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Interest Rate Risk of Zero-coupon Bond Prices on Bombay Stock Exchange (BSE) – Empirical Test of the Duration, Modified Duration, Convexity and Immunization Risk

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Abstract: Duration and convexity are important measures in fixed-income portfolio management and help develop methodologies in interest rate risk management .This article presents empirical test of duration and convexity of Zero-Coupon Bonds(ZCBs)at BSE in order to determine sensitivity of ZCBs prices on interest rate changes. The sensitivity of ZCBs in BSE on interest rate changes is tested and determined that convexity is more accurate measure as approximation of ZCBs prices changes than duration. The empirical results provide evidence that first duration is an increasing function of the interest rate and next there is no relationship between convexity and interest rate. The estimated percentage changes in ZCBs price using *duration* decrease by raising the percentage change in interest rate and we have non-parallel shift in lines for different level of duration. By non-parallel shifting of duration *downward,* the percentage change using only duration decreases and indicates higher negative difference and hence higher sensitivity at higher duration levels. As the interest rate increases the percentage change in ZCBs price using *duration and convexity* increases and again we have non-parallel shift in lines for different level of duration. By non-parallel shifting of duration *upward,* the percentage change using both duration and convexity increases and indicates higher *positive* difference and hence higher sensitivity at higher duration levels and by non-parallel shifting of duration *downward* , the percentage change using both duration and convexity decreases and indicates higher *negative* value and hence higher sensitivity at shorter duration levels. Also it has tested empirically whether convexity is return enhancing or return diminishing. Results of empirical tests over time periods under consideration show ZCBs convexity to be either significantly or negatively related to ex ante ZCBs returns. Further, the magnitude of ZCBs convexity is shown to be related indirectly and significantly to the immunization risk inherent in a bond portfolio. The main goal of this study is to determine how non-linear estimation models fit in case of ZCBs that are traded on BSE and to verify whether they offer reliable results compared with linear regression model on BSE. Also the most relevant contribution of the paper is to obtain a better curve estimation during the time period of March, 2009 -June, 2012 for duration and convexity exposures that contribute to the marginal increment of the coefficient of determination and the construction of a best nonlinear regression model that overcomes the linearity models.

Key words: Duration, convexity, corporate bonds, interest rate risk

INTRODUCTION

The price of any financial instrument is equal to the present value of the expected cash flow from the financial instrument (Damodaran, 2008). In order to determine intrinsic value of the bond we need to estimate expected cash flows and appropriate required rate of return (yield). The expected cash flows are determined from bond characteristics or bond contract. The required rate of return (yield) reflects the yield for financial instruments with comparable risk, or alternative investments (Brealey, 2006). Bond as a debt instrument requires from the issuer (debtor or borrower) to repay to the lender/investor the amount borrowed (principal) plus interest over a specified period of time. A key feature of a bond is the nature of the issuer, which is usually divided in three groups: government, municipalities and corporations (domestic and foreign).

The price of a bond is a function of the promised payments and the market-required rate of return. Since the promised payments are fixed, bond prices change in response to changes in the market determined required rate of return. For investors who hold bonds, the issue of how sensitive a bond's price is to changes in the required rate of return is important.

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There are four measures of bond price sensitivity that are commonly used: simple maturity, Macaulay duration (effective maturity), modified duration, and convexity. Each of these measures provides a more exact description of how a bond price changes relative to changes in the required rate of return (Conroy, 2008). Bond Investors care about the sensitivity of bonds prices to the changes in the required rate of return (Yield to Maturity). They use different measures such as Modified Duration and Convexity to capture the relationship between bond prices and yield to maturity. Both measures are used to calculate PercentageΔ in bond prices but with different accuracy (Fayad, 2011). Duration and convexity are important measurement tools for use in valuation and portfolio management strategies (Sarkar, 1999). As such, they are an integral part of the financial services landscape. The concepts of duration and convexity are commonly used in the field of asset-liability management, They are important because they provide key measures of sensitivity of the price of a financial instrument to changes in interest rates and they help develop methodologies in interest rate risk management(Gajek *et al* ,2005).

A financial institution or a nonfinancial firm may face market risk due to unexpected movements in interest rates. Such market risk arises as a result of positions in fixed income securities taken by traders, portfolio managers or financial managers. This risk may arise because rates move opposite of the forecast on which an active strategy is based (Fooladi *et al*, 2000).

Duration and convexity functions are available in numerous financial management software packages and through Microsoft excel. In the pre-computer days of Macaulay, Duration was conceived as a short-hand method of estimating price volatility as the result of changes in market yield. Today, the value of Duration is somewhat less evident, since computer-pricing programs are widely available which can indicate precisely the value of a bond with respect to all the important financial variables: coupon, yield and time. Still, Duration can be used by the bond investor to implement his investment strategy. If the investor believes that market yields are going to decline, he may wish to alter his bond mix to include bonds carrying higher Durations in order to leverage the increase in bond value. If an increase in yields is expected, he may elect to change the mix to include bonds of lower Duration to minimize the negative effect on his portfolio. Obviously, bonds are subject to risk beyond changes in the coupon-yield-maturity variables, e.g. the risk of default, but Duration is not intended to reflect risk; it measures interest rate sensitivity. (Fayad, 2011).

A well-developed capital market comprises deep and liquid equity and bond markets. While the equity capital market in India has grown manifold aided by progressive reforms, world-class infrastructure and a wide range of market participants, the corporate bond market in India is still relatively underdeveloped (Patil *et al*, 2010).The Corporate Debt Market in India is in its infancy, both in terms of microstructure as well as market outcomes. Primary issuance market is dominated by non-banking finance companies and relatively small amount of funds are raised through issuance of debt papers by manufacturing and other service industries. Bank finance is the most sought after path to fulfill the funding requirement of these companies. Secondary market activities in corporate bonds have not picked up. Efforts of Securities Exchange Board of India (SEBI) and the stock exchanges to bring the trading to stock exchange platforms have not yielded desired results. The Indian financial system is not well developed and diversified. One major missing element is an active, liquid, and large debt market. Historically, the most of corporate entities have been depending on loans from banks and institutions and they have not shown much interest to raise even a small part of the required long term resources from the market through bonds and other debt instruments. The corporate units which usually raise funds through public deposits also did not show much interest in issuing bonds although they could possibly raise more money through market borrowings than through public deposits. During the last decade significant quantum changes have taken place in the quality of the equity market. In terms of efficiency and transparency it is now ranked among one of the best markets globally. In contrast the corporate debt market continues to remain in a highly undeveloped state (FCCI team, 2005).

So all above discussions are why to choose this study in order to examine the Indian bond market especially in case of Zero-Coupon bond with more detailed. This article is analyzed Indian Zero-Coupon Bonds Debt Market (ICDM) (Zero-Coupon Bonds) that are quoted and traded on Bombay Stock Exchange (BSE) and conducted on historical data on Indian Zero-Coupon Bonds supplied by Bombay Stock Exchange (BSE) of India. The main objectives of this article are whether the Zero-Coupon Bond value P is sensitive on changes in the interest rate y, if estimation error of bond value P using both Duration and Convexity significantly reduces the estimation error compared to the case when only Duration is used. All sample Zero-Coupon Bonds are amortization bonds (with annuity payment of interest and principal) with different year's maturity between 1 and 20.It is made empirical test of duration, modified duration and convexity of Zero-Coupon Bond at BSE in order to determine sensitivity of Zero-Coupon Bond prices on interest rate changes and dependence of estimation errors using Duration and Convexity on changes in the interest rate in order to determine the measure that is better for Zero-Coupon Bond prices forecasting in BSE. It is analyzed annual data that covered the 2005-2012 sample periods.

The first section presents the mathematical demonstration of interest rate risk measures such as percentage change in bond price, duration and convexity in case of Zero-Coupon Bond. In section 2 is mentioned data to be collected and this demonstration is supplemented in Section 3 with an empirical analysis of the relationship between Zero-Coupon Bond prices and interest rate changes applying regression curve estimation in order finding how sensitive is Zero-Coupon Bond prices on interest rate changes. It is used different measures such as Modified Duration and Convexity to capture the relationship between bond prices and yield to maturity and then determined dependence of estimation errors of duration and convexity on the change in the interest rate .The consequences would be compared to find more accurate measure as approximation of Zero-Coupon Bond prices changes on interest rate changes. The final section presents the conclusions.

2. Background:

The value of an asset is the present value of the cash flows expected from the asset. A standard bond ("a vanilla bond") pays periodic coupons, normally one per semester, and at the time of maturity it returns the nominal value of the bond. Therefore, the value of a bond would be:

$$P(y) = \sum_{t=1}^{n} \frac{C_t}{(1+y)^t} + \frac{M}{(1+y)^n} = \frac{C_t}{y} \times \left(1 - \frac{1}{(1+y)^n}\right) + \frac{M}{(1+y)^n}$$

Where:

Ct = Payment at time *t*

y = Discount rate per period

M = Nominal value of the bond

n= Number of periods of the bond (Maturity)

t = period

If we define wt as the per unit weight of the total value, corresponding to the payment in period t, then the "Macauley Duration" D can be obtained through:

$$D = \sum_{t=1}^{N} t w_t$$

Modified Duration is:

$$ModD = \frac{-D}{1+y}$$

Modified Duration is important because it is the first derivative of the value of the bond *P*, as a function of the interest rate *y*.Therefore, knowing the Modified Duration, the variation of the value of the bond can be estimated for a small variation of the interest rate.

$\Delta P \cong ModD\Delta y$

Due to the convexity of the curve, for large variations of the interest rate the Modified Duration is not a good estimate. The convexity is a correction to estimate more accurately the value of the bond when large variations of the interest rate occur.

$$Convexity = C = \frac{\sum_{t=1}^{n}(t+t^2)w_t}{(1+y)^2}$$

Using a Taylor Series expansion, for a variation of the interest rate, the variation in the value of the bond can be estimated from:

$$\Delta P \cong (-ModD\Delta y) + \frac{C(\Delta y)^2}{2}$$

2.1 Zero-Coupon Bond:

The value of a zero coupon bond is:

$$P(y) = \frac{M}{(1+y)^n}$$

The Macaulay Duration of a zero-coupon bond is equal to its time to maturity. The convexity of a zero-coupon bond is then:

$$C = \frac{D + D^2}{(1+y)^2} = \frac{D(1+D)}{(1+y)^2}$$

2.2 Percentage change in Bond Price for n years by Fayad's Formula:

Fayad (2011) presents a geometric series used for annual coupon payment. Using the geometric series equations, the sum of the n terms of geometric series is

$$\Delta P\%(M=n)=\frac{-\Delta y}{(1+y_2)}+\frac{-\Delta y}{(1+y_2)^2}+\frac{-\Delta y}{(1+y_2)^3}+\cdots+\frac{-\Delta y}{(1+y_2)^n}$$

Therefore the sum of the above geometric series equals to:

$$\Delta P\%(M=n)=\frac{-\Delta y}{y_2}\left[1-\frac{1}{(1+y_2)^n}\right]$$

Where M = Bond's Maturity, CR =Coupon Rate, y_1 = required rate of return when the company issues the bond (equals to CR initially), y_2=New Required rate of return after t periods since issuance.

2.3 Some Previous studies:

Sarkar(1999) have derived closed-form expressions for duration and convexity of zero-coupon and were found both measures to be very different from those of straight bonds, in magnitude and in their response to parameter changes and duration is generally a decreasing function of the interest rate (except for the case of short maturity bonds) and for short maturity bonds, however, the convexity is an increasing function of interest rate.

Any bond, or a portfolio of bonds, can be synthesized with its yield to maturity, duration, and convexity. Llano-Ferro(2009) derives the parameters of the two zero-coupon bonds to model a bond, or bond portfolio. Then a comparison is made of different methods to estimate the variation of a bond value, under interest rate, and time to maturity variations and found that a single zero-coupon bond, even though it does not replicate the convexity of a vanilla bond, or a bond portfolio, is a simple and accurate model to estimate bond values under interest rate and time to maturity changes.

Sotos (2004) investigate risky-prices sensitivity to interest rate changes in the Spanish market and to see if sensitivity is lower than public debt. To contrast this hypothesis, this paper presents a model that analyzes the risky-prices sensitivity to interest rate changes through effective duration and convexity. This study is a great improvement to the usual methodology on the analysis of the risky fixed income because it shows that it is possible to change the volatility over time which introduces the non-linearity existence in the variance of the risky assets. Crack and Nawalkha (2007) represented the slope of the bond price-yield plot does not define bond duration, and the curvature of the bond price-yield plot does not define bond convexity. They demonstrate several common misunderstandings regarding duration and convexity, and offer a new bond return-yield plot for illustrating the roles of duration and convexity. In this article we use this approach to calculate the duration and convexity properly. Conroy (2008) demonstrated the price of a bond is a function of the promised payments and the market required rate of return, Because the promised payments are fixed, bond prices change in response to changes in the market required rate of return Fayad (2010) found Although Convexity provides a better measure than Modified Duration, still both measures viz., duration and convexity are complex and do not capture the true relationship and presented a new formula to measure actual Percentage∆ in bond price when YTM changes for this formula is much simpler than other measures and provides accurate results. In addition, the new formula can be applied to calculate directly modified duration, duration and new price of the bond without using complex formula of each.

Fonseca (2011) shows that convexity effect on the rates of return on bonds is not completely eliminated using empirical data. This empirical analysis was conducted on a database of European government bond indexes supplied by EuroMTS. Daily data for the period between 1 July 2003 and 30 June 2010 was used, which amounts to 1796 daily observation for each index. Each MTS bond index is defined by index value, coupon, maturity and yield to-maturity. The empirical analysis also shows that the influence of convexity on bond returns depends on the magnitude of interest rate changes. Ivanovski *et al* (2012) present valuation of Treasury Bonds (T-Bonds) on Macedonian Stock Exchange (MSE) and empirical test of duration, modified duration and convexity of the T-bonds at MSE in order to determine sensitivity of bonds prices on interest rate changes and determine how standard valuation models fit in case of T- Bonds that are traded on MSE and verify whether they offer reliable results compared with average bonds prices on MSE. They test the sensitivity of T- Bonds on MSE on interest rate changes and determine that convexity is more accurate measure as approximation of bond prices changes than duration.

Maleki Nia *et al* (2012) provided evidence that first duration is an increasing function of the interest rate and next the convexity is an increasing function of interest rate for short maturities and duration. The estimated percentage changes in ZCBs price using *duration* decrease by raising the percentage change in interest rate and we have non-parallel shift in lines for different level of duration. By non-parallel shifting of duration *downward* the percentage change increases and indicates higher difference and hence higher sensitivity at higher duration levels. As the interest rate increases the percentage change in ZCBs price using *duration and convexity* increases and again we have non-parallel shift in lines for different level of duration. By non-parallel shifting of duration *upward* the percentage change increases and indicates higher difference and hence higher sensitivity at higher duration levels. It has tested empirically whether convexity is return enhancing or return diminishing. Results of empirical tests over time periods under consideration show ZCBs convexity to be either insignificantly or negatively related to ex ante ZCBs returns. Further, the magnitude of ZCBs convexity is shown to be related indirectly and insignificantly to the immunization risk inherent in a bond portfolio.

3. Data:

Zero-coupon bond prices are obtained from the daily trade reported on Indian Corporate Bonds Trading Data from debt market of Bombay Stock Exchange (BSE). These Zero-coupon bonds are traded on the Exchange and all reports are closed reports. Our period of examination extends from January 2009 through June 2012; the entire period is divided into two-month sub periods.The first sample period includes March, 2009 and April, 2010, the second period includes April, 2010 and May, 2011 and the third period includes May, 2011 and June, 2012. Excluded are Zero-coupon bonds have missing data for calculating Zero-coupon bond exposures. So 6405 daily observation of all bond trades volume are included in our sample over the main period divided into three sub periods including 5522 trades volume in the first one, 400 trades volume in the second one and 483 trades volume in the last one and then all data required of Zero-coupon bonds extracted from this list. Zero-coupon bond list are illustrated in table 1:

4. Empirical test of the Duration, Mod-Duration and Convexity:

After selecting the sample data of Zero-Coupon bonds (ZCBs) with different maturities ranging from 1 to 15 the required parameters including yield to maturity, duration, Modified Duration and convexity are calculated and then percentage change of ZCBs price are estimated using only duration and also using both duration and convexity during the time period of investigation March, 2009 and April, 2010 in order to study the sensitivity of ZCBs prices to interest rate risk.

4.1 Actual Relationship Between Percentage Changes In Zcbs Price And Percentage Change In Interest Rates:

The relationship between interest rates and prices in the markets of Zero-coupon income are plotted in Figure 1and shows this relationship is as significant as Expected and as the interest rate increases the price changes decreases, (the coefficients value of determination would be shown in Table 4 and 5).

We spilt this relationship to Zero-Coupon Bond prices and yield to maturity time series during the time period under consideration.

As shown in Figure 2 the Zero-Coupon Bond prices decrease as moving from 2009 toward the 2012 and as shown in Figure 3 the Zero-Coupon Bond yield to maturity increases as moving from 2009 toward the 2012 and prove the inverse relationship between Zero-Coupon Bond prices and yield to maturity.

4.2 Non-Parallel Shift In Estimated Percentage Changes In Zcbs Price Using Duration:

The percentage changes in Zero –Coupon Bonds price using duration for different maturities between 1 and 15 and interest rate ranging from 0% to 0.2% are constructed. Either a small percentage changes in Zero –Coupon Bonds price or a large percentage changes in Zero –Coupon bonds price that deviates from one indicates a significant duration impact that causes Zero –Coupon Bonds to be under-priced or over-priced respectively. The results are plotted in Figure 4 with each line representing a different maturity, the bottom line being 15 years maturity and the top line corresponding to one year.

The experiment demonstrates the important role of maturity plays in the pricing of Zero –Coupon bonds. The Zero –Coupon bonds percentage change would be increased as the maturity and the interest rate increases. This result stems from longer streams of maturities and higher duration producing larger sensitivity of bonds price in interest rate change.

The sensitivity of Zero –Coupon bond price is larger when higher duration are considered with a D=15 because of steeper slope. Thus, the duration impact is economically significant and the estimated percentage change in Zero-coupon bond price by duration decrease by raising the percentage change in interest rate (except for D=1) because of the inverse relationship between these two variables as stated earlier. As a result it can be concluded percentage changes in Zero-Coupon Bonds by duration is generally a decreasing function of the interest rate.

Another important result is the impact of duration level by shifting downward on the relationship between percentage changes in Zero –Coupon bonds price and the percentage change in interest rate. The slope of each line for various duration levels is steeper for higher duration (D =15) and higher time to maturity, on the other hand duration is an increasing function of the interest rate Figure 5 and implies as the interest rate rises the duration rises (R^2 = 0.437) by the cubic regression model in Table 2) and percentage change in Zero –Coupon bonds decrease (R^2 = 0.643). In the same value of interest rate the Zero-Coupon Bonds with higher duration have higher percentage change in price and in the same value of percentage change in price; Zero-Coupon bonds with shorter duration have higher interest rate.

4.3 Non-Parallel Shift In Estimated Percentage Changes In Zcbs Price Using Duration And Convexity:

The percentage changes in Zero –Coupon Bonds price using duration and convexity for different maturities between 1 and 10 and interest rate ranging from 0% to 0.7% are constructed. The convexity of Zero-Coupon bonds is computed by convexity calculator for each bond during the period of time under consideration. A convexity could be negative, particularly for highly-leveraged firms (i.e., with high quasi-debt ratios). The results are plotted in Figure 6 with each line representing a different duration, the bottom line being one year maturity and the top line corresponding to 15 years.

The experiment demonstrates the important role of maturity and convexity plays in the pricing of Zero –Coupon bonds .The relationship between percentage changes in Zero –Coupon Bonds price using duration and convexity with interest rate changes are briefly summarized below.

As the interest rate increases the percentage change in Zero –coupon Bond price increases for D = 12,13,14 and 15 by shifting the lines upward that show higher duration, the percentage change increases and indicates higher positive difference and hence higher sensitivity. As the interest rate increases the percentage change in Zero –coupon Bond price decreases for D = 1, 7, 9 and 10 by shifting the lines downward that show higher duration, the percentage change decreases and indicates higher negative difference and hence higher sensitivity Another important result is to change the direction of relationship by using convexity move from negative to the positive one compared to the case as shown in Figure 4 by duration. In the same value of interest rate the Zero-Coupon Bonds have higher percentage change in price with higher duration in the positive region for long term ZCBs and on the other side have shorter duration in the negative region for short term ZCBs and in the same value of percentage change in price; Zero-Coupon bonds with shorter duration have higher interest rate.

As you see in Figure 7 the relationship between interest rate and convexity is not significant with R^2 =0.116 by cubic regression model as best curve estimation illustrated in Table 3.

4.4 Sensitivity Of Zcbs Bond Value On Changes In The Interest Rate:

As the pervious graphs show, due to the convexity of the curve, for large variations of the interest rate the Modified Duration is not a good estimate, while it is estimated the percentage change in price fairly well. The Modified Duration overestimates the percentage change in price for large variations of the interest rate and the convexity is a correction to estimate more accurately the value of the bond when large variations of the interest rate occur.

The introduction of convexity to initial model supposes an improvement over non-linear regression estimation without convexity terms because of higher R^2 (0.950) as illustrated in Table 4 .The empirical results also provide support of the existence of a non-linear relationship between interest rate risk and prices of the bonds. The use of the models quadratic and cubic implies an improvement in the price-yields models which introduces the non-linearity existence in the variance. As shown in second panel of Table 4 the marginal increment of the coefficient of determination is improved by applying the cubic model.

As stated earlier the non-linear regression is applied for estimating changes in bond values on changes in interest rate. The results of both quadratic and cubic models are shown in Table 5:

As a result the sensitivity of bond value *P* on changes in the interest rate *y* has calculated in a wide range of values for *y* between 0% and 0.14% as shown in Table 6 applying the following estimated cube regression curves as the best estimators because of higher marginal increment of R^2 in comparison with quadratic regression:

$$\%\,\Delta\, p = -6.095\Delta y - 34.022\Delta y^2 - 52.645\Delta y^3 - 0.172$$

$$\%\,\Delta P_D = -10.305\Delta y - 29.822\Delta y^2 - 353.789\Delta y^3 - 0.454$$

$$\%\,\Delta P_C = -6.715\Delta y - 18.351\Delta y^2 - 214.962\Delta y^3 - 0.279$$

Figure 8 gives the sensitivity of bond value *P* on changes in the interest rate *y*. Interest rate *y* is given on the *x*-axis, while ΔP is given on the *y* axis. First, % Δ *p* is calculated using the Fayyad's new formula and then it is estimated using duration denoted by % ΔP_D, and using both duration and convexity denoted by % ΔP_C. In a wide range of values for *y* between -0.1% and 0.15%, true change in bond value (circles) can be closely estimated using Duration (diamonds), and using Duration and Convexity (triangles).Dotted line with triangles almost completely is closer to bold line with circles since the true change in bond value can be estimated with high precision using Duration and Convexity. Clearly, bond value *P* using both Duration and Convexity significantly are more sensitive to changes in the interest rate *y* because of higher R^2 (0.950) compared to the case when only Duration is used with R^2 (0.671).

4.5 Dependence Of Estimation Errors Of Duration And Convexity On The Change In The Interest Rate of ZCBs:

The dependence between estimation errors of duration and convexity on the change in the interest rate in the markets of Zero-Coupon bonds is as significant as Expected, which along with the coefficients value of determination as shown in Table 7, and the introduction of convexity to initial model suppose an improvement over non-linear regression estimation without convexity terms because of higher R^2 (0.844). The empirical results also provide support of the existence of a non-linear relationship between estimation errors of duration and convexity with the change in the interest rate. The use of the models quadratic and cubic implies an improvement in the price-yields models which introduces the non-linearity existence in the variance. As shown in second panel of Table 7 the marginal increment of the coefficient of determination is improved by applying the cubic model.

So dependence of estimation errors of % ΔP_D and % ΔP_C on the change in the interest rate is estimated by nonlinear regressions as shown in Table 8. The quadratic and cubic models are applied to estimate best nonlinear regression curve.

The results of curve regression estimation indicates the cubic model is much better estimator than the quadratic one because of higher $R^2$and higher beta weights and also lower significant level. As a result the estimation errors has been calculated in a wide range of values for y between -0.1% and 0.4% as shown in Table 9 applying the following estimated cube regression curves:

$$Estimation\ error\ of\ \ Duration = 0.620\Delta y + 15.718\Delta y^2 - 162.265\Delta y^3 + 0.106$$

$$Estimation\ error\ of\ \ Convexity = -3.601\Delta y - 11.466\Delta y^2 + 139.064\Delta y^3 - 0.176$$

Figure 9 depicts how correct the estimation is. Dependence of estimation errors% ΔP_D and% ΔP_C on the change in the interest rate is shown Clearly, estimation of bond value P using both Duration and Convexity with R^2 = 0.844 (bold line with circles) significantly reduces the estimation error compared to the case when only Duration with R^2 = 0.299 (dotted one with triangles) is used.

4.6 On the Relationship of Immunization Risk and Return with Convexity:

The empirical tests of immunization risk require the construction of bond portfolios from the observed return data. Over each two-month period, portfolios are constructed to have equal durations ranging 7, 9 , 10, 12 , 13 , 14 and 15 (i.e., to be immunized with respect to duration) but different amounts of convexity exposure varying the desired convexity levels (i.e., 47.81, 6.6…, 201.57), so five different portfolios are formed. Each convexity level therefore defines a different bond portfolio.

We then compute for each of the five portfolios the portfolio's standard deviation of returns, where returns are defined as the portfolio's duration-immunized return by the following formula:

$$R_i = (\frac{C_t}{P(y)} \times 100) + \frac{P(y) - M}{n}$$

C_t = Coupon payment
$P(y)$ = Bond price
M = Maturity value
n = maturity date

For ZCBs the first term of the above equation is Zero and we have calculated the second term which is known as portfolio's return. The standard deviations are reported in Table 10, with the relationship between portfolio convexity and standard deviation shown graphically in Figure 10. In general, high positive-convexity is not associated with positive excess returns as you see in Table 10 for D=15 we have portfolio with R_i= - 4.88 shows a negative one; a conclusion is that increasing the level of positive convexity doesn't increase the return on a ZCBs bond portfolio.

Table 1: Summary of companies, issue and maturity date under consideration from BSE

Security	Issuer Name
0ICICIHF13	ICICI HOME FINANCE
0PFC22	POWER FINANCE
0MMFSL13A	MAHINDRA
0RRVPNNL22	RAJASTHAN RAJYA VIDYUT PRASARAN NIGAM
0RRVPNL27	RAJASTHAN RAJYA VIDYUT PRASARAN NIGAM
0RRVPNL24	RAJASTHAN RAJYA VIDYUT PRASARAN NIGAM
0RRVPNL2025	RAJASTHAN RAJYA VIDYUT PRASARAN NIGAM
0NABARD19	NATIONAL BANK FOR AGRICULTURE
0KMPL2012	KOTAK MAHINDRA PRIME
0RRVPNL26	RAJASTHAN RAJYA VIDYUT PRASARAN NIGAM
0RRVPNL27	RAJASTHAN RAJYA VIDYUT PRASARAN NIGAM
0RRVPNL21	RAJASTHAN RAJYA VIDYUT PRASARAN NIGAM
0KMPL12J	KOTAK MAHINDRA PRIME
0TATACAP13	TATA CAPITAL
0NABARD19	NATIONAL BANK FOR AGRICULTURE
0MMFSL12	MAHINRA AND MAHINDRA FINANCE
0INDIABULLS13	INDIABULLS FINANCIAL SERVICES
0TATACAPL13	TATA CAPITAL
0MANFL2012	MANAPPURAM FINANCE
0HDFC12C	HOUSING DEVELOPMENT FINANCE CORPORATION

Table 2: R^2 and the marginal increment of R^2 for correlation between Duration and interest rate of ZCBs

row	variables	Linear regression	quadratic model	cubic model
1	R^2	0.179	0.2	0.437
2	the marginal increment of R^2	-	0.021	0.258

Table 3: R^2 and the marginal increment of R^2 for correlation between Convexity and interest rate of ZCBs

row	variables	Linear regression	quadratic model	cubic model
1	R^2	0.003	0.11	0.116
2	the marginal increment of R^2	-	0.107	0.113

Table 4: R^2 and marginal increment of R^2 of ZCBs for nonlinear regressions compared to the linear regression

Row	variables	R^2		
		Linear regression	quadratic model	cubic model
1	% Δp	0.421	0.441	0.643
2	% ΔP_D	0.292	0.310	0.671
3	% ΔP_C	0.779	0.944	0.950
Row	variables	The marginal increment of R^2		
		Linear regression	quadratic model	cubic model
1	% Δp	-	0.02	0.222
2	% ΔP_D	-	0.018	0.379
3	% ΔP_C	-	0.165	0.171

Table 5: Sensitivity of bond value on changes in the interest rate of ZCBs applying nonlinear regression

row	variables	% Δp	
		quadratic model	cubic model
1	R^2	0.441	0.643
2	p-value(ANOVA)	0.000	0.000
3	η_1	-2.744	-6.715
4	η_2	7.276	18.351
5	η_3		214.962
6	*constant*	-0.347	-0.279
row	variables	% ΔP_D	
		quadratic model	cubic model
1	R^2	0.310	0.671
2	p-value(ANOVA)	0.000	0.000
3	η_1	-3.769	-10.305
4	η_2	11.593	29.822
5	η_3	-	353.789
6	*constant*	-0.566	-0.454
row	variables	% ΔP_C	
		quadratic model	cubic model
1	R^2	0.944	0.950
2	p-value(ANOVA)	0.000	0.000
3	η_1	-5.123	-6.095
4	η_2	31.310	34.022
5	η_3	-	52.645
6	*constant*	-0.189	-0.172

Table 6: Sensitivity of ZCBs value *P* on changes in the interest rate *y*

Row	Δy	% Δp	% ΔP_D	% ΔP_C
1	0.00	-0.28	-0.45	-0.17
2	0.02	-0.40	-0.65	-0.28
3	0.04	-0.50	-0.80	-0.36
4	0.08	-0.59	-0.91	-0.41
5	0.10	-0.55	-0.83	-0.39
6	0.12	-0.45	-0.65	-0.32
7	0.14	-0.27	-0.34	-0.21

Table 7: R^2 and the marginal increment of R^2 for dependence of estimation errors on the change in the interest rate of ZCBs

Row	variables	R^2		
		Linear regression	quadratic model	cubic model
1	% ΔP_D	0.123	0.136	0.299
2	% ΔP_C	0.483	0.719	0.844
Row	variables	The marginal increment of R^2		
		Linear regression	quadratic model	cubic model
1	% ΔP_D	-	0.013	0.176
2	% ΔP_C	-	0.236	0.361

Table 8: Dependence of estimation errors of Duration and Convexity on the change in the interest rate of ZCBs applying nonlinear regression

row	variables	Duration	
		quadratic model	cubic model
1	R^2	0.136	0.299
2	p-value(ANOVA)	0.000	0.000
3	η_1	-1.031	-3.601
4	η_2	4.301	11.466
5	η_3		139.064
6	*constant*	-0.220	-0.176
		Convexity	
row	variables	quadratic model	cubic model
1	R^2	0.719	0.844
2	p-value(ANOVA)	0.000	0.000
3	η_1	-2.378	0.620
4	η_2	24.078	15.718
5	η_3	-	-162.265
6	*constant*	0.158	0.106

Table 9: Dependence of the estimation errors of ZCBs value *P* using Duration and Convexity on the change in the interest rate

row	Δy	duration	convexity
1	0.00	-0.18	0.11
2	0.02	-0.24	0.12
3	0.04	-0.29	0.15
4	0.08	-0.32	0.17
5	0.10	-0.28	0.16
6	0.12	-0.20	0.13
7	0.14	-0.07	0.06
8	0.16	0.11	-0.06
9	0.18	0.36	-0.22
10	0.20	0.67	-0.44
11	0.25	1.81	-1.29
12	0.30	3.53	-2.67
13	0.35	5.93	-4.71
14	0.40	9.12	-7.52

Table 10: Convexity and Immunization Risk and Returns Exposures of ZCBs portfolios for different durations

Portfolio Duration Exposure (D)	Portfolio Convexity Exposure (C)	Portfolio Immunization Risk (σ_i)	Portfolio Return (R_i)
D =7	47.81	0.053	-6.08
D =9	72.25	0.037	-6.16
D =10	91.67	0.105	-5.99
D =12	130.4	0.012	-5.5
D =13	152.36	0.017	-5.28
D =14	175.95	0.006	-5.08
D =15	201.57	0.012	-4.88

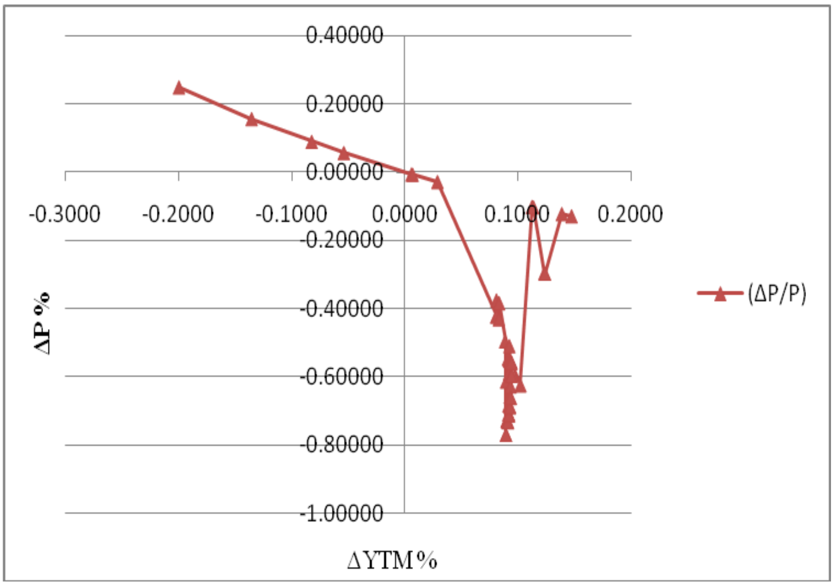

Fig. 1: Sensitivity of ZCBs price *P* changes in interest rate change *y*

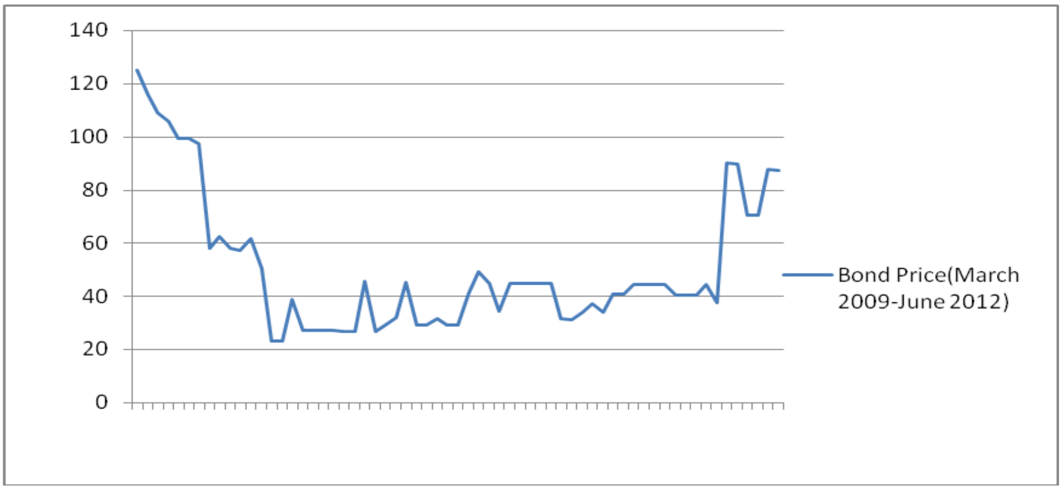

Fig. 2: ZCBs Price (March 2009 to April 2010)

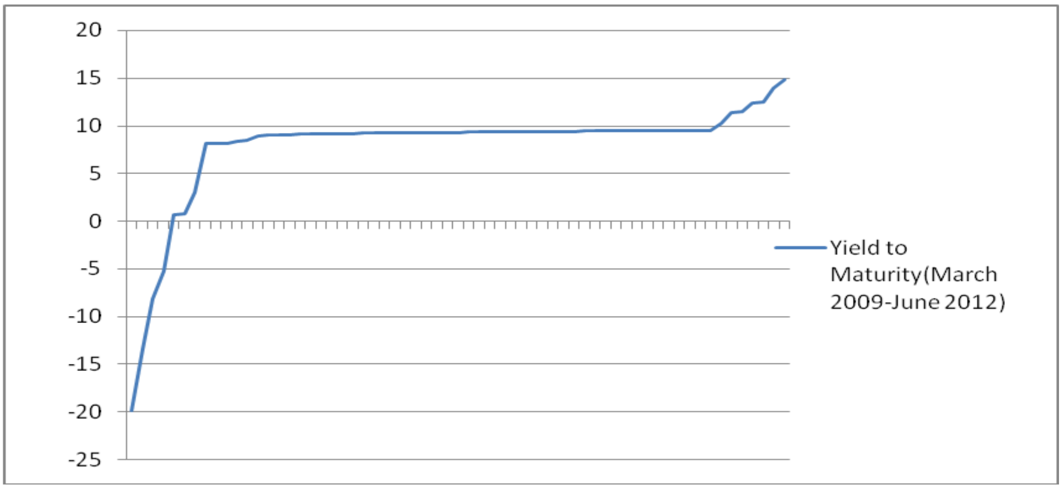

Fig. 3: ZCBs Yield to Maturity (March 2009 to April 2010)

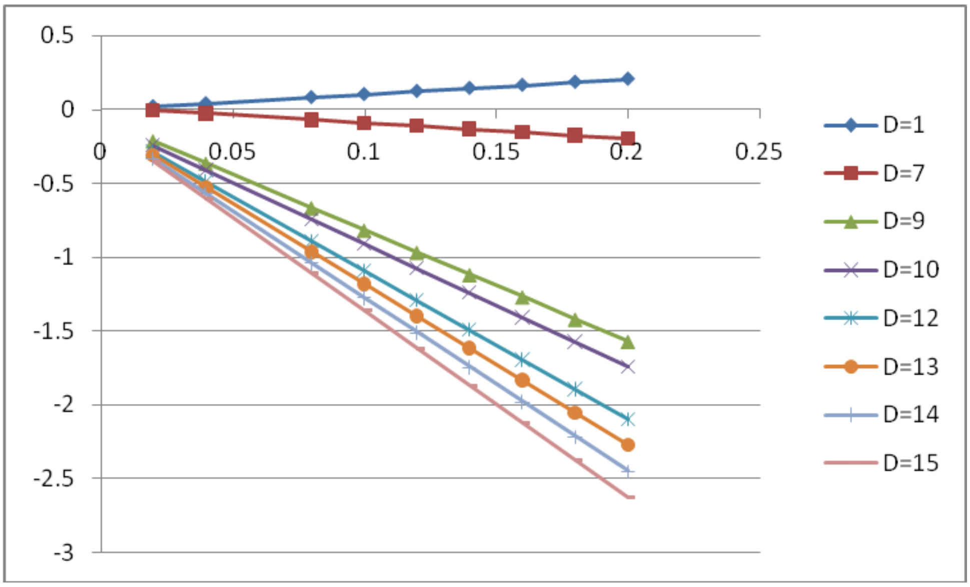

Fig. 4: Percentage changes in ZCBs price by duration for different Durations levels

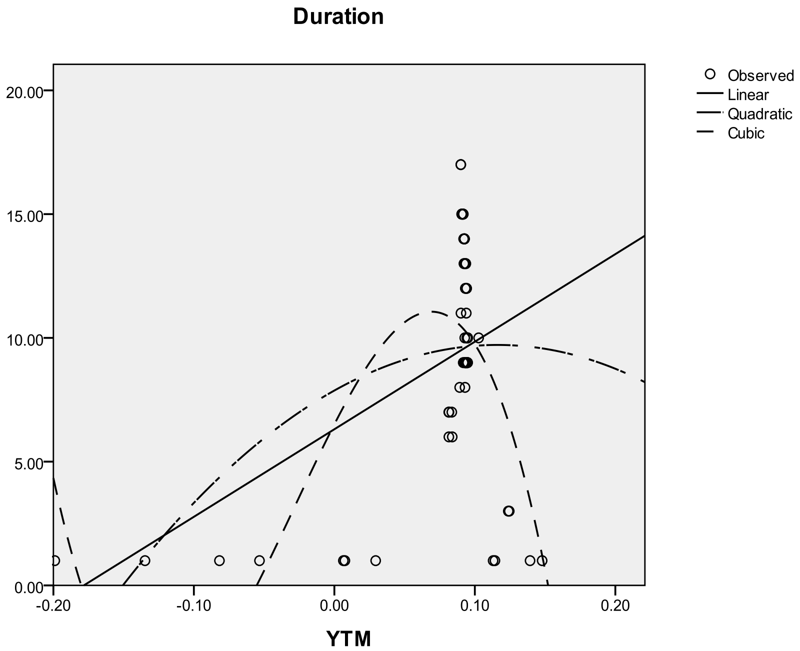

Fig. 5: The relationships between Duration and interest rate of ZCBs by linear and nonlinear regression

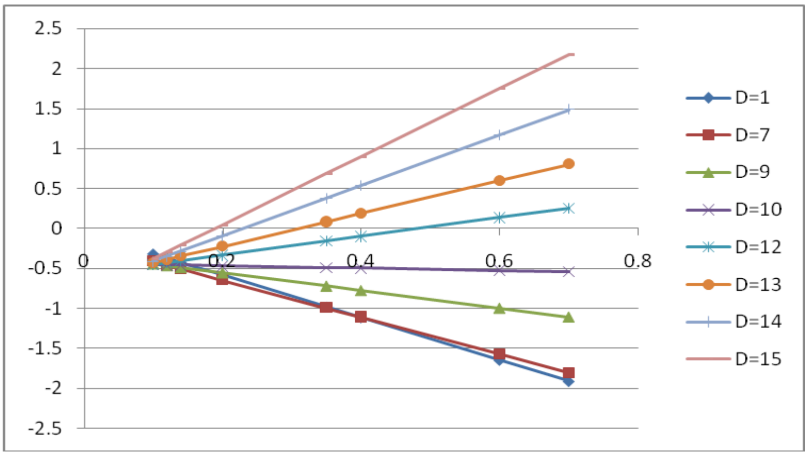

Fig. 6: Percentage changes in ZCBs price by both Duration and Convexity for different Durations levels

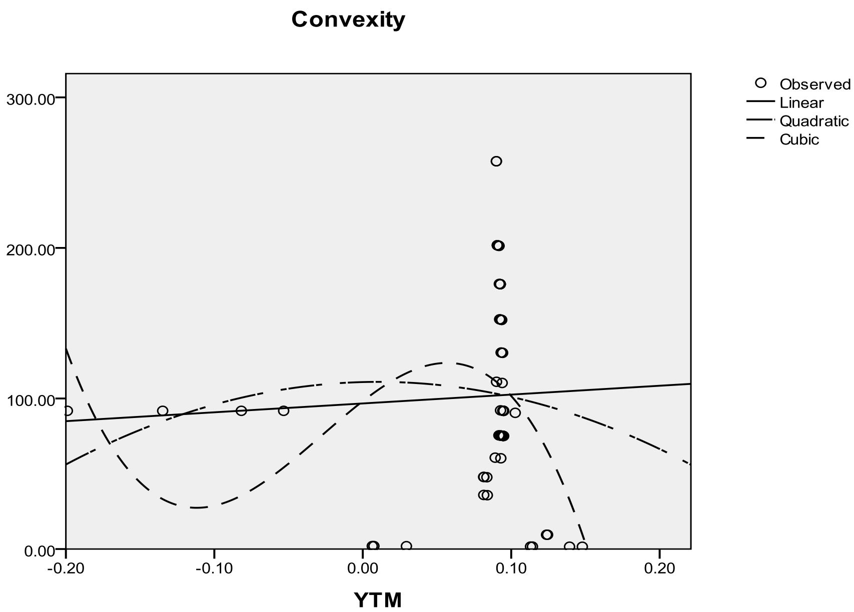

Fig. 7: The relationships between Convexity and interest rate of ZCBs by linear and nonlinear regression

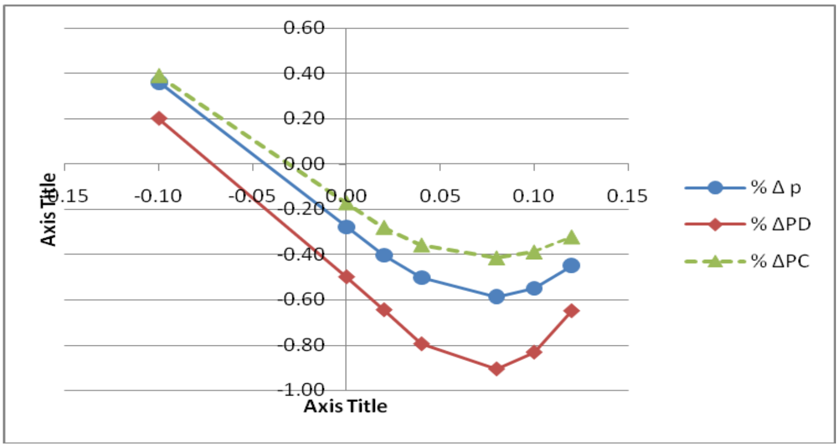

Fig. 8: Sensitivity of ZCBs value *P* on changes in the interest rate *y*

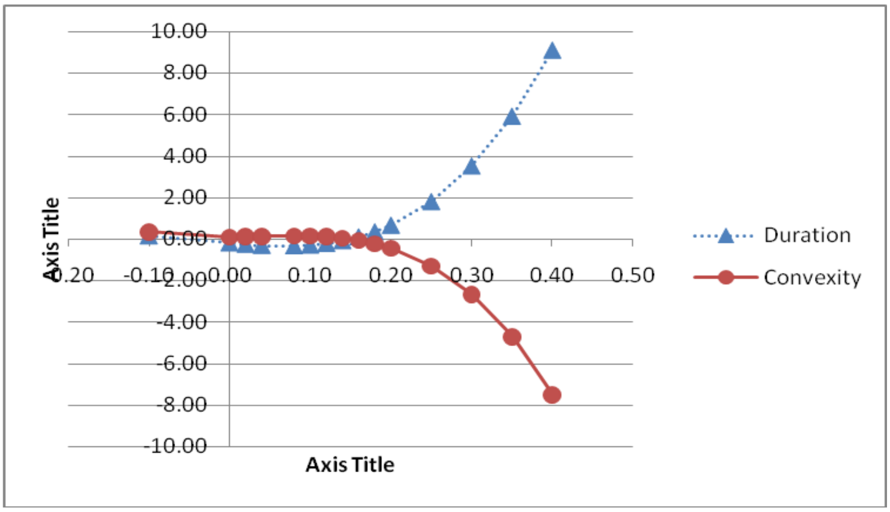

Fig. 9: Dependence of estimation errors of ZCBs on the change in the interest rate *y*

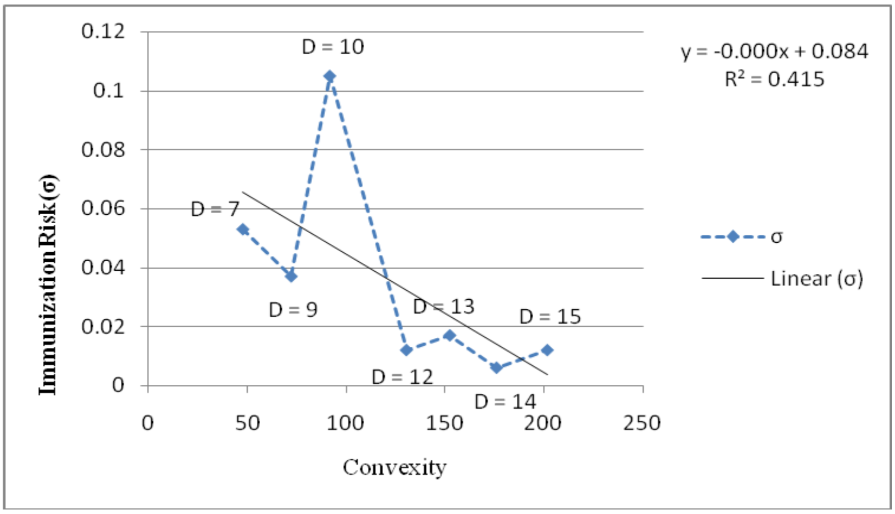

Fig. 10: The relationship between ZCBs Immunization Risk and convexity

A weak relationship is shown between portfolio convexity and immunization risk (R^2 = 0.4312), where risk is defined by the standard deviation of the portfolio's return. High convexity portfolios (positive) have the lowest risk, while low convexity portfolios (positive) have the highest risk. The degree of tilt between convexity and risk is as significant as expected. Thus, our results strongly support the view that the magnitude of convexity exposure decreases significantly immunization risk.

Conclusions:

An empirical test of duration, modified duration and convexity of the corporate bonds at BSE is conducted in order to determine sensitivity of Zero-Coupon bond prices on interest rate changes and tested the sensitivity of Zero-Coupon bonds on BSE on interest rate changes and determined the measure that is better for Zero-Coupon bond prices forecasting by calculating duration, modified duration and convexity.

We analyzed annual data that covered the January 2009 through June 2012 sample period. The negative and non-linear relationship between bond prices and interest rate is observed as significant as expected implies bond prices are inversely related to changes in market interest rates.

The empirical results provide evidence that first duration is an increasing function of the interest rate and implies as the interest rate raises the duration rises which is consistent with Sarkar study for short maturities and duration (1999) and next the relationship between convexity and interest rate is not significant.

The estimated percentage changes in Zero-coupon bond price using *duration* decrease by raising the percentage change in interest rate and we have non-parallel shift in lines for different level of duration. As the interest rate increases the percentage change in Zero –coupon Bond price using *duration and convexity* increases for long term maturities, decreases for short term maturities and again we have non-parallel shift in lines for different level of duration. By non-parallel shifting of duration upward the percentage change increases and indicates higher positive difference and hence higher sensitivity at higher duration levels (per duration is illustrated on separate line representing a different maturity) and By non-parallel shifting of duration downward the percentage change decreases and indicates higher negative value and hence higher sensitivity at shorter duration levels.

The empirical results provide also evidence that convexity is more accurate measure as approximation of Zero-Coupon bond prices changes than duration. We have tested empirically whether convexity is return enhancing or return diminishing. Results of empirical tests over time periods under consideration show ZCBs convexity to be either insignificantly or negatively related to ex ante ZCBs returns. Further, the magnitude of ZCBs convexity is shown to be related indirectly and significantly to the immunization risk inherent in a bond portfolio.

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