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New Results for Weighted Planar and Outer Planar of Graph

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Abstract: Let f be a family of graphs closed under taking sub graphs , such that , for some positive c, |E(G)|≤c(|V(G)|-1) for all G in f. let W be nonnegative weight function on the edges of some G in f , and let be a maximum a spanning forest in G. then W(f) ≥ $\frac{1}{c}$ w(G). Calinecsu and others (Peranen, T., 2004) is show that w(f)≥ $\frac{1}{c}$ w(p) and w(f)≥ $\frac{1}{2}$ w(p) for planar and outer planar of graph , respectively . We show that w (f) ≥ $\frac{1}{2}$ w (p) and w (f) ≥ $\frac{2}{3}$ w (p) for planar and outer planar graph which does not contain any triangle respectively and new claims for sparse and critical of graph.

Key words: Weighted planar and outer planar graph. Sparse, critical keywords. Weighted planar and outer planar, sparse, critical.

INTRODUCTION

The thickness of a graph G= (V, E) is the minimum number of planar sub graph into which the graph can be decomposed. Therefore, the thickness is one measure of the degree of non planarity of a graph. The thickness problem is NP-hard. The thickness problem has application in VLSI design. Outer planar graphs are a widely studied graph class with application in graph Drawing and with intersecting theoretical properties.

The outer thickness of the graph is the minimum number of outer planar into which the graph can be decomposed and outer thickness is one of the classical and standard measures of non- outer planarity of graphs. In this article we present algorithms have concerning simulated annealing algorithm and genetic algorithm for determining the outer thickness of a graph is given. In this paper we show that w (f) ≥ $\frac{1}{2}$ w (p) and w (f) ≥ $\frac{2}{3}$ w (p) for planar and outer planar graph which does not contain any triangle respectively and new claims for sparse and critical of graph.

Since the outer thickness is one of the classical and standard measure of non- outer planarity of graphs then we determining the outer thickness of a graph is very important problem.

2. Characterization of Planar and Outer Planar Graphs:

Def (2.1). A graph H is said to be homemorphic from G if either H= G or H is isomorphic to a subdivision of G. A graph G_1 is homemorphic with G_2 if there exists a graph a graph E_3 such that G_3 and G_2 are both homemorphic from G_3.

Corollary (2.1). If a graph G has sub graph that is homemorphic from K_5 or $K_{3.3}$, then G is non planar.

Proof. Trivial.

Theorem (2.1) (Kedlaya, K.S., 1996**),** A graph is planar if and only if it does not contain a sub graph which is homemorphic form K_5 or $K_{2.3}$

Theorem (2.2) (Chartland, G. and F. Harary, 1967**).** A graph is outer planar if and only if it does not contain a sub graph which is homemorphic from K_4 or $K_{2,3}$.

Corollary (2.2)(Chartland, G. and F. Harary, 1967**).** Let G be a connected planar graph with n vertices, m edge, and f

Faces, the we have for the plane embedding of G that n-m+f=2.

Corollary (2.3). (Chartland, G. and F. Harary, 1967**)**. If l(f_i) does not the length efface Ft in a plane graph G , then 2E=$\sum l(f_t)$.

Def (2.2). if a graph G' = (F, E") is a planar sub graph of G such that every graph G" obtained from G' by adding on edge from E\ E' is non planar , then G' is called a maximal planar sub graph of G.

Def (2.3). if there is no planar sub graph G"=(V,G") of G |E"| > |E'| , then G1 is a maximum planar sub graph.

3. Theoretical Bounds:

Theorem(3.1) (Harary, F., 1971) let G'= (V ,E') be a maximum planar sub graph of a graph G= (V ,E) then |E'|≤ 3|V|-6.

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Theorem (3.2) (Harary, F., 1971**)** let G' = (V,E') be a maximum planar sub graph of a graph which does not contain any triangle then $|E'| \leq 2|V|-4$.

Theorem (3.3) (Kedlaya, K.S., 1996**)** let G'=(V,E') be a maximum outer planar sub graph of a graph G=(V,E) then $|E'| \leq 2\,|V|-3$.

Theorem (3.4) (Kedlaya, K.S., 1996**)** let G'=(V,E') be a maximum outer planar sub graph of a graph G=(V,E) which does not contain any triangle then $|E'| \leq 3/2\,|V|-2$.

Theorem (3.5) (nash-williams)

The arboricity of graph G is the maximum, over all sub graph H of G with at least two vertices, of $\left\lceil \frac{E(H)}{|V(H)-1|} \right\rceil$

Lemma (3.1) (Calinescu, G., 2003**)** let f be a family of graphs closed under taking sub graphs, such that, for some positive c, $|E\ (G)| \leq c(|V(g)|-1)$ for all G in f . Let W be non- negative weight function on the edges of some G in f. And let f be a maximum a spanning forest in G. Then W (f) 1/c w (G).

Colollary (3.1). (Calinescu, G., 2003**)** let p a simple planar graph with a non-negative function w define on its edges. If F a maximum spanning forest in P, then $W\ (f) \geq 1/3\ W\ (P)$.

Lemma (3.2). (Calinescu, G., 2003**)** let p be an outer planar graph with a non – negative weight function w defined on its edges . Then a maximum spanning forest in P, then $W\ (f) \geq 1/2\ W\ (P)$.

4. New results:

Claim(4.1) let p be a simple planar graph with a non- negative weight function W defined on its edges which does not contain any triangle , if f is a maximum spanning forest in p , then $W(f) \geq 1/2\ W(P)$.

Proof. Apply Euler's formula and theorem (3.2) and lemma (3.1) $|E\ (G)| \leq 2(|V(g)|-1)$ with c=2 to family of simple planar graphs.

Claim (4.2) let p be simple outer planar graph with a non- negative weight function w on the edges does not contain any triangle. if F is a maximum spanning forest in O , then $W(f) \geq 3/2\ W(P)$.

Proof. apply Euler's formula and theorem (3.4) lemma (3.1) with C=3/2 to the family of all outer planar graphs.

Def (4.1) A graph G is a sparse if $\Delta_G(x) \geq 0$

For every non-empty subset x of vertices of a graph $\Delta_G(x)$ will denotes $C(|x|-1)-e_x$ where ex is the number of edge joining pairs of elements of x. (C.St. A Nask-Williams, 1964)

Def (4.2). A graph G is a critical if $\Delta_G(x) = 0$ for ever non- empty subset x of V(G).

Def (4.3) a graph is tree if it does not contain any cycle. a tree is a spanning tree , if there is a unique path between every pairs of vertices . For a graph with n vertices, as panning tree contains n-1 edges.

Def (4.4) a graph is a forest if each of its components is a tree.

Claim (4.3) let G be a sparse and let f be a maximum a spanning forest in G and w non- negative weight function on the edges of G inf then $W(f) \geq 1/cw(G)$ for some positive C.

Proof. Since G is a sparse then $\Delta_G(x) \geq 0$ i.e

$C(|V(G)|-1)-|E(G)| \geq 0 \rightarrow |E(G)| \leq C(|V(G)|-1)$

By according lemma (2.1) then $w(G) \geq 1/c\ W(G)$

Claim (4.4). let G be a critical and let f be a maximum spanning forest in G and w be non- negative function on the edge G in f then $W(f) \geq W(G)$.

Proof. Since G is a critical then $\Delta_G(x) = 0$ i.e.

$C\ (|V(G)|-1)-|E(G)| = 0 \rightarrow |E(G)| = C(|V(G)|-1)$

And since a spanning tree contains n-1 edges then c=1. Apply lemma (2.1) and Def(4.4) hence $W(f) \geq W(G)$.

Claim (4.5). if G be a tree them G is critical.

Proof. since a spanning tree contain n-1 edges then C=1 i.e $|E|=|v|-1 \rightarrow \Delta_G(x) = 0$ hence G is critical.

5. Conclusion:

In this paper we have present some results concerning the weighted planar and weighted outer planar of a graph. In particular, constants c=2 and 2=2/3 for planar and outer planar of graph which does not contain any triangle respectively.

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