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Partial Differential Equation Based Denoising Technique for MRI Brain Images

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ABSTRACT

Image denoising is a common pre-processing step in many Magnetic Resonance (MR) image processing and analysis tasks, such as segmentation, registration or parametric image synthesis. Image denoising remains a challenge for researchers because noise removal introduces artifacts and causes blurring of the images. In this paper four different methods are proposed to reduce the image artifacts and noise in the MRI images. These methods use Partial Differential Equations (PDE) to get better results in MRI images. The proposed methods are compared and evaluated based on the error rate and their quality of the image. The performance of the proposed denoising method is measured using quantitative performance as well as in terms of visual quality of the images.

INTRODUCTION

In medical image processing, medical images are corrupted by different type of noises. It is very important to get precise images to help exact observations for the given application (Nobi, M.N., M.A. Yousuf, 2011). Image denoising is an active area of interest for image processing researchers for a long period. The use of partial differential equations (PDEs) in image processing has grown over the past years and many of PDE based methods have particularly been proposed to tackle the problem of image denoising with a good preservation of edges, and to explicitly account for the intrinsic geometry (JenyRajan, K., M.R. Kannan, Kaimal, 2008). There have been several published algorithms and each approach has its assumptions, advantages, and limitations (Mukesh, C., Motwani, Mukesh C. Gadiya, 2004).

Image denoising is a procedure in digital image processing aiming at the removal of noise which may corrupt an image during its acquisition or transmission, while retaining its quality (Kavitha, S., Dr.V.S.Jayanthi, 2012). All denoising methods depend on a filtering parameter *h* (Buades, A., B. Coll, *et al*., 2005). The filtering value measures the degree of filtering applied to the image. For most methods, the value h depends on the estimation of the noise variance σ^2. One can define the result of a denoising method D_h as a decomposition of any image v as

$$v = D_h v + n(D_h, v)$$

where,

1. $D_h v$ is more smooth than v.
2. $n(D_h, v)$ is the noise guessed by the denoising method.

In order to get a high quality MRI image, many filtering techniques had introduced in image denoising field.

The rest of this paper is as follows: In section 2, the overview of methodologies and technical details of image filtering is described. In section 3, the experimental results and discussions made. Finally, the conclusions are given in section 4.

Methodology:

Non Local Means Filter:

Non-Local (NL) means algorithm is based on the natural redundancy of information in images to remove noise (Pierrick Coupe, Pierre Yger, Christian Barillot, 2006). The non-local means filter averages all observed pixels to recover a single pixel. The weight of each pixel is calculated by depending the distance between its

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intensity gray level vector and that of the target pixel (Antoni Buades, Bartomeu Coll, *et al*., 2005). The NLM filter is a neighborhood filter which achieves denoising by averaging similar image pixels according to their intensity similarity.

The key idea of the non-local means filter is that a given noisy image f: $\Omega \subset R^2 \mapsto R$ has filtered by

$$u(x) = \int \omega_f(x, y) f(y) d_y \tag{1}$$

where $u : \Omega \mapsto R$ is the denoised image and $w_f : \Omega \times \Omega \mapsto R^+$ is a normalized weight function written as

$$w_f(x, y) \Leftrightarrow \frac{e^{-\frac{d^2 f(x,y)}{h^2}}}{\int e^{-\frac{d^2 f(x,y)}{h^2}} dy}, \; for \;\; d^2 f(x, y) \Leftrightarrow \left\| f_x - f_y \right\|^2 G\sigma \tag{2}$$

where eqn (2) is the differences of f_x and f_y, weighted against a Gaussian window G_σ with standard deviation σ. The map $d_f(x, y)$ measures the different patches of f centered in x and y. If two patches are similar, then the corresponding weight $w_f(x, y)$ will be high. Otherwise, if the patches are dissimilar, the weight $w_f(x, y)$ will be small (but positive) (Yifei Lou, *et al*., 2009). While the parameter σ defines that the dimensions of the patch where it is used to measure the similarity between two patches, the parameter h regulates how strict or relaxed to consider the patches similarly. Finally, the result of the non-local means filter has several (similar) patch used to reconstruct another one.

Anisotropic Diffusion Filter:

The anisotropic filter is non-optimal for MR images with spatially varying noise levels of such sensitivity-encoded data and intensity inhomogeneity corrected images (Alexei, A., Samsonov, and Chris R. Johnson, 2004). The main aim of anisotropic diffusion filtering in image processing is to remove noise via a Partial Differential Equation (PDE), with respect to the image function is as follows:

$$\frac{\partial u}{\partial t} = div(k\Delta u) \tag{3}$$

where $u = u(x, y, t)$ the image is enhanced in the continuous domain in an instant t.

The initial condition for this equation of the input image is given by $u(x, y, 0)$ (Larrabide, I., A.A. Novotny *et al*., 2005). By applying values for this equation, it produces blurred image edges of the input image.

Bilateral Filtering:

The **bilateral filter** is an edge-preserving and noise reducing smoothing filter. This filtering technique is achieved by the combinations of two Gaussian filters: one filter works with spatial domain and the other one works with intensity domain (Devanand Bhonsle,Vivek Chandra,G.R. Sinha, 2012). The intensity value of each pixel in an image is replaced by a weighted average of intensity values from nearby pixels. The weight is mainly based on the Gaussian distribution.

Bilateral filtering is a non-linear filtering technique for removing noise without degrading important structures such as edges.

Several qualities of bilateral filter are listed below (Joakim Rydell, 2007):

1. It is simple to formulate it. Each pixel is replaced by a weighted average of its neighbors.
2. It depends only on two parameters that indicate the size and contrast of the features to keep.
3. It is a non-iterative method. This makes the parameters easy to set for their effort is not cumulative over several iterations.

This filtering is better than many techniques.

One of the advantages of the bilateral filter is that it smoothes the areas where the pixels are similar. This allows us to leave relatively unaffected edges in the image (Oliver, A. Nina, Bryan S. Morse).

ROF Filter:

ROF denoising method is based on total-variation, originally proposed by Rudin, Osher, and Fatemi. The total variation model is one of the most popular models. The total- variation method (Charles, A., Micchelli, Lixin Shen, 2011) is sensitive to geometric features of images but the lack of multiscale representation, which is crucial for developing efficient computational algorithms. This method can be iterated which generally results in a partial differential equation formulation the denoised image function are given as follows:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta\left(\dfrac{\Delta u}{|\Delta u|}\right) - 2\lambda(u - u_0), \\ u(x, y, t) = u_0(x, y) \end{cases}$$

This model preserves the edge of the image very well, whereas produces the block effect when dealing with the flat areas, thus the local details characteristics of the original image misplaced.

RESULTS AND DISCUSSIONS

In order to measure the image quality and performance of the algorithm, metrics such as PSNR, RMSE and SSIM to be calculated. **PSNR** is defined as the ratio of peak signal power to average noise power

$$PSNR(db) = 10\log_{10}\left(\frac{D^2 MN}{\sum_{i,j}(x(i,j) - y(i,j))}\right) \tag{5}$$

for $0 \le i \le M - 1 and 0 \le j \le N - 1$, where D is the maximum peak-to-peak swing of the signal (255 for 8-bit images). Assume that the noise $x(i, j) - y(i, j)$ is uncorrelated with the signal.

RMSE is often used to measure the difference between values predicted by a model or an estimator and the values actually observed. It has a good measure of accuracy (www.gabormelli.com/RKB/Root_Mean_Square_Error). These individual differences are also called residuals.

$$RMSE\left(\hat{\theta}\right) = \sqrt{MSE\left(\hat{\theta}\right)} = \sqrt{E((\hat{\theta} - \theta)^2)} \tag{6}$$

The RMSE of an estimator $\hat{\theta}$ with respect to the estimated value θ is defined as the square root of the mean square error.

Structural Similarity Index Metric is a method for measuring the similarity between two images. It is calculated on various windows of an image. This metric is designed to improve the methods like PSNR and MSE. The measure between two windows x and y of common size $N \times N$ is (Sathya, S., R. Manavalan, 2011):

MN=size of the reference image and filtered image.

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)} \tag{7}$$

where,

μ_x - the average of x;

μ_y - the average of y;

σ_x^2 - the variance of x;

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σ_y^2 - the variance of y;

σ_{xy} - the covariance of x and y

In this section, the performance of various de-noising techniques such as Non Local Means filter, Anisotropic Diffusion filter, bilateral filter and ROF filter are analyzed for the MRI images. The resultant images for various de-noising methods operated on original images are shown in Fig.1 and the corresponding performance metrics obtained such as Peak Signal to Noise Ratio (PSNR), Root Mean Square Error (RMSE) and Structured Similarity Index Metric (SSIM) values are tabulated in Table1. In general the PSNR for any filter should be high whereas the RMSE is very low. The value of SSIM for any filter should be high with respect to RMSE.

Conclusion:

MRI denoising is an important step, used to increase image quality and to improve performance for image analysis. The various denoising algorithms are applied to remove noise from MRI brain images. For obtaining the better denoising method in MRI images the performance metrics are evaluated. From the above table, the average value of performance metrics for the ROF denoising technique gives a higher quality of PSNR value, which is 39.4693 and gives lowest error rate 3.6091 and the SSIM is evaluated to measure the similarity between two images, which gives higher value compared to the other three algorithms for MRI brain images. The simulation result shows, the ROF algorithm has better denoising performance for the MRI images.

Table1: Performance Metrics for denoising methods

Methodology/Performance Metrics	PSNR	RMSE	SSIM
Non Local Means Filter	27.7361	8.3942	0.8526
Anisotropic Diffusion Filter	30.5399	7.3287	0.9201
Bilateral Filter	35.8799	4.7531	0.9650
ROF Filter	39.4693	3.6091	0.9852

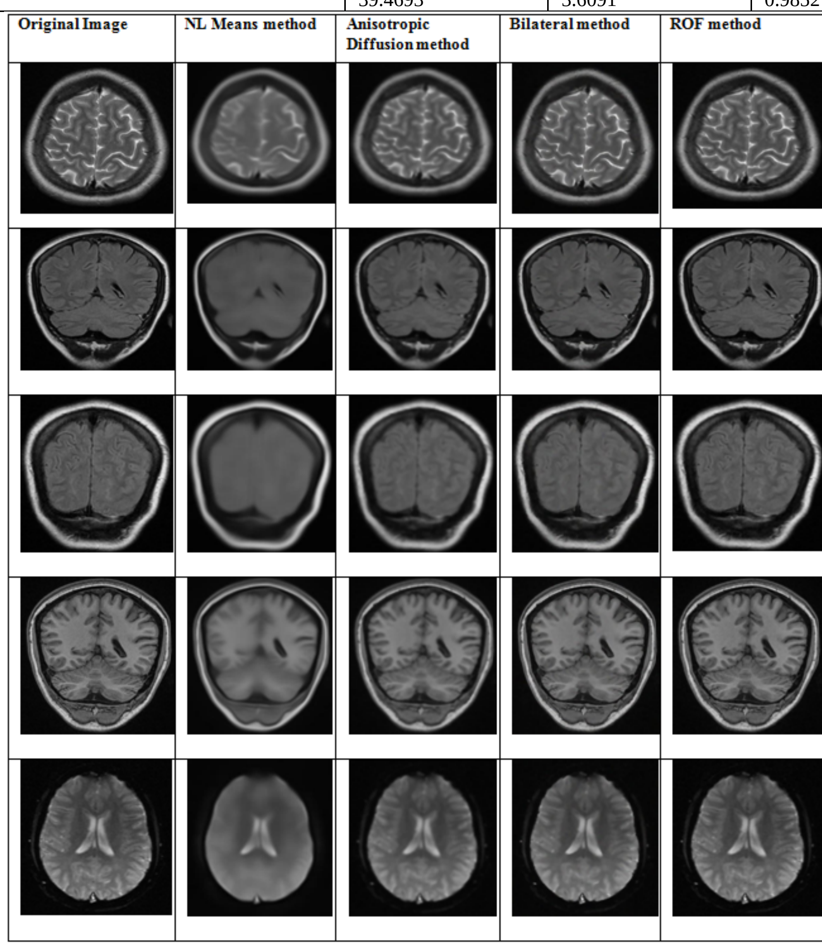

Fig. 1: Resultant Images for denoising methods

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