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The Effectiveness of MEWMA Control Chart with Mild Correlation

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ABSTRACT

Since it is undeniable that whenever conventional control charts are used there is the implied presumption that this observations tend to be independently as well as identically distributed with time. However, in reality, this kind of observations generated through continuous as well as discrete production procedures tend to be serially correlated, which violates the independence assumption of conventional control charts in addition to affect the performance of control charts negatively. In this paper, we investigate the performance of MEWMA control chart with autocorrelated data with mild correlation being controlled. The generated data were applied to MEWMA control chart procedure and showed an in-control state, while the generated observations were subjected to normality tests from the assumptions and sensitivities for departure to normality, and turned out to be normal by all standard. Hence, this gives an alternative for the quality practitioners to adopt for the continuous and discrete production processes also the autocorrelation has no effect on the performance of MEWMA control limits when the mild correlation has been controlled

INTRODUCTION

Statistical process control methods tend to be popular in industry to evaluate processes and enhance the quality of products, conventional statistical process control methodology assures that process data are statistically independent. One of the primary objectives of statistical process control would be to effectively reduce the variability in a process. This assumption holds with continuous and discrete processes, as stated by (D.H., Pignatiello *et al*. 2000) that many processes such as chemical manufacturing, electricity generation, water quality processing, waste water etc. generate autocorrelated data which violates the assumption of conventional control charts, leads to unnecessary large ARL values. (Bagshaw and Johnson 1975), discussed the effects of autocorrelation on the performance of cumulative sum (CUSUM) control charts. (Harris and Ross 1991)discussed the impact of autocorrelation on CUSUM and exponentially weighted moving average (EWMA) control charts and indicated that positive autocorrelation may also negatively impact the performance of these charts. (Woodall and Faltin 1993) discussed the effects of autocorrelation on the performance of control charts and made recommendations on how to deal with autocorrelation. (Zhang 1997) proposed a statistical control chart for stationary processes and compared its performances to some of the charts recommended for autocorrelated data.

The assumption of independence is not actually approximately satisfied in certain production processes, because the characteristics are measured in order of the production, which might introduce autocorrelation, that may have a significant effect on the performance of control chart procedure.

Within the univariate case, whenever substantial autocorrelation is actually noticed, the overall approach associated with process monitoring methods would be to fit a time series model towards the process data. The residuals, that are independent, will be accustomed to construct the control chart (Alwan and Roberts 1988; Lu and Renolds 1999) have extensively discussed it. Much more, when the model is not sufficient, the actual residuals might not be independent, consequently, there will be alarms.

Lots of approaches can be found in the literature for dealing with autocorrelation. (Montgomery and Mastrangelo 1991) reviewed various model-based methods, a model- free approach, and an engineering controller, and recommends model-based methods for getting rid of autocorrelated structures.

There are many associated with literatures which echoes about the performance of MEWMA control chart, for instance, (Lowry, Woodall *et al*. 1992; Borror, Montgomery *et al*. 1999; Testik, Runger *et al*. 2003) and so

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many other researchers but none yet have discuss the effect of autocorrelation on the performance of MEWMA using the mild level of correlation.

The research is intended to determine how effective will be the MEWMA control chart, when the data used in constructing the control chart is correlated with mild level in control but imposed with white noise(autocorrelation). In the literature, there has not been a study on the effects of the mild correlation when autocorrelated data was used to construct the MEWMA control chart. As a results, we decided to study the effectiveness of the autocorrelation on MEWMA under such condition.

The reality is that in SPC, the basic aim is to bring the quality improvement to the continuous and discrete data production both in small and large production scale and to reduce variability in the production processes.

In this article, we will investigate effect from the performance on the control limits for the multivariate exponentially weighted moving average (MEWMA) control procedure when observations are autocorrelated with mild level of correlation being controlled.

The outline of the rest of this article is as follows;

In the next section, we described the Multivariate exponentially weighted moving average and in section 3, we have talked on the materials and methods used in the analysis of data to explain our findings. Finally, we summarise our findings in section 4.

Multivariate Exponentially Weighted Moving Average Control Procedure:

The conventional Shewhart-type control charts such as the T^2 charts are pretty effective for detecting mean shifts. However, they are slow in reacting to small and moderate shifts in the process mean. In that regard, the MEWMA control chart was developed to provide more sensitivity to small mean shifts (Montgomery,2005).

The univariate EWMA chart is based on the values

$$Z_t = rX_i + (1-r)Z_{i-1} \tag{2.1}$$

$i= 1,2,...,$ where $Z_0 = \mu_0 = 0$ and $0< r \leq 1$

(Roberts 1959) showed that if $X_1, X_2,...$ are iid N(0,σ^2) random variables, then the mean of Z_i is 0 and the variance is

$$\sigma^2_{Zi} = \{r[1-(1-r)^{2i}]/(2-r)\sigma^2, \text{ i=1,2, ...,}$$

Thus, when the in-control value of the mean is 0, the control limits of the EWMA chart are often set at $\mp L\sigma_{Zi}$, where L and r are the parameters of the chart.

(Lucas and Saccucci 1990) discussed the choice of r and L from the univariate EWMA chart in details.

In the case of multivariate, a natural extension is to define the vectors of EWMA's

$$Z_i = RX_i + (1-R)Z_{i-1} \tag{2.2}$$

$i=1,2, ...,$ where $Z_0= 0$ and $R = diag(r_1, r_2, ..., r_p)$ $0<r_j \leq 1$, $j =1,2, ... ,p.$

The MEWMA chart gives an out of control signal as soon as

$$T_i^2 = Z_i' \Sigma_{Zi}^{-1} Z_i > h_4 \tag{2.3}$$

when h_4 (>0) is chosen to achieve a specified in control ARL and Σ_{zi} is the covariance matrix of Z_i is given as $\{\frac{r[1-(1-r)^{2t}]}{(2-r)}\}\Sigma_Z$.

Note from (2.2) that when Z_i is expanded recursively, we get

$$Z_i = rZ_i + r(1-r)Z_{i-1} + r(1-r)^2 Z_{i-2} + \cdots + r(1-r)^{t-1} Z_1 + (1-r)^t Z_0.$$

Thus, Z_i is a weighted average of the i quality measurements available with weights following a geometric form.

The ARL performance of the MEWMA chart depends only on the noncentrality parameter λ

In

$$\lambda = (\mu' \Sigma^{-1} \mu)^{1/2} \tag{2.4}$$

It is then much easier to make ARL comparisons among several multivariate control charts if all of the charts have this property (Lowry, Woodall *et al*. 1992).

However, as (MacGregor and Harris 1990) suggested for the univariate case, using the exact variance of the EWMA statistic leads to a natural fast initial response (FIR) for the EWMA charts, which is also true with the MEWMA control chart.

That leads to the assumption that for the chart design and the ARL comparisons the asymptotic (as i→ ∞) covariance matrix, then

$$\Sigma_{Zi} = \{r/(2-r)\}\,\Sigma \tag{2.5}$$

is used to calculate the MEWMA statistic.

(Lowry, Woodall *et al*. 1992) gave a table that contains ARL profiles of general MEWMA charts for various values of *r*, smaller values of *r* are more effective in detecting small shifts in the mean vector which is analogous to the univariate case.

This article talks on the application of MEWMA to the autocorrelated with mild level of correlation.

We generated a set of data from a multivariate random process for the 3-quality characteristics of interest by developing a(Mathworks. 2011) Mat lab source codes. As shown in Table 1 below:

MATERIALS AND METHODS

Table 1: The MEWMA scheme

Observations				MEWMA vector			MEWMA Statistic
i	X_1	X_2	X_3	Z_1	Z_2	Z_3	T^2_i
1	0.61	1.12	3.91	0.06	0.11	0.32	1.24
2	1.57	-1.69	3,95	0.21	-0.07	0.68	2.68
3	0.56	1.92	-3.33	0.25	0.13	0.28	0.61
4	0.40	3.64	-2.38	0.26	0.48	0.02	1.01
5	-0.33	-1.19	0.82	0.20	0.31	0.10	0.38
6	-0.30	1.25	-4.21	0.15	0.41	-0.34	1.54
7	0.79	2.71	2.31	0.22	0.64	-0.08	1.33
8	0.92	-1.93	-2.97	0.29	-1.35	-0.36	0.57
9	0.81	0.35	4.64	0.34	-1.18	0.14	0.28
10	1.39	0.31	-1.08	0.44	-1.03	0.01	0.07
Control Limit							h_4 = 8.66

Table 1 present the generated autocorrelated data for the three characteristics (X1,X2, X3) which was used to determine the MEWMA vector as well as the MEWMA statistic using (2.2) and (2.3) respectively.

The multivariate normal distribution is considered with unit variances and a correlation of 0.1, the process mean is on target (0,0,0) for the first 5 observations and then shifts to (1,2,3) for the last 5 observations. (X_1, X_2, X_3) are the observations in the table while Z_1, Z_2, Z_3 are the MEWMA vectors with r=0.1 also the values of T_i^2 were obtained using equation (2.3) with covariance matrix using equation (2.5) which provides the natural (HS) feature for the MEWMA chart. The value of h_4 was obtained using the simulation to provide in-control ARL's of 300. Table 1shows the data used to determined the MEWMA vector as well as the MEWMA statistic. A mat lab codes was developed to generate the desired data for the 3 characteristics of interest. The codes can be obtained from the authors based on request.

As observed by Testik, *et al* that quality practitioners should check the assumptions and the sensitivities to departures from normality before operational use of the multivariate control chart for the individual observations, if a process shows evidence of even moderate departure from the normality, the control limit may be entirely inappropriate. In view of their suggestion that we subject the generated autocorrelated for test of normality using the graphical and statistical methods, since there is not a direct test for multivariate normality, we generally test each variable individually and assume that they are multivariate normal if they are individually normal.

The 3- variables were subjected to normality test so that the data can be fit for the analysis, from the outcome of the test, it was found that the variables are normally distributed as shown in Table 2, the *Shapiro-Wilks's* significance values are all greater than 0.05. Also to support the S-W, the normality plot shows that all the 3- variables are normal as shown in Figures 2-4.

The autocorrelated values were now used to determine the MEWMA control chart with the usual procedures as spelt out by Lowry *et al* (1991), the control chart and T^2 Statistic were generated as shown in Figure1, which indicates that the control chart has the UCL of 10.81 and all the 10 observations were within the control limit, none is outside or showing an alarm.

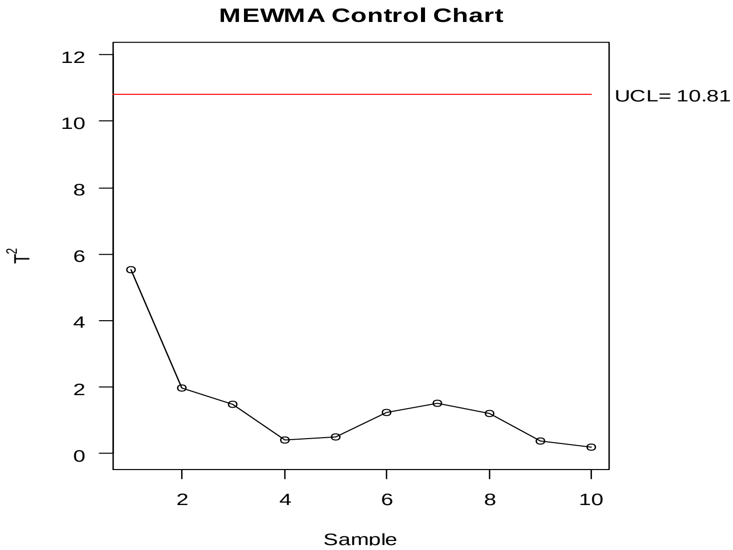

Fig. 1: The MEWMA control chart of the data

Figure 1 present the MEWMA control with the points/values lying within the control limits, with the upper control limit of 10.81 while the lower control limit being 0.

Table 2: Showing the Results of Normality Test

Tests of Normality						
	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
X1	.150	10	.200*	.934	10	.493
X2	.138	10	.200*	.957	10	.753
X3	.174	10	.200*	.913	10	.302
*. This is a lower bound of the true significance.						
a. Lilliefors Significance Correction						

Table 2 present the results of the test of normality showing the *Kolmogorov-Smirnov* and *Shapiro-Wilk* values, here since our samples is less than 50 we shall consider the Shapiro-Wilk's values instead of K-S value which is for sample size 50 and above. From S-W table all the values for the 3-characteristics on Significance column shows its values greater than 0.05, which is the rule of thumb for a variable to be normally distributed otherwise it is not normal.

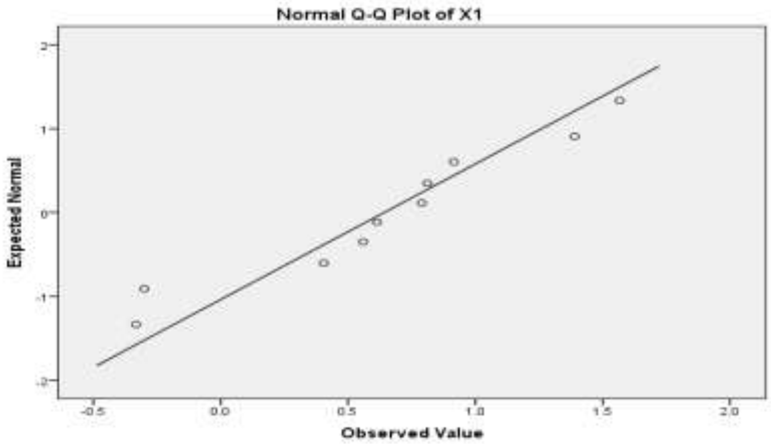

Fig. 2: The Q-Q Plot for X1

Figure 2 present the Q-Q plot for the first characteristic (X1), as we can see that the almost all points are attached to the fit line, which indicates the normality of characteristic under consideration.

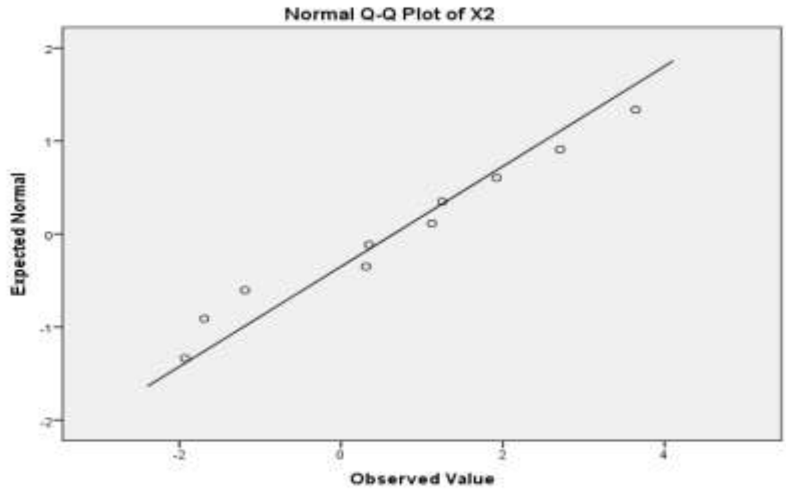

Fig. 3: The Q-Q Plot for X2

Figure 3 present the Q-Q plot for (X2), here also the points are almost attached to the fit line, that's indicates the normality of the variable under consideration.

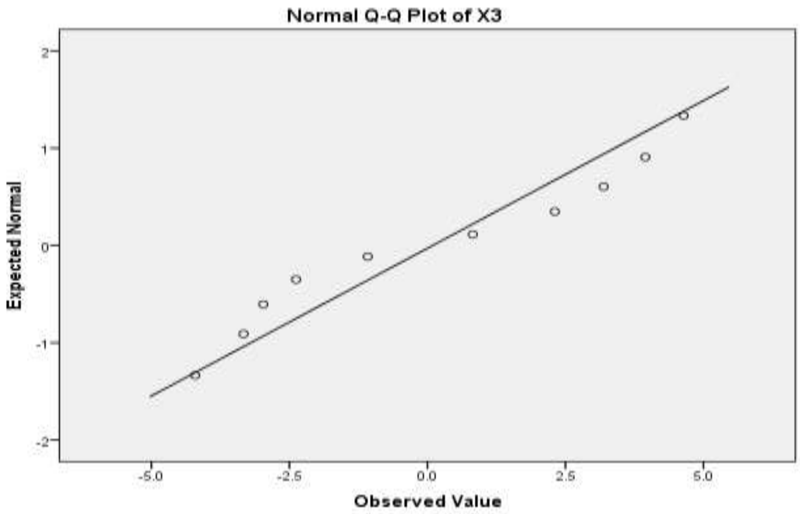

Fig. 4: The Q-Q Plot for X3

Figure 4 present also the Q-Q plot for the last characteristics (X3), the points are almost clustered to the fit line an indication of the normality.

Based on these data generated and plotted , it is observed that the use of autocorrelated data to the MEWMA when the correlated values are being controlled can lead to an in-control and it is a good alternative for the practitioners to use for the continuous data usage.

Conclusion:

From Figure 1, we can see that the generated autocorrelated data with the mild correlation applied on the MEWMA control chart has produced an in-control chart with all its values/points lying within the control limits with no point raising an alarm.

The Ch-square and T^2 charts are Shewhart-type control chart. That is, they use information only from the current sample, so consequently they are relatively insensitive to small and moderate shifts in the mean vector, as noted, that T^2 charts can be used in both phase I and phase II situations. Cumulative sum and EWMA control charts were developed to provide more sensitivity to small shifts in the univariate case, the multivariate version of these charts are a phase II procedure.

For the quality practitioner to operationally use the multivariate control charts, it has to be check for the assumptions and sensitivities to departures from the normality.

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The generated data were subjected to normality test which proves to be normal by all standard. With these results of this article it can be an alternative to other techniques for the quality practitioners to adopt for use in continuous data as well as the discrete data. With this findings its eminent to conclude that the autocorrelation has no effect on the performance of the MEWMA control limits when mild correlation is controlled.

As a result of the assumption of the traditional control charts being violated the independence by the autocorrelation. In this paper we have studied the autocorrelation being imposed on the MEWMA control procedure and observes that a this level of correlation it has not violates the independence assumption which resulted into the in-control state of the observations.

Finally, we conclude the discussion that the autocorrelated data with mild correlation controlled can result into the in-control process on multivariate exponentially weighted moving average, the above method was tested using 3 characteristics of interest but can be extended to higher characteristics desired.

We are recommending that the autocorrelated data with mild level of correlation being controlled should be applied to other statistical process control techniques.

Appendix A:

Derivation of the Covariance Matrix for Z_i

By repeated substitution of (2.2), it can be shown that

$Z_i = \sum_{j=1}^{i} R(I-R)^{i-j} X_j$.

Thus

$\Sigma_{Zi} = \sum_{j=1}^{i} var[R(I-R)^{i-j} X_j]$

$= \sum_{j=1}^{i} [R(I-R)^{i-j} \Sigma(I-R)^{i-j} R]$.

Because R and (I-R) are diagonal matrices, the (k,l)th element of Σ_{Zi} is

$r_k r_l [1-(1-r_k)^i(1-r_l)^i]/[r_k + r_l - r_k r_l]\sigma_{k,l}$, (A.1)

where $\sigma_{k,l}$ is the (k,l)th element of Σ. If $r_1 = r_2 = \ldots = r_p = r$, then the expression in (A.1) simplifies to $\{r[1-(1-r)^{2i}]/(2-r)\}\ \sigma_{k,l}$, so that

$\Sigma_{Zi}\ \ \{r[1-(1-r)2\mathrm{i}]/(2-r)\}\ \Sigma$. (A.2)

The covariance is derived here under the assumption that the control rule is ignored, but it can offers some guidance on the type of control rule to be used.{Lowry, 1992 }

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