Australian Journal of Basic and Applied Sciences, 7(9): 156-164, 2013
ISSN 1991-8178

# Bayesian Analysis of Weibull Distribution Using R Software

S.P. Ahmad and Kaisar Ahmad

Department of Statistics, University of Kashmir, Srinagar, Jammu and Kashmir, India-190006

**Abstract:** The Weibull distribution is perhaps the most widely used life time distribution model. Weibull distribution is widely employed in modeling and analyzing lifetime data. Its application in connection with lifetimes of many types of manufactured items has been widely advocated and it has been used as a model with diverse types of items such as vacuum tubes, ball bearings and electrical insulation. It is also widely used in biomedical applications, for example, in studies of the time to the occurrence of tumors in human population or in laboratory animals and in many other situations. In Bayesian parameter inference the choice of fitted prior distributions and loss functions has great importance. The present paper considers the estimation of the scale parameter of two parameter Weibull distribution with known shape. Bayes estimator is obtained using Jeffrey's and extension of Jeffrey's prior under linear exponential loss function and symmetric loss function. Maximum likelihood estimation is discussed. These methods are compared using mean square error through simulation study with varying sample sizes by using R software.

**Key words:** Weibull distribution, Priors, Bayesian estimation, loss functions and R software.

## INTRODUCTION

The Weibull distribution is one of the most widely used distributions for analyzing lifetime data. It is found to be useful in diverse fields ranging from engineering to medical sciences (see Lawless (2002), Martz and Waller (1982)). The Weibull family is a generalization of the exponential family and can model data exhibiting monotone hazard rate behavior, i.e. it can accommodate three types of failure rates, namely increasing, decreasing and constant. The probability density function of the Weibull distribution is given by:

$$f(x;\alpha,\beta) = \frac{\beta}{\alpha} x^{\beta-1} \exp\left(-\frac{x^{\beta}}{\alpha}\right) \qquad , x \geq 0 \tag{1.1}$$

where $\alpha > 0$ and $\beta > 0$ are the scale and shape parameters of the distribution. In Weibull lifetime analysis it is frequent case that the value of shape parameter is known. For example, the exponential and Rayleigh distributions are obtained when $\beta$=1 and $\beta$=2 respectively. Soland (1968) gives a justification for this situation. The Weibull distribution was studied by Weibull (1951) in connection with the strength of materials; Lieblein and Zelen (1956); Kao (1959) considerd application in reliability and Pike (1966) applications in medicine. Frank (1988) have assigned meaning and interpretations for the Weibull distribution. Hallinan (1993) has recently provided an excellent review of the Weibull distribution by presenting historical facts, and the many different forms of this distribution as used by practitioners and possible confusions, errors that arise due to this non-uniqueness.

Maximum Likelihood Estimation has been the most widely used method for estimating the parameters of the Weibull distribution. Recently Bayesian estimation approach has received great attention by most researchers among them are, Ahmed *et al*. (2011). They considered Bayesian Survival Estimator for Weibull distribution with censored data while Al-Aboud (2009) studied Bayesian estimation for the extreme value distribution using progressive censored data and asymmetric loss. Bayes estimator for exponential distribution with extension of Jeffreys' prior information was considered by Al-Kutubi (2009). Others including, Pandey *et al*. (2011), Al-Atari (2011), and Hossain and Zimmer (2003) did some comparative studies on the estimation of Weibull parameters using complete and censored samples and Lye *et al*. (1993) determined Bayes estimation of the Extreme-Value Reliability Function.

The paper is arranged as follows: Section 2, discusses the Maximum Likelihood estimator and Bayesian methodology using Jeffrey's prior and extension of Jeffrey's prior information under LINEX and squared error loss functions for estimation of the scale parameter of Weibull distribution with known shape. Section 3, focuses in the simulation study and results to compare the estimators and finally section 4 is the conclusion of the paper.

## MATERIAL AND METHODS

**Corresponding Author:** S.P. Ahmad, Department of Statistics, University of Kashmir, Srinagar, Jammu and Kashmir, India-190006
E-mail: sprvz@yahoo.com

***Maximum Likelihood Estimation:***

Maximum likelihood estimation of the parameters of Weibull distribution is well discussed in literature (see Cohen, (1965) and Mann *et al*. (1975)).

Let $(x_1, x_2, \ldots, x_n)$ be a random sample of size n having the probability density function as

$$f(x;\alpha,\beta) = \frac{\beta}{\alpha} x^{\beta-1} \exp\left(-\frac{x^{\beta}}{\alpha}\right) \qquad , x \geq 0$$

The likelihood function is given by

$$L(x \mid \alpha) = \frac{\beta^n}{\alpha^n} \prod_{i=1}^{n} x_i^{\beta-1} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\beta}}{\alpha}\right) \therefore \qquad \log L(x \mid \alpha) = n\log\beta - n\log\alpha + \sum_{i=1}^{n} x_i^{\ \beta} - \frac{\sum_{i=1}^{n} x_i^{\beta}}{\alpha}$$

As the shape parameter β is assumed to be known, the ML estimator of α is obtained by solving the

$$\frac{\partial}{\partial\alpha}\log L(x \mid \alpha) = 0$$

$$\Rightarrow \qquad -\frac{n}{\alpha} + \frac{\sum_{i=1}^{n} x_i^{\beta}}{\alpha^2} = 0 \qquad \Rightarrow \hat{\alpha} = \frac{\sum_{i=1}^{n} x_i^{\beta}}{n} \qquad (2.1)$$

***Bayesian Estimation of Weibull Distribution Under Jeffrey's Prior by Using Different Loss Functions:***

Prior Distribution**:**

Quite often, the derivation of the prior distribution based on information other than the current data is impossible or rather difficult. Moreover, the statistician may be required to employ as little subjective inputs as possible, so that the conclusion may appear solely based on sampling model and the current data.

Jeffrey's (1946) proposed a formal rule for obtaining a non-informative prior as

$$g(\theta) \ \ \alpha \ \ \sqrt{\det(I(\theta))}$$

Where θ is k-vector valued parameter and I(θ) is the Fisher's information matrix of order k × k. In particular if θ is a scalar parameter, Jeffrey's non- informative prior for θ is $\sqrt{\det(I(\theta))}$ Thus, in our problem we consider the prior distribution of α to be

$$g(\alpha) \ \ \alpha \ \ \sqrt{\det(I(\alpha))}$$

$$\Rightarrow \qquad g(\alpha) = k\frac{1}{\alpha}$$

where k is a constant.

The posterior distribution of α is given by

$$p(\alpha \mid x) \quad \alpha \quad L(x \mid \alpha) g(\alpha)$$

$$\therefore \qquad p(\alpha \mid x) \ \ \alpha \ \ \frac{\beta^n}{\alpha^{n+1}} \prod_{i=1}^{n} x_i^{\beta-1} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\beta}}{\alpha}\right)$$

$$\Rightarrow \qquad p(\alpha \mid x) = \frac{k}{\alpha^{n+1}} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\beta}}{\alpha}\right)$$

Where k is independent of α.

and $k^{-1} = \int_0^{\infty} \frac{1}{\alpha^{n+1}} \exp\left( -\frac{\sum_{i=1}^{n} x_i^{\beta}}{\alpha} \right) d\alpha$

$$\Rightarrow \qquad k^{-1} = \frac{\Gamma n}{\left( \sum_{i=1}^{n} x_i^{\beta} \right)^n}$$

$$\Rightarrow \qquad k = \frac{\left( \sum_{i=1}^{n} x_i^{\beta} \right)^n}{\Gamma n}$$

Hence the posterior distribution of α is given by

$$p(\alpha \mid x) = \frac{\left( \sum_{i=1}^{n} x_i^{\ \beta} \right)^n}{\alpha^{n+1} \Gamma n} \exp\left( -\frac{\sum_{i=1}^{n} x_i^{\beta}}{\alpha} \right)$$

***Estimation Under LINEX Loss Function:***

For determining the Bayes estimate of scale parameter α we will introduce a very useful asymmetric linex loss function given by

$L(\sigma) = \exp(a\sigma) - a\sigma - 1$

Where $\sigma = \frac{\hat{\alpha}}{\alpha} - 1$ , $\quad a \neq 0$

To obtain the Bayes estimator, we minimize the posterior expected loss given by

$$\rho = E[L(\sigma)] = \int_0^{\infty} L(\sigma) p(\alpha \mid x) d\alpha$$

$$\Rightarrow \qquad \rho = \int_0^{\infty} \{\exp(a\sigma) - a\sigma - 1\} \frac{\exp\left( -\frac{\sum_{i=1}^{n} x_i^{\beta}}{\alpha} \right) \left( \sum_{i=1}^{n} x_i^{\beta} \right)^n}{\alpha^{n+1} \Gamma n} d\alpha$$

$$\Rightarrow \qquad \rho = \int_0^{\infty} \left\{ \exp\left( a\left( \frac{\hat{\alpha}}{\alpha} - 1 \right) \right) - a\left( \frac{\hat{\alpha}}{\alpha} - 1 \right) - 1 \right\} \frac{\exp\left( -\frac{t}{\alpha} \right) (t)^n}{\alpha^{n+1} \Gamma n} d\alpha$$

Where $\sigma = \frac{\hat{\alpha}}{\alpha} - 1$ and $t = \sum_{i=1}^{n} x_i^{\beta - 1}$

$$\Rightarrow \qquad \rho = \frac{t^n}{\Gamma n} \int_0^{\infty} \frac{1}{\alpha^{n+1}} \exp\left( -\left( \frac{t - a\hat{\alpha}}{\alpha} \right) \right) \exp(-a) d\alpha - \frac{t^n}{\Gamma n} a\hat{\alpha} \int_0^{\infty} \frac{1}{\alpha^{(n+1)+1}} \exp\left( -\frac{t}{\alpha} \right) d\alpha$$
$$+ \frac{at^n}{\Gamma n} \int_0^{\infty} \frac{1}{\alpha^{n+1}} \exp\left( -\frac{t}{\alpha} \right) d\alpha - \frac{t^n}{\Gamma n} \int_0^{\infty} \frac{1}{\alpha^{n+1}} \exp\left( -\frac{t}{\alpha} \right) d\alpha$$

$$\Rightarrow \qquad \rho = \frac{t^n \exp(-a)}{(t - a\hat{\alpha})^n} - \frac{t^n}{\Gamma n} a\hat{\alpha} \frac{\Gamma(n+1)}{(t)^{n+1}} + \frac{at^n}{\Gamma n} \frac{\Gamma n}{t^n} - \frac{t^n}{\Gamma n} \frac{\Gamma n}{t^n}$$

$$\Rightarrow \qquad \rho = \frac{t^n \exp(-a)}{(t - a\hat{\alpha})^n} - \frac{na\hat{\alpha}}{t} + a - 1$$

Now solving $\frac{\partial \rho}{\partial \hat{\alpha}} = 0$, we obtain the Bayes estimator as

$$t^n \exp(-a)(-n(t-a\hat{\alpha})^{-n-1})(-a) - \frac{na}{t} = 0$$

$$\Rightarrow \quad nat^n \exp(-a)(t-a\hat{\alpha})^{-n-1} = \frac{na}{t}$$

$$\Rightarrow \quad t^{n+1}\exp(-a) = (t-a\hat{\alpha})^{n+1}$$

$$\Rightarrow \quad t(\exp(-a))^{\frac{1}{n+1}} = (t-a\hat{\alpha})$$

$$\Rightarrow \quad a\hat{\alpha} = (t - t(\exp(-a))^{\frac{1}{n+1}})$$

$$\Rightarrow \quad \hat{\alpha} = \frac{t}{a}(1-(\exp(-a))^{\frac{1}{n+1}})$$

$$\Rightarrow \quad \hat{\alpha}_{BL} = \frac{\sum_{i=1}^{n} x_i^{\beta}}{a}\left(1-\exp\left(-\frac{a}{na}\right)\right) \tag{2.2}$$

***Estimation Under Squared Error Loss Function:***

The squared error loss function (SELF) was proposed by Legendre (1805) and Gauss to develop least square theory. Later, it was used in estimation problems when unbiased estimations of θ were evaluated in terms of the risk function R(θ, a) which becomes nothing but the variance of the estimator.

In our problem SELF is given by

$l(\hat{\alpha}-\alpha) = c(\hat{\alpha}-\alpha)^2$

By using the squared error loss function $l(\hat{\alpha}-\alpha) = c(\hat{\alpha}-\alpha^2)$, The Risk function is given by

$$\rho = R(\hat{\alpha}-\alpha) = El(\hat{\alpha}-\alpha)^2$$

$$\rho = E(L(\alpha)) = \int_0^{\infty} l(\hat{\alpha}-\alpha)\ p(\alpha \mid x)d\alpha \quad \Rightarrow \quad \rho = \int_0^{\infty} c(\hat{\alpha}-\alpha)^2 \frac{\left(\sum_{i=1}^{n} x_i^{\beta}\right)^n}{\Gamma n}\frac{1}{\alpha^{n+1}}\exp\left(-\frac{\sum_{i=1}^{n} x_i^{\beta}}{\alpha}\right)d\alpha$$

$$\Rightarrow \quad \rho = \frac{c\hat{\alpha}^2 t^n}{\Gamma n}\int_0^{\infty}\frac{1}{\alpha^{n+1}}\exp\left(-\frac{t}{\alpha}\right)d\alpha - 2\frac{c\hat{\alpha}t^n}{\Gamma n}\int_0^{\infty}\frac{1}{\alpha^{n}}\exp\left(-\frac{t}{\alpha}\right)d\alpha + \phi(\alpha)$$

Where $\phi(\alpha) = \frac{ct^n}{\Gamma n}\int \frac{1}{\alpha^{n-1}}\exp\left(-\frac{t}{\alpha}\right)d\alpha$ and $t = \sum_{i=1}^{n} x_i^{\beta-1}$

$$\Rightarrow \quad \rho = \frac{c\hat{\alpha}^2 t^n}{\Gamma n}\frac{\Gamma n}{t^n} - 2\frac{c\hat{\alpha}t^n}{\Gamma n}\frac{\Gamma(n-1)}{t^{n-1}} + \phi(\alpha)$$

$$\Rightarrow \quad \rho = c\hat{\alpha}^2 - 2\frac{c\hat{\alpha}t}{n-1} + \phi(\alpha)$$

Now solving $\frac{\partial \rho}{\partial \hat{\alpha}} = 0$ , we obtain the Bayes estimator as

$$\frac{\partial}{\partial \hat{\alpha}}(c\hat{\alpha}^2 - 2\frac{c\hat{\alpha}t}{n-1} + \phi(\alpha)) = 0$$

$$\Rightarrow \quad 2c\hat{\alpha} - 2\frac{ct}{n-1} = 0$$

$$\Rightarrow \quad \hat{\alpha}_{BS} = \frac{t}{n-1}$$

$$\Rightarrow \qquad \hat{\alpha}_{BS} = \frac{\sum_{i=1}^{n} x_i^{\beta}}{n-1} \tag{2.3}$$

***Bayesian estimation of Weibull distribution under Extension of Jeffrey's prior by using different Loss Functions:***

Let $(x_1, x_2, \ldots, x_n)$ be a random sample of size n having the probability density function as

$$f(x;\alpha,\beta) = \frac{\beta}{\alpha} x^{\beta-1} \exp\left(-\frac{x^{\beta}}{\alpha}\right) \qquad , x \geq 0$$

The likelihood function is given by

$$L(x\,|\,\alpha) = \frac{\beta^n}{\alpha^n} \prod_{i=1}^{n} x_i^{\beta-1} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\beta}}{\alpha}\right)$$

Thus, in our problem we consider the prior distribution of $\alpha$ to be

$$g(\alpha) \;\; \alpha \;\; [\det|I(\alpha)|]^c \qquad , c \in R^+$$

$$\Rightarrow \qquad g(\alpha) = k\frac{1}{\alpha^{2c}}$$

where k is a constant.

The posterior distribution of α is given by

$$p(\alpha\,|\,x) \quad \alpha \quad L(x\,|\,\alpha) g(\alpha)$$

$$\therefore \qquad p(\alpha\,|\,x) \;\; \alpha \;\; \frac{\beta^n}{\alpha^{n+2c}} \prod_{i=1}^{n} x_i^{\beta-1} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\beta-1}}{\alpha}\right)$$

$$\Rightarrow \qquad p(\alpha\,|\,x) = \frac{k}{\alpha^{n+2c}} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\beta-1}}{\alpha}\right)$$

Where k is independent of α.

and $k^{-1} = \int_0^{\infty} \frac{1}{\alpha^{n+2c}} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\beta-1}}{\alpha}\right) d\alpha$

$$\Rightarrow \qquad k^{-1} = \int_0^{\infty} \frac{1}{\alpha^{n+2c-1+1}} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\beta-1}}{\alpha}\right) d\alpha$$

$$\Rightarrow \qquad k^{-1} = \frac{\Gamma(n+2c-1)}{\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{n+2c-1}}$$

$$\Rightarrow \qquad k = \frac{\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{n+2c-1}}{\Gamma(n+2c-1)}$$

After using the value of k , we get the posterior distribution as

$$\Rightarrow \qquad p(\alpha \mid x) = \frac{\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{n+2c-1}}{\alpha^{n+2c}\Gamma(n+2c-1)} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\beta}}{\alpha}\right)$$

***Estimation Under LINEX Loss Function:***

For determining the Bayes estimate of scale parameter α we will introduce a very useful asymmetric linex loss function given by

$$L(\sigma) = \exp(a\sigma) - a\sigma - 1$$

Where $\sigma = \frac{\hat{\alpha}}{\alpha} - 1$ , $\qquad a \neq 0$

To obtain the Bayes estimator, we minimize the posterior expected loss given by

$$\rho = E[L(\sigma)] = \int_0^{\infty} L(\sigma)\, p(\alpha \mid x) d\alpha$$

$$\Rightarrow \qquad \rho = \int_0^{\infty} \{\exp(a\sigma) - a\sigma - 1\} \frac{\exp\left(-\frac{\sum_{i=1}^{n} x_i^{\beta-1}}{\alpha}\right)\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{n+2c-1}}{\alpha^{n+2c}\Gamma(n+2c-1)} d\alpha$$

$$\Rightarrow \qquad \rho = \int_0^{\infty} \left\{\exp\left(a\left(\frac{\hat{\alpha}}{\alpha} - 1\right)\right) - a\left(\frac{\hat{\alpha}}{\alpha} - 1\right) - 1\right\} \frac{\exp\left(-\frac{t}{\alpha}\right)(t)^{n+2c-1}}{\alpha^{n+2c}\Gamma(n+2c-1)} d\alpha$$

Where $\sigma = \frac{\hat{\alpha}}{\alpha} - 1$ and $t = \sum_{i=1}^{n} x_i^{\beta}$

$$\Rightarrow \rho = \frac{t^{n+2c-1}}{\Gamma(n+2c-1)} \int_0^{\infty} \frac{1}{\alpha^{(n+2c-1)+1}} \exp\left(-\left(\frac{t - a\hat{\alpha}}{\alpha}\right)\right) \exp(-a) d\alpha - \frac{t^{n+2c-1}}{\Gamma(n+2c-1)} a\hat{\alpha} \int_0^{\infty} \frac{1}{\alpha^{n+2c+1}} \exp\left(-\frac{t}{\alpha}\right) d\alpha$$

$$+ \frac{at^{n+2c-1}}{\Gamma(n+2c-1)} \int_0^{\infty} \frac{1}{\alpha^{(n+2c-1)+1}} \exp\left(-\frac{t}{\alpha}\right) d\alpha - \frac{t^{n+2c-1}}{\Gamma(n+2c-1)} \int_0^{\infty} \frac{1}{\alpha^{(n+2c-1)+1}} \exp\left(-\frac{t}{\alpha}\right) d\alpha$$

$$\Rightarrow \qquad \rho = \frac{t^{n+2c-1}\exp(-a)}{(t - a\hat{\alpha})^{n+2c-1}} - \frac{(n+2c-1)a\hat{\alpha}}{t} + a - 1$$

Now solving $\frac{\partial \rho}{\partial \hat{\alpha}} = 0$ , we obtain the Bayes estimator as

$$t^{n+2c-1}\exp(-a)(-(n+2c-1))(t - a\hat{\alpha})^{-n-2c})(-a) - \frac{(n+2c-1)a}{t} = 0$$

$$\Rightarrow \qquad t^{n+2c-1}\exp(-a)(t - a\hat{\alpha})^{-n-2c}(n+2c-1)a = \frac{(n+2c-1)a}{t}$$

$$\Rightarrow \qquad t^{n+2c}\exp(-a) = (t - a\hat{\alpha})^{n+2c}$$

$$\Rightarrow \qquad t(\exp(-a))^{\frac{1}{n+2c}} = (t - a\hat{\alpha})$$

$$\Rightarrow \qquad a\hat{\alpha} = (t - t(\exp(-a))^{\frac{1}{n+2c}})$$

$$\Rightarrow \qquad \hat{\alpha} = \frac{t}{a}(1 - (\exp(-a))^{\frac{1}{n+2c}})$$

$$\Rightarrow \qquad \hat{\alpha}_{BL} = \frac{t}{a}\left(1 - \left(\exp\left(-\frac{a}{n+2c}\right)\right)\right)$$

$$\Rightarrow \qquad \hat{\alpha}_{BL} = \frac{\sum_{i=1}^{n} x_i}{a}\left(1-\left(\exp\left(-\frac{a}{n+2c}\right)\right)\right) \qquad (2.4)$$

Which is the required Bayes estimate of scale parameter α under extension of Jeffrey's prior.

***Estimation Under Squared Error Loss Function:***

In our problem SELF is given by

$$l(\hat{\alpha}-\alpha) = c(\hat{\alpha}-\alpha)^2$$

By using the squared error loss function $l(\hat{\alpha}-\alpha) = c\left(\hat{\alpha}-\alpha^2\right)$, The Risk function is given by

$$\rho = R(\hat{\alpha}-\alpha) = El(\hat{\alpha}-\alpha)^2$$

$$\Rightarrow \qquad \rho = E(L(\alpha)) = \int_0^\infty l(\hat{\alpha}-\alpha)\; p(\alpha \mid x)d\alpha$$

$$\Rightarrow \qquad \rho = \int_0^\infty c(\hat{\alpha}-\alpha)^2 \frac{\left(\sum_{i=1}^{n} x_i^{\beta}\right)^{n+2c-1}}{\Gamma(n+c-1)} \frac{1}{\alpha^{n+2c}} \exp\left(-\frac{\sum_{i=1}^{n} x_i^{\beta}}{\alpha}\right)d\alpha$$

$$\Rightarrow \qquad \rho = \frac{c\hat{\alpha}^2 t^{n+2c-1}}{\Gamma(n+2c-1)}\int_0^\infty \frac{1}{\alpha^{n+2c}}\exp\left(-\frac{t}{\alpha}\right)d\alpha - 2\frac{c\hat{\alpha}t^{n+2c-1}}{\Gamma(n+2c-1)}\int_0^\infty \frac{1}{\alpha^{n+2c-1}}\exp\left(-\frac{t}{\alpha}\right)d\alpha + \psi(\alpha)$$

Where $\psi(\alpha) = \frac{ct^{n+2c-1}}{\Gamma(n+2c-1)}\int_0^\infty \frac{1}{\alpha^{n+2c-2}}\exp\left(-\frac{t}{\alpha}\right)d\alpha$ and $t = \sum_{i=1}^{n} x_i^{\beta}$

$$\Rightarrow \qquad \rho = c\hat{\alpha}^2 - 2\frac{c\hat{\alpha}t}{n+2c-2} + \psi(\alpha)$$

Now solving $\frac{\partial \rho}{\partial \hat{\alpha}} = 0$, we obtain the Bayes estimator as

$$\frac{\partial}{\partial \hat{\alpha}}(c\hat{\alpha}^2 - 2\frac{c\hat{\alpha}t}{n+2c-2} + \psi(\alpha)) = 0$$

$$\Rightarrow \qquad 2c\hat{\alpha} - 2\frac{ct}{n+2c-2} = 0$$

$$\Rightarrow \qquad \hat{\alpha}_{BS} = \frac{t}{n+2c-2}$$

$$\Rightarrow \qquad \hat{\alpha}_{BS} = \frac{\sum_{i=1}^{n} x_i^{\beta}}{n+2c-2} \qquad (2.5)$$

Which is required Bayes estimator of α under extension of Jeffrey's prior.

***Simulation Study:***

In our simulation study, we chose a sample size of n=25, 50 and 100 to represent small, medium and large data set. The scale parameter is estimated for Weibull distribution with Maximum Likelihood and Bayesian using Jeffrey's & extension of Jeffrey's prior methods. For the scale parameter we have considered α= 0.5 and 1.5. The Shape parameter β has been fixed at 0.8 and 1.2. The values of Jeffrey's extension were c = 0.4 and 1.4. The value for the loss parameter a = ± 0.6 and ±1.6. This was iterated 1000 times and the scale parameter for each method was calculated. A simulation study was conducted R-software to examine and compare the performance of the estimates for different sample sizes with different values for the Extension of Jeffreys' prior and the loss functions. The results are presented in tables for different selections of the parameters and c extension of Jeffrey's prior.

## RESULTS AND DISCUSSION

**Table 1:** Mean Squared Error for $(\hat{\alpha})$ under Jeffrey's prior.

| n | A | β | $\hat{\alpha}_{ML}$ | $\hat{\alpha}_{BL}$ | $\hat{\alpha}_{BL}$ $a=-0.6$ | $\hat{\alpha}_{BL}$ | $\hat{\alpha}_{BL}$ | $\hat{\alpha}_{BS}$ |
|---|---|---|---|---|---|---|---|---|

| | | | | $a=0.6$ | | $a=1.6$ | $a=-1.6$ | |
|---|---|---|---|---|---|---|---|---|
| 25 | 0.5 | 0.8 | 0.06714 | 0.06382 | 0.06531 | 0.06261 | 0.06658 | 0.06994 |
| | 0.5 | 1.2 | 0.05943 | 0.05649 | 0.05781 | 0.05542 | 0.05894 | 0.06191 |
| | 1.5 | 0.8 | 0.08946 | 0.08503 | 0.08702 | 0.08343 | 0.08872 | 0.09319 |
| | 1.5 | 1.2 | 0.05943 | 0.05649 | 0.05781 | 0.05781 | 0.05542 | 0.06191 |
| 50 | 0.5 | 0.8 | 0.04461 | 0.04347 | 0.04399 | 0.04305 | 0.04442 | 0.04551 |
| | 0.5 | 1.2 | 0.03797 | 0.03701 | 0.03746 | 0.03665 | 0.03782 | 0.03875 |
| | 1.5 | 0.8 | 0.10740 | 0.10467 | 0.10591 | 0.10369 | 0.10696 | 0.10959 |
| | 1.5 | 1.2 | 0.18145 | 0.17685 | 0.17895 | 0.17513 | 0.18072 | 0.18516 |
| 100 | 0.5 | 0.8 | 0.05302 | 0.05234 | 0.05266 | 0.05209 | 0.05292 | 0.05356 |
| | 0.5 | 1.2 | 0.04451 | 0.04393 | 0.04419 | 0.04371 | 0.04441 | 0.04495 |
| | 1.5 | 0.8 | 0.13590 | 0.13415 | 0.13495 | 0.13349 | 0.13562 | 0.13727 |
| | 1.5 | 1.2 | 0.14073 | 0.13893 | 0.13975 | 0.13824 | 0.14045 | 0.14215 |

ML= Maximum Likelihood, BL= LINEX Loss Function, BS=Squared Error Loss Function

**Table 2:** Mean Squared Error for ($\hat{\alpha}$) under extension of Jeffrey's prior.

| n | α | c | β | $\hat{\alpha}_{ML}$ | $\hat{\alpha}_{BL}$ $a=0.6$ | $\hat{\alpha}_{BL}$ $a=-0.6$ | $\hat{\alpha}_{BL}$ $a=1.6$ | $\hat{\alpha}_{BL}$ $a=-1.6$ | $\hat{\alpha}_{BS}$ |
|---|---|---|---|---|---|---|---|---|---|
| 25 | 0.5 | 0.4 | 0.8 | 0.06714 | 0.06431 | 0.06582 | 0.06308 | 0.06712 | 0.07052 |
| | 0.5 | 0.4 | 1.2 | 0.05943 | 0.05693 | 0.05827 | 0.05584 | 0.05941 | 0.06243 |
| | 0.5 | 1.4 | 0.8 | 0.06714 | 0.05973 | 0.06103 | 0.05867 | 0.06215 | 0.06506 |
| | 0.5 | 1.4 | 1.2 | 0.05943 | 0.05287 | 0.54033 | 0.05194 | 0.05502 | 0.05759 |
| | 1.5 | 0.4 | 0.8 | 0.08946 | 0.08569 | 0.08770 | 0.08405 | 0.08943 | 0.09397 |
| | 1.5 | 0.4 | 1.2 | 0.13369 | 0.12805 | 0.13106 | 0.12561 | 0.13364 | 0.14043 |
| | 1.5 | 1.4 | 0.8 | 0.08946 | 0.07959 | 0.08132 | 0.07818 | 0.08281 | 0.08669 |
| | 1.5 | 1.4 | 1.2 | 0.13369 | 0.11893 | 0.12153 | 0.11683 | 0.12375 | 0.12954 |
| 50 | 0.5 | 0.4 | 0.8 | 0.04461 | 0.04364 | 0.04416 | 0.04322 | 0.04461 | 0.04571 |
| | 0.5 | 0.4 | 1.2 | 0.03797 | 0.037161 | 0.03761 | 0.03679 | 0.03797 | 0.03891 |
| | 0.5 | 1.4 | 0.8 | 0.04461 | 0.042003 | 0.04248 | 0.04161 | 0.04288 | 0.04391 |
| | 0.5 | 1.4 | 1.2 | 0.03797 | 0.03576 | 0.03616 | 0.03542 | 0.03651 | 0.03737 |
| | 1.5 | 0.4 | 0.8 | 0.10741 | 0.10508 | 0.10633 | 0.10406 | 0.10739 | 0.11004 |
| | 1.5 | 0.4 | 1.2 | 0.18145 | 0.17755 | 0.17966 | 0.17581 | 0.18144 | 0.18592 |
| | 1.5 | 1.4 | 0.8 | 0.10741 | 0.101131 | 0.10228 | 0.10018 | 0.10326 | 0.10571 |
| | 1.5 | 1.4 | 1.2 | 0.18145 | 0.17086 | 0.17861 | 0.16925 | 0.17446 | 0.17861 |
| 100 | 0.5 | 0.4 | 0.8 | 0.05302 | 0.05245 | 0.05276 | 0.05219 | 0.05302 | 0.05367 |
| | 0.5 | 0.4 | 1.2 | 0.04451 | 0.04401 | 0.04428 | 0.04379 | 0.04450 | 0.04504 |
| | 0.5 | 1.4 | 0.8 | 0.05302 | 0.05143 | 0.05173 | 0.05118 | 0.05198 | 0.05261 |
| | 0.5 | 1.4 | 1.2 | 0.04451 | 0.04316 | 0.04341 | 0.04295 | 0.04362 | 0.04414 |
| | 1.5 | 0.4 | 0.8 | 0.13590 | 0.13442 | 0.13522 | 0.13376 | 0.13590 | 0.13755 |
| | 1.5 | 0.4 | 1.2 | 0.14073 | 0.13920 | 0.14244 | 0.13851 | 0.14073 | 0.14244 |
| | 1.5 | 1.4 | 0.8 | 0.13590 | 0.13181 | 0.13258 | 0.131178 | 0.13323 | 0.13482 |
| | 1.5 | 1.4 | 1.2 | 0.14073 | 0.13650 | 0.13730 | 0.13584 | 0.13797 | 0.13962 |

ML= Maximum Likelihood, BL= LINEX Loss Function, BS= Squared Error Loss Function

In table 1, Bayes estimation with LINEX Loss function under Jeffrey's prior provides the smallest values in most cases especially when loss parameter a is 1.6. Similarly, in table 2, Bayes estimation with LINEX Loss function under extension of Jeffrey's prior provides the smallest values in most cases especially when loss parameter a is1.6 whether the extension of Jeffrey's prior is 0.4 or 1.4.

***Conclusion:***

In this paper, we have addressed the problem of Bayesian estimation for the Weibull distribution, under asymmetric and symmetric loss functions and that of Maximum Likelihood Estimation. From the results, we observe that in most cases, Bayesian Estimator under Linear Exponential Loss function (LINEX) has the smallest Mean Squared Error values for both prior's i.e, Jeffrey's and an extension of Jeffrey's prior information.

## REFERENCES

Ahmed, A.O.M., N.A. Ibrahim, J. Arasan and M.B. Adam, 2011. Extension of Jeffrey's Prior Estimate for Weibull Censored data using Lindley's approximation. Australian Journal of Basic and Applied Sciences, 5(12): 884-889.

Al-Aboud, F.M., 2009. Bayesian estimations for the extreme value distribution using progressive censored data and asymmetric loss. International Mathematical Forum, 4(33): 1603-1622.

Al-Athari, F.M., 2011. Parameter estimation for the double-pareto distribution. Journal of Mathematics and Statistics, 7(4): 289-294.

Al-Kutubi, H.S. and N.A. Ibrahim, 2009. Bayes estimator for exponential distribution with extension of Jeffery prior information. Malaysian Journal of Mathematical Sciences, 3(2): 297-313.

Cohen, A.C., 1965. Maximum likelihood estimation in Weibull distribution based on complete and on censored samples. Technometrics, 7: 579-588.

Franck, J.R., 1988. A simple explanation of the Weibull distribution and its applications. Reliability Review, 8(3): 93-116.

Hallinan, A.J., 1993. A review of the Weibull distribution. Journal of Quality Technology, 25: 282-284.

Hossain, A. and W. Zimmer, 2003.Comparison of estimation methods for Weibull parameters: complete and censored samples. Journal of Statistical Computation and Simulation, 73(2): 145-153.

Jefferys, H., 1946. An invariant form for the prior probability in estimation problems; proceedings of the royal Society of London, Series. A., 186: 453-461.

Kao, J.H.K., 1959. A graphical estimation of mixed Weibull parameters in life testing of electron tubes. Technometrics, 1: 389-407.

Lawless, J.F., 2002. Statistical Models and Methods for Lifetime Data. 2nd ed., Wiley, New York.

Legendre, A., 1805. Nouelles Methods Pour La Determination Des Orbites Dis Cometes, Coucier, Pairs.

Lieblein, J. and M. Zelen, 1956. Statistical investigation of the fatigue life of deep groove ball bearing. J. Res. Nat. Bur. Stand., 57: 273-316.

Lye, L.M., K.P. Hapuarachchi and S. Ryan, 1993. Bayes estimation of the extreme-value reliability. IEEE Transactions on Reliability, 42(4): 641-644.

Mann, N.R., R.E. Schaffer and N.D. Singpurwala, 1974. Methods for Statistical Analysis of Reliability and Life Data. John Wiley & Sons, New York.

Martz, H.F. and R.A. Waller, 1982. Bayesian Reliability Analysis. Wiley, New York.

Pandey, B.N., N. Dwividi and B. Pulastya, 2011. Comparison between Bayesian and maximum likelihood estimation of the scale parameter in Weibull distribution with known shape under linex loss function. Journal of Scientific Research, 55: 163-172.

Pike, M.C., 1966. A method of analysis of certain classes of experiments in carcinogenesis. Biometrics, 22: 142-161.

Soland, R.M., 1968. Bayesian analysis of the Weibull process with unknown scale parameter and its application to acceptance sampling. IEEE Transactions on Reliability, 17: 84-90.

Weibull, W., 1951. A statistical distribution function of wide applicability. Appl. Mech., 18: 293-297.