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Converting Linear Programming Problem with Fuzzy Coefficients into Multi Objective Linear Programming Problem

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Abstract: This paper introduces a method to solve fuzzy linear programming problem when the constraint matrix and the coefficients in the objective function are fuzzy in nature and expressed as fuzzy numbers, with the help of multi objective linear programming problem. A suitable introduction and basic definitions are presented before introducing the solution method. A numerical example is solved to illustrate the method.

Key words: Fuzzy Sets, Fuzzy Numbers, Fuzzy Linear Programming.

INTRODUCTION

Many real world problems have been solved using linear programming which assumes that the data are precisely known and the constraints form a crisp set of feasible decisions and those criteria are well defined and can be easily formalized. However in many situations that is not true. Vagueness and imprecise data oppose us in solving many optimization problems. Uncertainty appears in different ways, due to lack of information or the future state of the system under consideration might not be completely known. This type of uncertainty has been handled by probability theory. However if the information about the uncertain quantity is based on a large number of measured data their relative frequencies form the probability distribution of a random variable. On the other hand, if the knowledge consists of a few expert opinions, it's more suitable to use a fuzzy variable instead.

Many researchers succeed in dealing with vague and imprecise data by using fuzzy linear programming. The concept of a fuzzy decision making was introduced by Bellman and Zadeh (1970). In the last few years a lot of work was devoted to the use of fuzzy linear programming in modeling and solving real life problems. Zimmermann (1976), (1978) introduced an application of fuzzy optimization techniques to linear programming problems with multiple objectives. Tanaka *et al*. (1991) presented a fuzzy approach to multi objective linear programming problems. Zhang *et al*. (2003) formulated a fuzzy linear programming problem as four objective constrained optimization problem where the cost coefficients are fuzzy.

In this section we are going to present some basic definitions that we found them very useful in dealing with fuzzy linear programming problems.

Definition (1): Let E be a referential set (for example R or Z), a fuzzy subset A will be defined by its characteristic function, called the membership function, which takes its value in the interval [0, 1] instead of the binary set { 0,1}

$x \in E : \quad \mu_A(x) \in [0, 1]$

Definition (2): A fuzzy subset A$\in$ R is convex if and only if

$x_1, x_2 \in R \quad \mu_A[\lambda x_1 + (1- \lambda)x_2] \geq \mu_A(x_1) \in \mu_A(x_2)$

$\lambda \in [0, 1]$

Definition (3): A fuzzy subset A $\in$ R is normal if and only if

$x \in R \qquad \in_x \ \mu_A(x) = 1$

Definition (4): A fuzzy number is a fuzzy subset that is convex and normal.

There are many different types of fuzzy numbers; here we will focus our attention on triangular fuzzy numbers and trapezoidal fuzzy numbers as they will be used in forming our fuzzy linear programming problem.

1- Triangular fuzzy number

A fuzzy number A is a triangular fuzzy number denoted by (a_1, a_2, a_3) if its membership function μ_A is given by

$$\mu_A(x) = \begin{cases} (x - a_1)/(a_2 - a_1) & a_1 \leq x \leq a_2 \\ 1 & x = a_2 \\ (a_3 - x)/(a_3 - a_2) & a_2 \leq x \leq a_3 \end{cases}$$

This membership function is illustrated in figure (1).

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2- Trapezoidal fuzzy number

A fuzzy number A is a trapezoidal fuzzy number denoted by (a_1, a_2, a_3, a_4) if its membership function μ_A is given by

$$\mu_A(x) = \begin{cases} (x - a_1) / (a_2 - a_1) & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ (a_4 - x) / (a_4 - a_3) & a_3 \le x \le a_4 \end{cases}$$

This membership function is illustrated in figure (2).

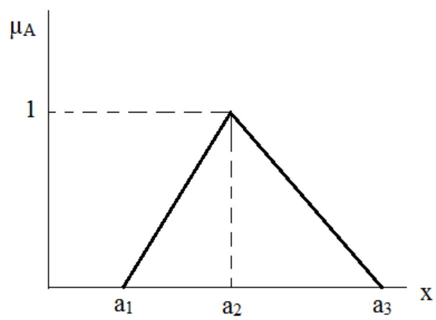

Fig. 1: Triangular fuzzy number
A = (a_1, a_2, a_3)

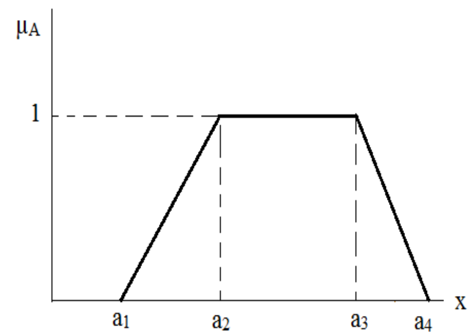

Fig. 2: Trapezoidal fuzzy number
A = (a_1, a_2, a_3, a_4)

For any triangular fuzzy numbers A = (a_1, a_2, a_3) and B = (b_1, b_2, b_3), A ≤ B if and only if $a_1 \le b_1$, $a_2 - a_1 \le b_2 - b_1$ and $a_1 + a_3 \le b_1 + b_3$. Obviously ordering of fuzzy numbers is also important, Campose and Verdegay (1989) stated that a comparison between fuzzy numbers is in fact a comparison between alternatives, Yager (1981) has proposed several ranking functions, one of them represents the centroid of the area under the membership function curve. The second index measures the consistency of the given fuzzy number with the linear fuzzy set $\mu(x) = x$. Here we will use Yager's associated number which was proposed as follows: If U^α is the α- level set of the fuzzy number and if $M(U^\alpha)$ is the mean value of elements of U^α then Yager's index will be calculated as $P = \int M(U^\alpha)\, d\alpha$.

Fuzzy Linear Programming Problem:

We consider the fuzzy linear programming problem (FLPP) where the coefficients in both the objective function and constraints are fuzzy numbers

$$Max\ Z = \sum_{j=1}^{n} C_j x_j$$

Subject to

$$\sum_{j=1}^{n} A_{ij}\, x_j \le \ B_i \qquad 1 \le i \ \le m$$

$x_j \ge 0$

Considering A_{ij} and B_{ij} are triangular fuzzy numbers with membership functions as illustrated in figure (1).

$A_{ij} = (a_1, a_2, a_3)_{ij}$

$B_{ij} = (b_1, b_2, b_3)_{ij}$

And coefficients C_j appear in the objective function are trapezoidal fuzzy numbers with membership functions as illustrated in figure (2).

$C_j = (c_1, c_2, c_3, c_4)_j$

Our work provided a method for solving FLPP by converting the problem into equivalent crisp linear problem which can be solved by standard optimization methods. The FLPP with fuzzy coefficients in the objective function will be converted into multi objective linear programming problem (MOLPP).

In general, MOLPP refers to those linear programming problems in which multiple objectives are to be achieved. By considering the weighting factor, the objective function in our FLPP may be converted to

Max {w_1Z_1, w_2Z_2, ……….,w_kZ_k}

Subject to the constraints of the original problem. Converting these fuzzy constraints into crisp ones using the principles of ordering fuzzy numbers, sowe obtain crisp linear programming problem which can be solved using standard optimization methods.

The following numerical example illustrate the method, it was solved by converting each constraints with triangular fuzzy numbers as coefficients into three crisp constraints, then checking the solution obtained using Yager's ranking approach.

Numerical Example:

The management of a certain company is trying to determine the amount of each of two products to produce over the coming planning period. The information concerns labor availability, labor utilization, and product profitability is presented in Table (1). Due to the changes in work circumstances all the information given to the decision maker are fuzzy numbers.

Table 1: Labor availability, labor utilization, and product profitability

Department	Product (hours/unit)		Labor hours available
	(1)	(2)	
A	(0.8, 1, 1.1)	(0.2, 0.35, 0,4)	(75, 100, 105)
B	(0.2, 0.3, 0.5)	(0.1, 0.2, 0.25)	(40, 56, 60)
C	(0.1, 0.2, 0.25)	(0.4, 0.5, 0.6)	(38, 50, 55)
Profit / unit	(28, 30, 32, 35)	(12, 15, 17, 20)	

Let x_1 : number of produced units of product (1)

x_2 : number of produced units of product (2).

The optimization problem can be formulated as following

Solve the following FLPP

Max $Z(x_1, x_2) = (28, 30, 32, 35)\ x_1 + (12, 15, 17, 20)\ x_2$

Subject to the constraints

$(0.8, 1, 1.1)\ x_1 + (0.2, 0.35, 0{,}4)\ x_2 \leq (75, 100, 105)$

$(0.2, 0.3, 0.5)\ x_1 + (0.1, 0.2, 0.25)\ x_2 \leq (40, 56, 60)$

$(0.1, 0.2, 0.25)\ x_1 + (0.4, 0.5, 0.6)\ x_2 \leq (38, 50, 55)$

$x_1 \geq 0$

$x_2 \geq 0$

We noticed that the coefficients in the objective function are trapezoidal fuzzy numbers while the coefficients in the constraints are triangular fuzzy numbers.

Now converting the given FLPP into the equivalent MOLPP

Max $\{w_1 (28\ x_1 + 12\ x_2),\ w_2 (30\ x_1 + 15\ x_2),$

$w_3 (32\ x_1 + 17\ x_2),\ w_4 (35\ x_1 + 20\ x_2)\}$

Subject to

$0.8\ x_1 + 0.2\ x_2 \leq 75$

$0.2x_1 + 0.1\ x_2 \leq 40$

$0.1\ x_1 + 0.4\ x_2 \leq 38$

$0.2\ x_1 + 0.15\ x_2 \leq 25$

$0.1\ x_1 + 0.1\ x_2 \leq 16$

$0.1\ x_1 + 0.1\ x_2 \leq 12$

$1.9\ x_1 + 0.6\ x_2 \leq 180$

$0.7\ x_1 + 0.35x_2 \leq 100$

$0.35\ x_1 + x_2 \leq 93$

$x_1 \geq 0$

$x_2 \geq 0$

The convex set of all feasible solutions is closed and bounded, thus an optimal solution is obtained. Regarding the weight appeared in the multi objective function, Thakre *et al*. (2009) reported that the solution is independent of weights. Table (2) lists the solution of the MOLPP for various values of weights

Table 2: Optimal solution of the MOLPP for various values of weights

w_1	0.5	0	0.5	0.4	0.2	0
w_2	0	0.5	0.5	0.4	0.2	0.4
w_3	0.5	0	0	0.2	0.4	0.2
w_4	0	0.5	0	0	0.2	0.4
Optimal solution	(55, 60)	(55, 60)	(55, 60)	(55, 60)	(55, 60)	(55, 60)

Checking the obtained optimal solution by resolving the FLPP using Yager's associated number of the fuzzy coefficients in the constraints as a powerful ranking tool. The MOLPP will be subjected to the following constraints

$0.975\ x_1 + 0.325\ x_2 \leq 95$
$0.325\ x_1 + 0.1875\ x_2 \leq 53$
$0.1875\ x_1 + 0.5\ x_2 \leq 48.25$
$x_1 \geq 0$
$x_2 \geq 0$

We obtained the following results listed in Table (3) which is nearly the same regarding that Yager's approach considered an approximated method to be used in solving linear programming

Table 2: Optimal solution of the MOLPP using Yager's approach

w_1	0.5	0	0.5	0.4	0.2	0
w_2	0	0.5	0.5	0.4	0.2	0.4
w_3	0.5	0	0	0.2	0.4	0.2
w_4	0	0.5	0	0	0.2	0.4
Optimal solution	(53, 59)	(53, 59)	(53, 59)	(53, 59)	(53, 59)	(53, 59)

Conclusion:

We successfully deal with fuzzy linear programming problem where the constraints matrix and the objective function's coefficients are fuzzy numbers by converting it into a multi objective constrained linear programming problem, then checking the solution by one of the reported ranking approaches.

Appendix A:

List of Symbols

R	Set of Real Numbers
Z	Set of Integers
□	Minimum
□	Maximum
$\mu_A(x)$	Membership Function for Element x with Respect to Fuzzy Number A.
FLPP	Fuzzy Linear Programming Problem
MOLPP	Multi Objective Linear Programming Problem

Appendix B:

List of Figures

Figure (1)	Triangular Fuzzy Number
Figure (2)	Trapezoidal Fuzzy Number

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