L	2.366	2.332	2.281	2.395	2.322	2.367

Entries are ratio of probabilities under ERSS relative to SRS for rejecting H_0 when the random sample is actually from the stated alternatives distribution.

Table 5: Power Efficiency of Test Statistics for the EP using ERSS Relative to SRS (Using Largest Order Statistics) with Significance Level $\alpha = 0.01$

Sample Size r	Set sizem	Test Statistics	Alternatives					
			Gamma(3)	Exp.(2)	U(0,1)	N(0,1)	LN(0,1)	Logistic(2)
	3	D	3.411	3.406	3.393	3.321	3.396	3.445
		V	5.163	5.233	5.217	4.969	5.073	5.158
		W^2	3.661	3.725	3.682	3.654	3.607	3.702
		U^2	5.888	5.278	5.864	5.836	5.781	6.131
		A^2	3.084	3.149	3.143	3.099	3.051	3.155
		L	3.171	3.285	3.299	3.261	3.208	3.289
10	5	D	5.476	5.637	5.580	5.258	5.429	5.454
		V	5.428	5.506	5.482	5.208	5.342	5.422
		W^2	3.720	3.794	3.734	3.709	3.679	3.768
		U^2	8.271	8.844	7.864	7.901	8.057	8.484
		A^2	3.123	3.195	3.177	3.139	3.104	3.203
		L	3.284	3.401	3.408	3.357	3.331	3.404
	7	D	5.476	5.637	5.580	5.258	5.429	5.459
		V	5.429	5.507	5.482	5.208	5.342	5.423
		W^2	3.720	3.794	3.734	3.709	3.679	3.768
		U^2	13.253	13.454	12.581	12.935	13.055	13.661
		A^2	3.123	3.195	3.177	3.139	3.104	3.203
		L	3.285	3.401	3.408	3.358	3.330	3.403
	3	D	7.133	6.411	7.081	6.702	7.238	6.668
		V	7.133	6.411	7.081	6.693	7.227	6.641
		W^2	4.324	4.072	4.505	4.401	4.448	4.317
		U^2	13.525	14.671	14.421	15.675	15.231	14.143
		A^2	3.759	3.546	3.808	3.762	3.774	3.660
		L	4.234	4.044	4.280	4.324	4.364	4.461
20	5	D	7.143	6.418	7.092	6.711	7.246	6.676
		V	7.143	6.418	7.092	6.702	7.236	6.649
		W^2	4.325	4.072	4.505	4.401	4.448	4.318
		U^2	15.006	15.971	15.714	17.123	16.716	15.519
		A^2	3.759	3.546	3.808	3.762	3.774	3.660
		L	4.234	4.045	4.281	4.325	4.367	4.163
	7	D	7.143	6.418	7.092	6.711	7.246	6.676
		V	7.143	6.418	7.092	6.702	7.236	6.649
		W^2	4.325	4.072	4.505	4.401	4.448	4.318

		U^2	15.015	15.971	15.974	17.123	16.722	15.528
		A^2	3.759	3.546	3.808	3.762	3.774	3.660
		L	4.234	4.045	4.281	4.325	4.367	4.163

Entries are ratio of probabilities under ERSS relative to SRS for rejecting H_0 when the random sample is actually from the stated alternatives distribution.

Table 5: (Continued)

Sample Size r	Set sizem	Test Statistics		Alternatives				
			Gamma(3)	Exp.(2)	U(0,1)	N(0,1)	LN(0,1)	Logistic(2)
		D	6.964	7.022	6.964	7.496	6.585	6.578
		V	6.906	6.944	6.916	7.429	6.568	6.552
	3	W^2	4.042	4.136	3.971	4.344	4.000	3.925
		U^2	16.296	15.391	15.524	16.668	14.687	14.663
		A^2	3.663	3.679	3.531	3.882	3.579	3.531
		L	3.934	3.876	3.820	4.149	3.855	3.900
		D	6.964	7.022	6.964	7.496	6.585	6.579
		V	6.906	6.944	6.916	7.429	6.570	6.553
30	5	W^2	4.042	4.136	3.971	4.344	4.000	3.925
		U^2	16.611	15.625	15.625	16.949	14.925	14.881
		A^2	3.663	3.679	3.531	3.882	3.579	3.531
		L	3.934	3.876	3.820	4.149	3.855	3.900
		D	6.964	7.022	6.964	7.496	6.585	6.579
		V	6.906	6.944	6.916	7.429	6.570	6.553
	7	W^2	4.042	4.136	3.971	4.344	4.000	3.925
		U^2	16.611	15.625	15.733	16.949	14.925	14.793
		A^2	3.663	3.679	3.531	3.882	3.579	3.531
		L	3.934	3.876	3.820	4.149	3.855	3.900
		D	6.775	6.784	7.092	7.257	6.510	7.013
		V	6.631	6.702	7.022	7.153	6.435	6.916
	3	W^2	4.517	4.440	4.355	4.480	4.413	4.566
		U^2	18.338	17.875	19.412	18.756	18.608	18.838
		A^2	3.834	3.715	3.628	3.704	3.712	3.748
		L	3.855	3.861	3.768	3.931	3.931	3.879
		D	6.775	6.784	7.092	7.257	6.510	7.013
		V	6.631	6.702	7.022	7.153	6.435	6.916
40	5	W^2	4.517	4.440	4.355	4.480	4.413	4.566
		U^2	18.382	18.248	19.455	18.797	18.657	18.868
		A^2	3.834	3.715	3.628	3.704	3.712	3.748
		L	3.855	3.861	3.768	3.931	3.931	3.879
		D	6.775	6.784	7.092	7.257	6.510	7.013
		V	6.631	7.702	7.022	7.153	6.435	6.916
	7	W^2	4.517	4.440	4.355	4.480	4.413	4.566
		U^2	18.338	17.921	19.455	19.455	18.608	18.838
		A^2	3.834	3.715	3.628	3.704	3.712	3.748
		L	3.855	3.861	3.768	3.931	3.931	3.879

Entries are ratio of probabilities under ERSS relative to SRS for rejecting H_0 when the random sample is actually from the stated alternatives distribution.

Table 6: Power Efficiency of Test Statistics for the EP using ERSS Relative to SRS (Using Largest Order Statistics) with Significance Level $\alpha = 0.05$

Sample Size r	Set sizem	Test Statistics		Alternatives				
			Gamma(3)	Exp.(2)	U(0,1)	N(0,1)	LN(0,1)	Logistic(2)
		D	2.970	3.014	2.967	2.954	2.952	2.981
		V	2.938	3.039	2.918	2.967	2.924	2.951
	3	W^2	2.322	2.387	2.338	2.333	2.292	2.307
		U^2	4.151	4.321	4.172	4.160	4.124	4.117
		A^2	2.012	2.074	2.048	2.047	1.989	2.009
		L	2.213	2.276	2.271	2.272	2.221	2.227
		D	3.027	3.077	3.016	3.005	3.008	3.038
		V	2.992	3.106	3.034	3.021	2.982	3.008
10	5	W^2	2.332	2.403	2.346	2.345	2.309	2.322
		U^2	3.254	3.361	3.180	3.244	3.272	3.263
		A^2	2.019	2.083	2.053	2.053	2.002	2.017
		L	2.237	2.306	2.297	2.301	2.257	2.254
		D	3.027	3.077	3.016	3.005	3.008	3.038
		V	2.992	3.106	2.969	3.021	2.982	3.008
	7	W^2	2.332	2.403	2.346	2.345	2.309	2.322
		U^2	5.168	5.363	5.176	5.152	5.165	5.144
		A^2	2.019	2.083	2.053	2.053	2.002	2.017
		L	2.237	2.306	2.297	2.301	2.257	2.254
		D	3.339	3.163	3.339	3.297	3.360	3.270
		V	3.324	3.151	3.297	3.269	3.340	3.255
	3	W^2	2.559	2.480	2.571	2.541	2.588	2.547
		U^2	5.865	5.464	5.641	5.830	5.663	5.447
		A^2	2.266	2.149	2.243	2.297	2.349	2.244
		L	2.297	2.206	2.352	2.312	2.334	2.320
		D	3.339	3.163	3.339	3.297	3.360	3.270
		V	3.324	3.151	3.297	3.270	3.340	3.255
20	5	W^2	2.559	2.480	2.571	2.541	2.588	2.547
		U^2	6.031	5.593	5.807	5.959	5.814	5.593
		A^2	2.266	2.149	2.243	2.297	2.349	2.244
		L	2.297	2.206	2.352	2.312	2.334	2.320
		D	3.339	3.163	3.339	3.297	3.360	3.270
		V	3.324	3.151	3.297	3.270	3.340	3.255
	7	W^2	2.559	2.480	2.571	2.541	2.588	2.547
		U^2	6.031	5.593	5.807	5.959	5.814	5.593
		A^2	2.266	2.149	2.243	2.297	2.349	2.244
		L	2.297	2.206	2.352	2.312	2.334	2.320

Entries are ratio of probabilities under ERSS relative to SRS for rejecting H_0 when the random sample is actually from the stated alternatives distribution.

Table 6: (Continued)

Sample Size r	Set sizem	Test Statistics		Alternatives				
			Gamma(3)	Exp.(2)	U(0,1)	N(0,1)	LN(0,1)	Logistic(2)
		D	3.300	3.342	3.213	3.470	3.205	3.220

	3	V	3.347	3.362	3.243	3.506	3.232	3.232
		W^2	2.639	2.611	2.548	2.738	2.537	2.611
		U^2	6.084	5.862	5.959	6.527	5.650	5.891
		A^2	2.234	2.239	2.178	2.299	2.197	2.212
		L	2.328	2.335	2.269	2.403	2.271	2.312
30	5	D	3.300	3.342	3.213	3.470	3.205	3.220
		V	3.347	3.362	3.243	3.506	3.332	3.232
		W^2	2.639	2.611	2.548	2.738	2.537	2.611
		U^2	6.098	5.862	5.981	6.527	5.545	5.903
		A^2	2.234	2.239	2.178	2.299	2.197	2.212
		L	2.328	2.335	2.271	2.403	2.271	2.312
	7	D	3.300	3.342	3.213	3.470	3.205	3.220
		V	3.347	3.362	3.243	3.506	3.332	3.232
		W^2	2.639	2.611	2.548	2.738	2.537	2.611
		U^2	6.098	5.862	5.981	6.527	5.663	5.903
		A^2	2.234	2.239	2.178	2.299	2.197	2.212
		L	2.328	2.335	2.271	2.403	2.271	2.312
	3	D	3.439	3.413	3.344	3.365	3.324	3.477
		V	3.499	3.467	3.415	3.446	3.394	3.536
		W^2	2.706	2.614	2.610	2.660	2.621	2.637
		U^2	5.675	5.806	6.141	6.179	5.754	5.859
		A^2	2.264	2.234	2.217	2.311	2.229	2.262
		L	2.366	2.332	2.281	2.395	2.322	2.367
40	5	D	3.439	3.413	3.344	3.365	3.324	3.477
		V	3.499	3.467	3.415	3.415	3.394	3.536
		W^2	2.706	2.614	2.610	2.660	2.621	2.637
		U^2	5.675	5.806	6.143	6.180	5.754	5.859
		A^2	2.264	2.234	2.217	2.305	2.229	2.262
		L	2.366	2.332	2.281	2.395	2.322	2.367
	7	D	3.439	3.413	3.344	3.365	3.324	3.477
		V	3.499	3.467	3.415	3.446	3.394	3.536
		W^2	2.706	2.614	2.610	2.660	2.621	2.637
		U^2	5.675	5.807	6.141	6.179	5.754	5.859
		A^2	2.264	2.234	2.217	2.311	2.229	2.262
		L	2.366	2.332	2.281	2.395	2.322	2.367

Entries are ratio of probabilities under ERSS relative to SRS for rejecting H_0 when the random sample is actually from the stated alternatives distribution.

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