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# General Fault Admittance Method Solution of a Line-to-Line-to-Ground Fault

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**Abstract:** Line-to-line faults either involving or not involving ground are, in the classical approach, usually analysed using connection of symmetrical component sequence networks. For a line-to-line fault, the sequence networks are solved separately; the positive and negative sequence networks are then combined in parallel and solved to obtain the phase quantities. When the ground is involved, i.e. for a line-to-line-to-ground fault, all the three sequence networks are connected in parallel. In this classical approach, the solution proceeds by identifying the connection of the sequence networks at the fault point and then solving for the symmetrical component currents and voltages. These are then used to determine the symmetrical component voltages at the other bus bars and hence the symmetrical component currents in the lines. This method requires that the connection of the sequence networks must be known for the common fault types. However, a solution by the general method of fault admittance matrix does not require prior knowledge of how the sequence networks are connected. It is therefore more versatile than the classical methods. The paper presents a procedure for simulating the short circuit, which is a requirement for using the general fault admittance method. The results obtained are as accurate as those obtained using the classical approaches.

**Key words:** Unbalanced faults analysis, Line-to-line-to-ground fault, Fault admittance matrix, delta-earthed-star transformer.

## INTRODUCTION

The paper presents a method for solving the line-to-line-to-ground fault using the general fault admittance method. The general fault admittance method differs from the classical approaches based on symmetrical components in that it does not require prior knowledge of how the sequence components of currents and voltages are related. In the classical approach, knowledge of how the sequence components are related is required because the sequence networks must be connected in a prescribed way for a particular fault. Then the sequence currents and voltages at the fault are determined, after which symmetrical component currents and voltages in the rest of the network are calculated. Phase currents and voltages are found by transforming the respective symmetrical component values (Elgerd, O.I., 1971) (Sadat, H., 2004) (Das, J.C., 2002) (El-Hawary, M., 1995) (Zhu, J., 2004) (Wolter, M., Oswald, B.R., 2007) (Anderson, P.M., 1995) (Oswald, B.R., Panosyan, A., 2006). The fault admittance method is general in the sense that any fault impedances can be represented, providing the special case of a zero impedance fault is catered for. This paper discusses a procedure for simulating short circuits for the line-to-line-to-ground fault.

### *Background:*

A line-to-line-to-ground fault presents low value impedances; with zero value for a direct short circuits or metallic faults, between two phases and between the two phases and ground at the point of fault in the network. In general, a fault may be represented as shown in Figure 1.

In Figure 1, a fault at a bus bar is represented by fault admittances in each phase, i.e. the inverse of the fault impedance in the phase, and the admittance in the ground path. Note that the fault admittance for a short-circuited phase is represented by an infinite value, while that for an open-circuited phase is a zero value. In a line-to-line-to-ground fault, the fault is assumed to be between phases *b* and *c* and between them and ground. Thus for a line-to-line-to-ground fault the admittance $Y_{af}$ is zero while $Y_{bf}$, $Y_{cf}$ and $Y_{gf}$ are infinite.

A systematic approach for using a fault admittance matrix in the general fault admittance method is given in (Sakala J.D., Daka J.S.J., 2007). The method is based on the work in (Elgerd, O.I., 1971). The method is summarized in this paper to give the reader a comprehensive view of the methodology.

The general fault admittance matrix is given by:

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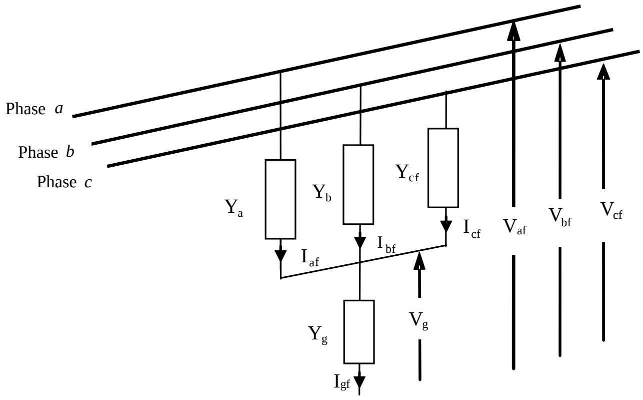


**Fig. 1:** General fault representation

$$Y_f = \left(\frac{1}{Y_{af} + Y_{bf} + Y_{cf} + Y_{gf}}\right) \times \begin{bmatrix} Y_{af}\left(Y_{bf} + Y_{cf} + Y_{gf}\right) & -Y_{af}Y_{bf} & -Y_{af}Y_{cf} \\ -Y_{af}Y_{bf} & Y_{bf}\left(Y_{af} + Y_{cf} + Y_{gf}\right) & -Y_{bf}Y_{cf} \\ -Y_{af}Y_{cf} & -Y_{bf}Y_{cf} & Y_{cf}\left(Y_{af} + Y_{bf} + Y_{gf}\right) \end{bmatrix} \quad (1)$$

Equation (1) is transformed using the symmetrical component transformation matrix *T*, and its inverse $T^{-1}$, where

$$T = \begin{bmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{bmatrix}$$

and

$$T^{-1} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix}$$

in which $\alpha = 1\angle 120^{\circ}$ is a complex operator.

The symmetrical component fault admittance matrix is given by the product

$Y_{fs} = T^{-1} Y_f T$ .

The general expression for $Y_{fs}$ is given in (Elgerd, O.I., 1971, Sakala J.D., Daka J.S.J., 2007) as

$$Y_{fs} = \frac{1}{Y_{af} + Y_{bf} + Y_{cf} + Y_{gf}} \begin{bmatrix} Y_{fs11} & Y_{fs12} & Y_{fs13} \\ Y_{fs21} & Y_{fs22} & Y_{fs23} \\ Y_{fs31} & Y_{fs32} & Y_{fs33} \end{bmatrix} \quad (2)$$

where

$$Y_{fs11} = Y_{fs22} = \frac{1}{3}Y_{gf}\left(Y_{af} + Y_{bf} + Y_{cf}\right) + Y_{af}Y_{bf} + Y_{af}Y_{cf} + Y_{bf}Y_{cf}$$

$$Y_{fs33} = \frac{1}{3}Y_{gf}\left(Y_{af} + Y_{bf} + Y_{cf}\right)$$

$$Y_{fs12} = \frac{1}{3}Y_{gf}\left(Y_{af} + \alpha^2 Y_{bf} + \alpha Y_{cf}\right) - \left(Y_{bf}Y_{cf} + \alpha Y_{af}Y_{bf} + \alpha^2 Y_{af}Y_{cf}\right)$$

$$Y_{fs21} = \frac{1}{3}Y_{gf}\left(Y_{af} + \alpha Y_{bf} + \alpha^2 Y_{cf}\right) - \left(Y_{bf}Y_{cf} + \alpha^2 Y_{af}Y_{bf} + \alpha Y_{af}Y_{cf}\right)$$

$$Y_{fs13} = Y_{fs32} = \frac{1}{3}Y_{gf}\left(Y_{af} + \alpha Y_{bf} + \alpha^2 Y_{cf}\right)$$

and

$$Y_{fs31} = Y_{fs23} = \frac{1}{3}Y_{gf}\left(Y_{af} + \alpha^2 Y_{bf} + \alpha Y_{cf}\right)$$

The above expressions simplify considerably depending on the type of fault. For example, considering a balanced three-phase fault with $Y_{af} = Y_{bf} = Y_{cf} = Y$:

$$Y_{fs} = \frac{1}{3Y + Y_{gf}} \begin{bmatrix} Y(3Y + Y_{gf}) & 0 & 0 \\ 0 & Y(3Y + Y_{gf}) & 0 \\ 0 & 0 & YY_{gf} \end{bmatrix}$$

$$= \begin{bmatrix} Y & 0 & 0 \\ 0 & Y & 0 \\ 0 & 0 & \dfrac{YY_{gf}}{3Y + Y_{gf}} \end{bmatrix} \tag{3a}$$

There is no coupling between the positive, negative and zero sequence networks. Since there are no negative and zero sequence voltages before the fault there will be no corresponding currents during and after the fault. Note that in the case that the ground is not involved $Y_{gf} = 0$ and the symmetrical component fault admittance matrix reduces to

$$Y_{fs} = \begin{bmatrix} Y & 0 & 0 \\ 0 & Y & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3b}$$

For a line-to-line fault

$Y_{af} = Y_{gf} = 0$, $Y_{bf} = Y_{cf} = 2Y$, i.e. $Z_{af} = Z_{gf} = \infty$

$$Y_{fs} = \frac{1}{2Y + 2Y} \begin{bmatrix} 2Y \times 2Y & -(2Y \times 2Y) & 0 \\ -(2Y \times 2Y) & 2Y \times 2Y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= Y \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

For a line-to-line-to-ground fault

$Y_{af} = 0$, $Y_{bf} = Y_{cf} = 2Y$, $Y_{gf} = \infty$ i.e. $Z_{af} = \infty$

$$Y_{fs} = \frac{1}{2Y_f + Y_{gf}} \begin{bmatrix} \frac{2}{3}Y_{gf}Y + Y^2 & -\frac{1}{3}Y_{gf}Y - Y^2 & -\frac{1}{3}Y_{gf}Y \\ -\frac{1}{3}Y_{gf}Y - Y^2 & \frac{2}{3}Y_{gf}Y + Y^2 & -\frac{1}{3}Y_{gf}Y \\ -\frac{1}{3}Y_{gf}Y & -\frac{1}{3}Y_{gf}Y & \frac{2}{3}Y_{gf}Y \end{bmatrix} \tag{4a}$$

When $Y_{gf} = Y$ i.e. fault impedance in ground path equals fault impedance in faulted phases equation (4a) becomes:

$$Y_{fs} = \frac{Y}{9} \begin{bmatrix} 5 & -4 & -1 \\ -4 & 5 & -1 \\ -1 & -1 & 2 \end{bmatrix} \tag{4b}$$

For a line-to-ground fault

$Y_{af} = Y$, $Y_{bf} = Y_{cf} = 0$, $Y_{gf} = \infty$ i.e. $Z_{gf} = 0$

$$Y_{fs} = \frac{YY_{gf}}{3(Y + Y_{gf})} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \frac{Y}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \tag{5}$$

Note that although for a metallic short circuit $Y$ is infinite the analysis is performed by means of a limit study.

***Currents in the Fault:***

At the faulted bus bar, say bus bar $j$, the symmetrical component currents in the fault are given by:

$$I_{fsj} = Y_{fs}\left(U + Z_{sjj}Y_{fs}\right)^{-1}V_{sj}^{\ 0} \tag{6}$$

where $U$ is the unit matrix

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $Z_{sjj}$ is the $jj^{th}$ component of the symmetrical component bus impedance matrix

$$Z_{sjj} = \begin{bmatrix} Z_{sjj+} & 0 & 0 \\ 0 & Z_{sjj-} & 0 \\ 0 & 0 & Z_{sjj0} \end{bmatrix}$$

The element $Z_{sjj+}$ is the Thevenin's positive sequence impedance at the faulted bus bar, $Z_{sjj-}$ is the Thevenin's negative sequence impedance at the faulted bus bar, and $Z_{sjj0}$ is the Thevenin's zero sequence impedance at the faulted bus bar. Note that as the network is balanced the mutual terms are all zero.

In equation (6) $V_{sj}^{\ 0}$ is the prefault symmetrical component voltage at bus bar $j$ the faulted bus bar

$$V_{sj}^{\ 0} = \begin{bmatrix} V_{sj+} \\ V_{sj-} \\ V_{sj0} \end{bmatrix} = \begin{bmatrix} V_{+} \\ 0 \\ 0 \end{bmatrix}$$

where $V_+$ is the positive sequence voltage before the fault. The negative and zero sequence voltages are zero because the system is balanced prior to the fault.

The phase currents in the fault are then obtained by transformation:

$$I_{fpj} = \begin{bmatrix} I_{afj} \\ I_{bfj} \\ I_{cfj} \end{bmatrix} = TI_{fsj} \tag{7}$$

***Voltages at the Bus bars:***

The symmetrical component voltage at the faulted bus bar $j$ is given by:

$$V_{fsj} = \begin{bmatrix} V_{j+} \\ V_{j-} \\ V_{j0} \end{bmatrix} = \left(U + Z_{sjj}Y_{fs}\right)^{-1}V_{sj}^{\ 0} \tag{8}$$

The symmetrical component voltage at a bus bar $i$ for a fault at bus bar $j$ is given by:

$$V_{fsi} = \begin{bmatrix} V_{i+} \\ V_{i-} \\ V_{i0} \end{bmatrix} = V_{si}^{\ 0} - Z_{sij}Y_{fs}\left(U + Z_{sjj}Y_{fs}\right)^{-1}V_{sj}^{\ 0} \tag{9}$$

Where $V_{si}^{\ 0} = \begin{bmatrix} V_{i+}^{\ 0} \\ 0 \\ 0 \end{bmatrix}$

gives the symmetrical component prefault voltages at bus bar $i$. The negative and zero sequence prefault voltages are zero.

In equation (9), $Z_{sij}$ gives the $ij^{th}$ components of the symmetrical component bus impedance matrix, the mutual terms for row $i$ and column $j$ (corresponding to bus bars $i$ and $j$)

$$Z_{sij} = \begin{bmatrix} Z_{sij+} & 0 & 0 \\ 0 & Z_{sij-} & 0 \\ 0 & 0 & Z_{sij0} \end{bmatrix}$$

The phase voltages in the fault, at bus bar $j$, and at bus bar $i$ are then obtained by transformation:

$$V_{fpj} = \begin{bmatrix} V_{afi} \\ V_{bfi} \\ V_{cfi} \end{bmatrix} = TV_{fsj}$$

and

$$V_{fpi} = \begin{bmatrix} V_{afpi} \\ V_{bfpi} \\ V_{cfpi} \end{bmatrix} = TV_{fpi} \tag{10}$$

***Currents in Lines, Transformers and Generators:***

The symmetrical component currents in a line between bus bars *i* and *j* is given by:

$$I_{fsij} = Y_{fsij}\left(V_{fsi} - V_{fsj}\right) \tag{11}$$

where

$$Y_{fsij} = \begin{bmatrix} Y_{fsij+} & 0 & 0 \\ 0 & Y_{fsij-} & 0 \\ 0 & 0 & Y_{fsij0} \end{bmatrix}$$

is the symmetrical component admittance of the branch between bus bars *i* and *j*.

Equation (11) is applicable to transformers, when there is no phase shift between the terminal quantities or when the phase shift is catered for when assembling the phase quantities. In the latter case, the positive sequence values are phase shifted forward and the negative sequence values are phase shifted backwards by the phase shift (usually $\pm 30^{\circ}$). The line currents on the delta-connected side of a delta-star transformer should have the appropriate phase to line conversion factor.

The equation also applies to a generator where the source voltage will be the prefault induced voltage and the receiving end bus bar voltage is the postfault voltages at the bus bar.

The phase currents in the branch are found by transformation:

$$I_{fpij} = \begin{bmatrix} I_{aifj} \\ I_{bifj} \\ I_{cfij} \end{bmatrix} = TI_{fsij} \tag{12}$$

***Line-To-Line-To-Ground Fault Simulation:***

Equation (4b) gives the symmetrical component fault admittance matrix for a line-to-line-to-ground fault when the fault impedances in the faulted phases and ground path are equal. It is restated here for easy of reference:

$$Y_{fs} = \frac{Y}{9}\begin{bmatrix} 5 & -4 & -1 \\ -4 & 5 & -1 \\ -1 & -1 & 2 \end{bmatrix} \tag{4b}$$

The value *Y* is the fault admittance in the faulted phases and in the admittance of the ground path.

The symmetrical component fault admittance matrix may be substituted in equation (6) to obtain a simplified value of $I_{fsj}$ given as:

$$I_{fsj} = \frac{V_j^0}{\frac{1}{Y} + Z_{sjj+} + Z_{sjj-}}\begin{bmatrix} 1 \\ -\frac{Z_{sjj0}}{Z_{sjj-} + Z_{sjj0}} \\ -\frac{Z_{sjj-}}{Z_{sjj-} + Z_{sjj0}} \end{bmatrix} \tag{13}$$

in which $V_j^0$ is the prefault voltage on bus bar *j*. The simplified formulation in equation (13), for the line-to-line-to-ground fault, is useful for checking the accuracy of the symmetrical component currents in the fault when the general form is used.

The impedances required to simulate the line-to-line-to-ground fault in general terms are the impedances in the faulted phases and the ground path. In the current work, various impedances were considered namely: purely resistive, resistive, inductive, and purely inductive. The impedances in the faulted phases and ground path are assumed equal. In practice, this is not significant as the general form allows use of different fault admittance values.

***Computation Of The Line-To-Line-To-Ground Fault:***

A computer program has been developed, based on the equations (1) to (13), to solve unbalanced faults for a general power system using the fault admittance matrix method. The program is then applied on a simple power system comprising of three bus bars to solve for a line-to-line-to-ground fault. A simple system is chosen because it is easy to check the results against those that are obtained by hand. Once the program is validated on a simple system then it can be used on large systems and ultimately on practical systems with confidence.

Figure 2 shows a simple three bus bar power system with one generator, one transformer and one transmission line. The system is configured based on the simple power system that is used in (Sadat, H., 2004).

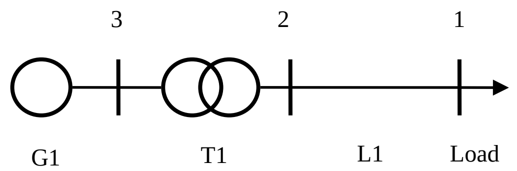


**Fig. 2:** Sample Three Bus Bar System

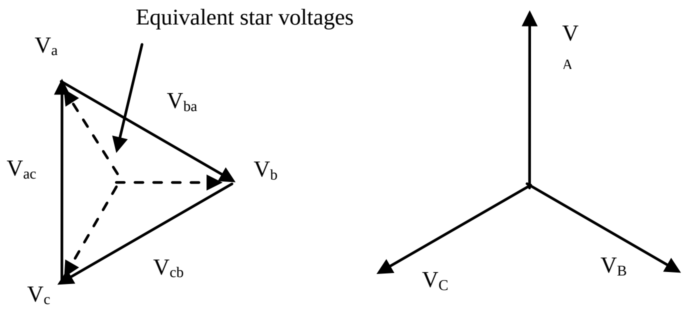


**Fig. 3:** Delta-Star Ttransformer Voltages for Yd11

**Table 1:** Power System Data

| Item | $S_{base}$ (MVA) | $V_{base}$ (kV) | $X_1$ (pu) | $X_2$ (pu) | $X_0$ (pu) |
|---|---|---|---|---|---|
| $G_1$ | 100 | 20 | 0.15 | 0.15 | 0.05 |
| $T_1$ | 100 | 20/220 | 0.10 | 0.10 | 0.10 |
| $L_1$ | 100 | 220 | 0.25 | 0.25 | 0.7125 |

The power system per unit data is given in Table 1, where the subscripts *1*, *2*, and *0* refer to the positive, negative and zero sequence values respectively. The neutral point of the generator is grounded through a zero impedance.

The transformer windings are delta connected on the low voltage side and earthed-star connected on the high voltage side, with the neutral solidly grounded. The phase shift of the transformer is $30^\circ$, i.e. from the generator side to the line side. Figure 3 shows the transformer voltages for a Yd11 connection, which has a $30^\circ$ phase shift.

The computer program incorporates an input program that calculates the sequence admittance and impedance matrices and then assembles the symmetrical component bus impedance matrix for the power system. The symmetrical component bus impedance incorporates all the sequences values and has $3n$ rows and $3n$ columns where $n$ is the number of bus bars. In general, the mutual terms between sequence values are zero as a three-phase power system is, by design, balanced. The power system is assumed to be at no load before the occurrence of a fault. In practice the pre-fault conditions, established by a load flow study may be used. For developing a computer program the assumption of no load, and therefore voltages of 1.0 per unit at the bus bars and in the generator, is adequate.

The line-to-ground fault is assumed to be at bus bar *1*, the load bus bar. Various impedances to simulate the line-to-line-to-ground fault are considered: a purely resistive, a resistive and an inductive combination, and a purely inductive value. The line-to-line-to-ground fault is described by the impedances in the respective phases and in the ground path. In the general fault admittance method, the impedances to be input are those in the *b* and *c* phases and the ground path. The open circuit values for the *a* phase is not input since its respective fault admittance is zero.

The initial fault impedance values were of the order of $10^{-3}\Omega$. The sequence fault currents were calculated for the initial value. A second value of the fault impedances was used, obtained by multiplying the initial value by a factor of $10^{-1}$. The second value of sequence fault currents is calculated. The absolute value of the change in the positive sequence current is compared against a tolerance of $10^{-8}$, and if smaller the solution is considered, converged. If the absolute value of the change is larger than the tolerance, the fault impedance is again reduced and then another value calculated. The iterative process is repeated until either convergence or non-convergence. Note that the order of the initial value of the fault impedances is much smaller than any of the components positive sequence impedances.

The presence of the delta-earthed-star transformer poses a challenge in terms of its modelling. In the computer program, the transformer is modelled in one of two ways: as a normal star-star connection, for the positive and negative sequence networks or as a delta-star transformer with a phase shift. In the former model, the phase shifts are incorporated when assembling the sequence currents to obtain the phase values. In particular on the delta connected side of the transformer the positive sequence currents' angles are increased by the phase shift while the angle of the negative sequence currents are reduced by the same value. The zero sequence currents, if any, are not affected by the phase shifts. Both models for the delta star transformer give same results. The $\sqrt{3}$ line current factor is used to find the line currents on the delta side of the delta star transformer.

## RESULTS AND DISCUSSIONS

### *Fault Simulation Impedances:*

The Thevenin's self sequence impedances of the network seen from the faulted bus bar are:

$$\begin{bmatrix} j0.5 & 0 & 0 \\ 0 & j0.5 & 0 \\ 0 & 0 & j0.8125 \end{bmatrix}$$

In the classical solution the sequence currents due to a line-to-line fault are equal but of opposite sign and are found by inverting the sum of the positive and negative sequence elements. Thus the sequence (positive, negative and zero) currents due to a line-to-line-to-ground fault at the faulted bus bar are:

$$-j\begin{bmatrix} 1.2353 \\ -0.7647 \\ -0.4706 \end{bmatrix}$$

In the general fault admittance method, the values of fault impedances that give accurate values of sequence currents are given in Table 2. Case 1 in the table is for a resistive fault, Case 2 is for a resistive and inductive fault while Case 3 is for an inductive fault. The resistive fault impedance gives a better convergence, since the current tolerance is met by a relatively higher fault impedance than for the other two cases.

**Table 2:** Solution convergence characteristics

| Fault impedance | | | Current tolerance | Current difference |
|---|---|---|---|---|
| Case | Phase a | Ground path | | |
| 1 (*r+j0*) | $5x10^{-6}$ | $5x10^{-6}$ | $1x10^{-8}$ | $5x10^{-9}$ |
| 2 (*r+jx*) | $(5+j5)x10^{-10}$ | $(5+j5)x10^{-10}$ | $1x10^{-8}$ | $3.7x10^{-9}$ |
| 3 (*0+jx*) | $j5x10^{-15}$ | $j5x10^{-15}$ | $1x10^{-8}$ | 0 |

There is a limit as to how low the fault impedance should be. When the value becomes too low the matrix $\left(U + Z_{sjj}Y_{fs}\right)^{-1}$ in equation (6) may not compute, and the solution for the symmetrical component currents may break down. Before solution breakdown, the values of the symmetrical component currents become inaccurate, depending on how much of the effect of the unity matrix in the equation is lost.

The computation results for purely resistive fault impedance for a total fault impedance of $10^{-9}\ \Omega$, are listed in simulation results section below.

### *Simulation Results:*

### *Unbalanced Fault Study Results:*

General Fault Admittance Method – Delta-Star Transformer Model
Number of bus bars = 3
Number of transmission lines = 1
Number of transformers = 1
Number of generators = 1
Faulted bus bar = 1
Fault type = 2

Line to Line to Ground Fault
Phase b resistance = 5.0000e-010
Phase b reactance = 0.0000e+000
Phase c resistance = 5.0000e-010
Phase c reactance = 0.0000e+000
Ground resistance = 5.0000e-010
Ground reactance = 0.0000e+000

Fault Admittance Matrix
Real and imaginary parts of Fault Admittance Matrix

| | | |
|---|---|---|
| 1.1111e+009 +j 0.0000e+000 | -8.8889e+008 +j 0.0000e+000 | -2.2222e+008 +j 0.0000e+000 |
| -8.8889e+008 +j 0.0000e+000 | 1.1111e+009 +j 0.0000e+000 | -2.2222e+008 +j 0.0000e+000 |
| -2.2222e+008 +j 0.0000e+000 | -2.2222e+008 +j 0.0000e+000 | 4.4444e+008 +j 0.0000e+000 |

Thevenin's Symmetrical Component Impedance Matrix of Faulted Bus bar
Real and imaginary parts of Symmetrical Component Impedance Matrix

| | | |
|---|---|---|
| 0.0000 +j 0.5000 | 0.0000 +j 0.0000 | 0.0000 +j 0.0000 |
| 0.0000 +j 0.0000 | 0.0000 +j 0.5000 | 0.0000 +j 0.0000 |
| 0.0000 +j 0.0000 | 0.0000 +j 0.0000 | 0.0000 +j 0.8125 |

Fault current in Symmetrical Components
Real and imaginary parts

| | Simplified Method | General method |
|---|---|---|
| +ve | 0.0000 +j -1.2353 | 0.0000 +j -1.2353 |
| -ve | -0.0000 +j 0.7647 | 0.0000 +j 0.7647 |
| zero | -0.0000 +j 0.4706 | 0.0000 +j 0.4706 |

Magnitude and angle

| | Simplified Method | | General method | |
|---|---|---|---|---|
| | magn | angle | magn | angle |
| +ve | 1.2353 | -90.0000 | 1.2353 | -90.0000 |
| -ve | 0.7647 | 90.0000 | 0.7647 | 90.0000 |
| zero | 0.4706 | 90.0000 | 0.4706 | 90.0000 |

Fault current in phase components
In Rectangular and Polar Coordinates

| | real | imag | magn | angle |
|---|---|---|---|---|
| Phase a | 0.0000 +j | 0.0000 | 0.0000 | 90.0000 |
| Phase b | -1.7321 +j | 0.7059 | 1.8704 | 157.8271 |
| Phase c | 1.7321 +j | 0.7059 | 1.8704 | 22.1729 |

Symmetrical Component Voltages at Faulted Bus bar
In Rectangular and Polar Coordinates

| | real | imag | magn | angle |
|---|---|---|---|---|
| +ve | 0.3824 | 0.0000 | 0.3824 | 0.0000 |
| -ve | 0.3824 | -0.0000 | 0.3824 | -0.0000 |
| zero | 0.3824 | -0.0000 | 0.3824 | -0.0000 |

Phase Voltages at Faulted Bus bar

| | real | imag | magn | angle |
|---|---|---|---|---|
| Phase a | 1.1471 | 0.0000 | 1.1471 | 0.0000 |
| Phase b | -0.0000 | -0.0000 | 0.0000 | 218.9062 |
| Phase c | -0.0000 | 0.0000 | 0.0000 | 143.3063 |

Postfault Voltages at Bus bar number = 1

| | | | | |
|---|---|---|---|---|
| Phase a | 1.1471 | 0.0000 | 1.1471 | 0.0000 |
| Phase b | -0.0000 | -0.0000 | 0.0000 | 218.9062 |
| Phase c | -0.0000 | 0.0000 | 0.0000 | 143.3063 |

Postfault Voltages at Bus bar number = 2

| | | | | |
|---|---|---|---|---|
| Phase a | 0.9294 | 0.0000 | 0.9294 | 0.0000 |
| Phase b | -0.3941 | -0.4330 | 0.5855 | 227.6923 |
| Phase c | -0.3941 | 0.4330 | 0.5855 | 132.3077 |

Postfault Voltages at Bus bar number = 3

| | | | | |
|---|---|---|---|---|
| Phase a | 0.8049 | 0.3500 | 0.8777 | 23.5013 |
| Phase b | 0.0000 | -0.7000 | 0.7000 | -90.0000 |
| Phase c | -0.8049 | 0.3500 | 0.8777 | 156.4987 |

Postfault Currents in Lines

| | | | Phase a | | Phase b | | Phase c | |
|---|---|---|---|---|---|---|---|---|
| Line No. | SE Bus | RE Bus | Current Magn | Current Angle Deg. | Current Magn | Current Angle Deg. | Current Magn | Current Angle Deg. |
| 1 | 2 | 1 | 0.0000 | 90.0000 | 1.8704 | 157.8271 | 1.8704 | 22.1729 |
| 1 | 1 | 2 | 0.0000 | -90.0000 | 1.8704 | -22.1729 | 1.8704 | 202.1729 |

Postfault Currents in Transformers

| | | | Phase a | | Phase b | | Phase c | |
|---|---|---|---|---|---|---|---|---|
| Transf No. | SE Bus | RE Bus | Current Magn | Current Angle Deg. | Current Magn | Current Angle Deg. | Current Magn | Current Angle Deg. |
| 1 | 3 | 2 | 1.8704 | -22.1729 | 3.4641 | 180.0000 | 1.8704 | 22.1729 |
| 1 | 2 | 3 | 0.0000 | 118.9409 | 1.8704 | -22.1729 | 1.8704 | 202.1729 |

Postfault Currents in Generators

| | | | Phase a | | Phase b | | Phase c | |
|---|---|---|---|---|---|---|---|---|
| Gen No. | SE Bus | RE Bus | Current Magn | Current Angle Deg. | Current Magn | Current Angle Deg. | Current Magn | Current Angle Deg. |
| 1 | 4 | 3 | 1.8704 | -22.1729 | 3.4641 | 180.0000 | 1.8704 | 22.1729 |

A summary of the transformer phase currents is shown in Figure 4.

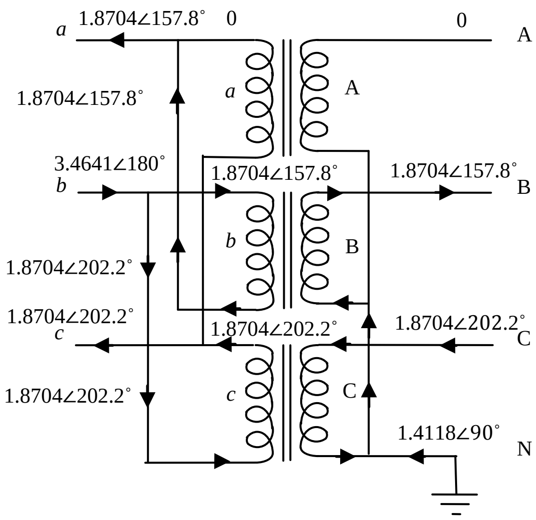


**Fig. 4:** Transformer currents for a line-to-line-to-ground fault

***Fault Admittance Matrix And Sequence Impedances At The Faulted Bus Bar:***

The symmetrical component fault admittance matrix obtained from the program for the line-to-line-to-ground is in agreement with the theoretical value, obtained using equation (4a). The self-sequence impedances at the faulted bus bar obtained from the program are equal to the theoretical values.

***Fault Currents:***

The symmetrical component fault currents obtained from the program using equations (6) and (13) are in agreement with the theoretical values. In particular, the positive sequence current is equal and of opposite sign to the sum of the negative and zero sequence currents. This is consistent with the classical approach that connects the positive, negative and zero sequence networks in parallel. In such a connection the negative and zero sequence networks are in parallel and in series opposition with the positive sequence; for purposes of calculating sequence currents

The phase currents in the fault obtained from the program are in agreement with the theoretical values. In particular, the current in the healthy phase is zero and the real parts of the phase *b* and phase *c* currents are equal and of opposite sign. The sum of the phase *b* and phase *c* current is *j1.4118*, which is the value of the current flowing out of the fault into the ground. The reactive current in the ground flows from the faulted point into the ground and from the ground into the neutral of the star connected side of the transformer.

The phase currents in the transmission line are equal to the currents in the fault. Note that the current at the receiving end of the line is by convention considered as flowing into the line, rather than out of it.

Figure 4 shows the transformer phase currents. The currents on the line side are equal to the currents in the line, after allowing for the sign change due to convention. The current in the neutral of the transformer is equal to the current that flows into the ground at the fault point. Note that the fault currents only flow in the windings of the faulted phases on the earthed-star connected side. The currents at the sending end of the transformer, the delta connected side, flow into the phase *a* and phase *c* terminals of the transformer, and return through the phase *b* terminal. However, the current in the phase *a* winding is zero, which is consistent with the ampere-turn balance requirement, as there is no current in the phase *a* winding on the earthed-star connected side.

The phase fault currents flowing from the generator are equal to the phase currents into the transformer. Phase fault currents flow out in phases *a* and *c* of the generator, and return in phase *b*. It is a feature of the delta earthed-star connection that a line-to-line load on the star side results in currents in all three phases on the delta side of the transformer, although the windings on the unloaded phase do not carry any currents.

***Fault Voltages:***

The symmetrical component voltages at the fault point obtained from the program using equation (9) are in agreement with the theoretical values. In particular, the positive, negative and zero sequence voltages for a line-to-line-to-ground fault are equal, consistent with the concept of all the three networks being connected in parallel.

The phase voltage of the healthy phase is *115%* at the fault, *15 %* more than the prefault value, while the voltages at the fault point in the faulted phases are zero due to the involvement of the ground. The phase voltages magnitudes in the faulted phases at bus bar *2* are *59%* of the prefault value while the voltages in the healthy phase is *93%* of the prefault value. At bus bar 3, the phase voltages lead the phase voltages at bus bar *2* by $23.5^{\circ}$, $42.3^{\circ}$ and $24.2^{\circ}$ in the *a*, *b* and *c* phases respectively. The magnitudes of the voltages in the *a* and *c* phases are equal to *87.8%* while that of the *b* phase is *70%* of the prefault value.

***Conclusions:***

A procedure for simulating the fault impedances of a metallic line-to-line-to-ground fault has been proposed and tested. The results show that a purely resistive fault impedance gives the best convergence. For the system studied, a value of $10^{-9}\ \Omega$ is found suitable. The method allows the estimated fault impedances to be reduced until the convergence to a preset tolerance is reached. In cases where convergence is not as good as for the purely resistive fault impedances the tolerance is reduced and process repeated until convergence is obtained.

The line-to-line-to-ground fault is interesting for studying the delta earthed star transformer arrangement. It is seen that although only two phases carry the fault current on the earthed star side the currents on the delta-connected side are in all three phases, although the winding of the healthy phase does not carry any current. The current in the neutral of the earthed star is equal to the current that flows into the ground at the fault point. Both the involvement of the ground and the phase shifts in the transformer are demonstrated in the results. The results give an insight in the effect that a delta earthed star transformer has on a power system during line-to-line-to-ground faults.

The main advantage of the general fault admittance method is that the user is not required to know beforehand how the sequence networks should be connected at the fault point in order to obtain the sequence currents and voltages. The user can deduce the various relationships from the results. The method is therefore easier to use and teach than the classical approach in which each network is solved in isolation and then the results combined to obtain the phase quantities.

## REFERENCES

Anderson, P.M., 1995, *Analysis of Faulted Power Systems,* IEEE Press.

Das, J.C., 2002 *Power System Analysis, Short circuit load flow and harmonics*, pp: 1-71, Marcel Dekker Inc.

Elgerd, O.I., 1971, *Electric Energy Systems Theory, an Introduction*, pp: 430-476, McGraw Hill Inc., El-Hawary, M., 1995, *Electrical Power Systems Design and Analysis*; pp 469-540, IEEE Press Power Systems Engineering Series.

Oswald, B.R., A. Panosyan, 2006. *A New Method for the Computation of Faults on Transmission Lines,* IEEE PES Transmission and Distribution Conference, Caracas, Venezuela.

Sadat, H., 2004. *Power System Analysis*, second edition, pp 353-459, McGraw Hill, International Edition.

Sakala J.D., J.S.J. Daka, 2007. *Unbalanced Fault Analysis by the General Fault Admittance Method,* Proceedings of the 10th Botswana Institution of Engineers International Biennial Conference, Gaborone, Botswana, pp: 17-19.

Wolter, M., B.R. Oswald, 2007. *Modelling of Switching Operations Using Fault Matrix Method,* Proceedings of the 2nd IASME/WSEAS International Conference on Energy and Environment (EE'07), Portoroz, Slovenia, 15-17: 181-184.

Zhu, J., 2004, *Analysis of Transmission System Faults in the Phase Domain,* MSc. Thesis, Texas A&M University.