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Relation Between Thickness and k-PCB Model

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Abstract: The thickness if a graph is the minimum number k such that G is the union of k planar graphs. Determining the thickness of a given graph is known too NP-complete problem [1, 2, 3], in this article we show relation between thickness and K-PCB model model and an open problem for this model.

Key words: thickness, K-PCB model

INTRODUCTION

The thickness problem, asking for the thickness of a given graph G, is NP-hard, so there is little hope to find a polynomial –time algorithm for the thickness problem a general graphs. However, for some graph classes, the thickness can be determined in polynomial time. For example, the thickness is known for complete and complete bipartite graphs, the thickness problem has application in VISI design. In electronic circuits, components are joined by means of conducting strips. These may not cross, since this would lead to undesirable signals in this case, an isolated wire must be used. For that reason, circuits with a large number of crossings are decomposed into several layers without crossing, which are then pasted together. The goal is to use as few layers as possible. In this application it would be desirable to know the thickness of a hypercube whose nodes are cells to be placed and whose hyper edges correspond to the nets connecting the cells. If the thickness problem could be solved for graphs, it graphs, it would be useful engineering tool in the layout of electronic circuits. in this article we show relation between thickness and K-PCB model and K-PCB model and an open problem for this model.

2. Characterization of Planar Graphs:

Del (2.1) a graph H is said to be homeomorphic from G if either $H \cong G$ or H is isomorphic to a subdivision of G. A graph G_1 is homeomorphic with G_2 if there exists a graph G_3 such that G_1 and G_2 are both homeomorphic from G_3.

Theorem (2.1) [A] A graph is planar if and only if it does not contain a sub graph which is homeomorphic K_5 or $K_{3,3}$.

Theorem (2.2) [5] G be a connected planar graph with n vertices, m edge, and f faces, then we have for the plane embedding of G that n - m + f=2

Corollary (2.2) [1] if L(fi) denotes the length of face fi in a planar graph G , than

$$\sum L(fi) = 2E$$

Theorem (2.3) if G is a simple planar graph with at least three vertices , then $E(G) \leq 3n-6$ .if also G is triangle –free , then $E(G) \leq 2n-4$.

Proof. It suffices to Consider connected graphs, otherwise we could add edges. Euler's formula will relate n and E if we can dispose if f. theorem (2.2) provides an in equality between E and f. Every face boundary in a simple contain at least three edges (if n≥3). Letting {fi} be the list of face lengths, this yields $2E=\sum fi \geq 3f$

But $n-E+f=2 \rightarrow f=2-n+E \rightarrow 2E \geq 6-3n-3E \rightarrow E \leq 3n-6$

When G is triangle –free, the faces have length at least 4.

In this case $2E=\sum fi \geq 4f$

But $n-E+f=2 \rightarrow f=2-n+E \rightarrow 2E \geq 8-4n+4E \rightarrow E \leq 2n-4$

3. Thickness:

Del (3.1) the thickness of a graph is the minimum number K such that G is the union of L planar Graphs.

Here, by "union of K planar Graphs" we mean that the edges of G can be partitioned into K sets so that the graph induced by each set is planar.

Theorem (3.1) if G = (V, E) is a graph with (|V| = n > 2) and |E| = m, then:

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i) θ (G) $\geq \left\lceil \frac{m}{3n-6} \right\rceil$

ii) θ (G) $\geq \left\lceil \frac{m}{2n-4} \right\rceil$, if G has no triangle.

Proof. by theorem (2.3), the denomination is the maximum size of each planar sub graph. The pigeonhole principle then yields the inequality.

Corollary (3.1) [7] the thickness of the complete graph K_n is

θ (K_n) $= \left\lfloor \frac{n+7}{6} \right\rfloor$, for n=9.10 θ (K_9) = θ (K_{10}) =3

Corollary (3.2) [8] the thickness of the complete bipartite graph $K_{m,n}$ is

θ ($K_{m,n}$)= $\left\lceil \frac{mn}{2(m+n-2)} \right\rceil$ except if m and n are both odd, m $\leq$ n and there is an integer K satisfying n= $\left\lfloor \frac{2k(m-2)}{m-2k} \right\rfloor$

Theorem (3.3) [7] the thickness of the complete bipartite graph $K_{n,n}$ is θ ($k_{n,n}$) = $\left\lfloor \frac{n+5}{4} \right\rfloor$

Theorem (3.4) [10, 11, 12] if G= (V, E) is a graph with m adage and maximal degree d. then

i) θ (G) $\leq \left\lfloor \sqrt{\frac{m}{3}} + \frac{7}{6} \right\rfloor$

ii) θ (G) $\leq \left\lceil \frac{d}{2} \right\rceil$

4. Embeddings for VLSI:

An "embedded adage" of G consists of a path of horizontal an/or vertical segments with the restriction that the only intersection allowed among these results are those in which the horizontal segment of an embedded edge intersects the vertical segment of another embedded edge or vice versa.

An embedded if a graph G in a One- layer grid is a mapping of the nodes of G to paths in the Grid [6, 9]. The paths repressing edge are disjoint except possibly at their end points. An example is shown in figure (4.1).

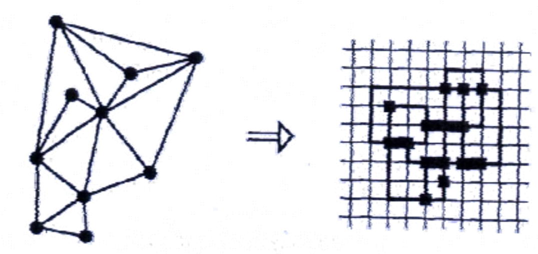

Fig. 4.1: A grid embedding of a graph.

(4.1) K-PCB Model:

This model consists of K grid layer with each mode of G embedded in the same position an each layer, and edge embedded as paths in the grid which may change layers at contact cuts.

An edge path may begin and end on any layer. But, as before, within each layer the paths must not intersect each other except possibly are their endpoints. (PCB stands for printed circuit board)/ The K-PCB model corresponds to K-layer printed circuit boards in which points of a mounted chip are present on all layers. Figure (4.2) shows an embedding of K_5 in the 2-PCB model.

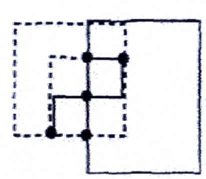

Fig. 4.2: 2-PCB embedding of K_5

4.2. Relation Between Thickness and K-PCB Model:

A graph can be embedded in the K-PCB model without contact cuts if and only if it has thickness at most K.

Proof. It is clear that the number of layers needed to embed a graph G is at least its thickness, and a simple homotopy argument proves the opposite direction.

5. Conclusion:

In this paper we present result have concerning the thickness and K-PCB model. In particular for every K-PCB model in some sense free contact cuts for each edge at its endpoint since each edge can connect to its endpoint nodes on any layer.

6. Open problem:

Whether the use of K-PCB model can applied complete bipartite $k_{3,3}$?

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