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Takagi Sugeno Fuzzy Sliding Mode Controller Design for a Class of Nonlinear System

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Abstract: This paper presents with the synthesis of fuzzy sliding mode controller nonlinear system based on Takagi Sugeno (T-S) fuzzy model. The concepts of state feedback control and Lyapunov functional approach is utilized in deriving the sufficient condition for robust stable which then formulated in the forms of linear matrix inequalities (LMI). Therefore the global asymptotically stability of sliding mode control (SMC) for a class of nonlinear systems via Takagi Sugeno fuzzy model is guaranteed. An example is given to demonstrate the feasibility and effectiveness of this approach.

Key words: Nonlinear system, T-S Fuzzy Model, Sliding Mode Control, LMI.

INTRODUCTION

Over the last decades, there has been a numerous research on nonlinear modeling. It's commonly known that an efficiency of the designed controller is based on the model representing the plant or system itself. Therefore it's crucial to model the system as exactly as the system. Classic controller designs are based on linearizing the model, thus exhibit substantial stability problem when the system is highly nonlinear. Ever since Takagi and Sugeno (1985) and Zadeh (1996) presented an ingenious method to represents a nonlinear systems as a group of linear time invariant (LTI) model mixed with nonlinear functions to form a fuzzy model representation, thus the used of Takagi-Sugeno (T-S) fuzzy model approach has been applied in numerous applications and attracted numerous studied and researches such as A.O. Hamadameen and Z.M. Zainuddin (2013), Hameed *et al* (2012), Park *et al* (2007), Guerra *et al* (2001) and Chiang *et al* (2001). This is due to its efficiency to control highly and complex nonlinearity in the system. Utilizing the state feedback approach, the same principles is applied to parallel distributed compensation (PDC) which then interpolated with the feedback gains in each of the determined Takagi-Sugeno (T-S) fuzzy rules as stated in Tanaka *et al* (1998). Furthermore the global linearized fuzzy model is made up from set of local linearized models which are derived from set of membership functions.

In T-S fuzzy model, the stability analysis approach is based on determining a common positive definite matrix P which normally transforms in to linear matrix inequalities (LMIs) as in Nachidi *et al.* (2008), Park (2003) and Boyd *et al*. (1994). However the task of determined the positive definite matrix is still difficult when involved a large number of fuzzy rules. Nevertheless in recent years there has been numerous interest in the studied of stability issues of nonlinear system based on T-S fuzzy system as published by Xiao and Zeng (2000), Tanaka and Sano (1996), Wang *et al*. (1996) and Kim *et al.* (1995).

Sliding mode control systems theory have been widely been studied to cater the nonlinear dynamic control problems arise from uncertainty parameter, time varying delay and external disturbances as in Huang *et al.* (2008), Young *et al.* (1999) and Utkin (1977). The main concept of SMC is designing a control law which guides the system state to reach and remain on the switching surface.

In this paper, we attempt to synthesize the concepts of state feedback and Lyapunov functional approach to obtain a sufficient stability condition for designing a robust sliding mode plane. By our approach, it can be seen that the derivation of the controller is straight forward and the approach of finding the required parameters are reduced to solving linear matrix inequalities (LMIs). We proposed yet another alternative by means of improving the T-S fuzzy model based control for a class of nonlinear system. To be precise, we present a systematic design procedure of T-S fuzzy model based control with guaranteed stabilization for a class of nonlinear system. A numerical example is presented using the proposed algorithms to show the effectiveness and the feasibility of the controller.

This paper is organized as follows. Section II presents the problem formulation of this intended paper. Section III presents the main results of the studied problem of sliding mode control for a class of nonlinear systems via Takagi Sugeno fuzzy model. Section IV presents the numerical example of the presented control approach with the simulation results and analysis. Finally section V presents the conclusion of this paper.

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Problem Formulation:

Consider a Takagi and Sugeno fuzzy model representing a class of uncertain nonlinear systems as:

Model Rule i:
IF $z_i(t)$ is M_i^1 and … and $z_p(t)$ is M_p^i
THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t) + f_i(x,t) \tag{1}$$

where $i = 1,2 \ldots, r$. r is the number of IF-THEN rules and $x(t) \in R^n$ is the state vector of the systems. $u(t) \in R^m$ and $y(t) \in R^q$ is the input vector and the output vector of the systems respectively. $B_i \in R^{n\times m}$ and $A_i \in R^{n\times n}$ are the system input matrices and the systems matrices and respectively. ΔA_i represents the parameters uncertainties and $f_i(x,t)$ is bounded external disturbance. $M_j^i, j = 1,2 \ldots, p$ is denoted as the j^{ith} fuzzy set for the i^{ith} rule and $z_1(t),..,z_p(t)$ are the known variables functions of state variables. $w_j^i(z_j)$ is denoted as the membership function for j^{ith} fuzzy set M_j^i of i^{ith} and $w_i(z(t) = \prod_{j=1}^{p} M_j^i\,(z(t))\; i = 1,..,r$

Let's denotes the pair of $x(t), u(t)$, as the fuzzy systems output represented by Wang *et al*. (1996) as:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i\,(t)\big((A_i x(t)) + B_i u(t) + f_i(x,t\big)}{\sum_{i=1}^{r} w_i\,(t)} \tag{2}$$

$z(t)$ is the premise vector for $z(t) = z_1, z_2 \cdots z_p$

As the $z(t)$ is regards as the combination of linear and state vector, thus the weight function can be represented as:

$$h_i\big(z(t)\big) = \frac{w_i(z(t)}{\sum_{i=1}^{r} w_i\,(t)} \quad , i = 1,\cdots,r \tag{3}$$

for all *t*.

$w_j^i\big(z_j\big)$ denoted as the membership grade for $z_j(t)$ in M_j^i from (2), noting

$$\begin{cases} \sum_{i=1}^{r} h_i\big(z(t)\big) = 1 \\ \quad h_i\big(z(t)\big) \geq 0 \end{cases} \qquad i = 1,\cdots,r$$

Therefore the Takagi Sugeno fuzzy model is represents as:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i\,(t)\,\Big(\big(A_i\,x(t)\big) + B_i\,u(t) + f_i(x,t\Big) \tag{4}$$

and the output of Takagi Sugeno fuzzy model is represents as:

$$y(t) = \sum_{i=1}^{r} h_i\,(t)\,C_i\,x(t) \tag{5}$$

Before proceeding, the following assumptions are needed.

Assumption 1: There exists $K \in R^{m\times n}$ for the pair (A_i, B) is stabiliseable such that $\overline{A_\iota} = A_i - BK$ is stable.

Assumption 2: $B_1 = B_2 = \cdots = B_n := B$ and B is a full column rank matrices.

Assumption 3: The external disturbances satisfying

$$f_i(x,t) = B\overline{f_\iota}(x,t), \tag{6}$$
$$\big\|\overline{f_\iota}(x,t)\big\| \leq \delta(t), \tag{7}$$

The first step is to determine the plane for the sliding mode in order to design the fuzzy sliding mode control Therefore we choose the sliding plane to be:

$$S = B^T P x(t) \tag{8}$$

where $P \in R^{m\times n}$ is a determined positive definite matrix.

The main purpose of this section is to predetermine the stability condition for the nonlinear Takagi Sugeno fuzzy model in two parts. The first part is to derive an appropriate sliding plane that guaranteed the trajectories of the systems at any given initial states values can converge into sliding plane within finite time. Second part is to determine the conditions that is sufficient to achieve an asymptotically stability of the control system.

Main Results:

The studied of derived sliding plane using Lyapunov functional approach for SMC theory has been studied proposed by Utkin (1977). Therefore we applied this concept of approach via Takagi Sugeno fuzzy model for a class of nonlinear systems.

Theorem 1: Noting on the assumptions 1-4, the trajectories for a class of nonlinear system at any given initial states, are brought within the sliding plane within a finite time via Takagi Sugeno fuzzy model with a given control as:

$$u(t) = u_{eq} + u_n \tag{9}$$

where u_{eq} is the equivalent control described as:

$$u_{eq} = -\sum_{i=1}^{n} h_i\left(z(t)\right)(B^T PB)^{-1}[B^T PA_i x(t)] \tag{10}$$

and u_n is the switching control described as:

$$u_n = -\sum_{i=1}^{n} h_i\left(z(t)\right)\{(B^T PB)^{-1}[\|B^T PB\| \cdot \delta + \varepsilon_0]sgn(s)\} \tag{11}$$

where ε_0 is a small positive constant.

Proof: Consider the Lyapunov function

$$V = 0.5S^T S \tag{12}$$

Thus derivative of the Lyapunov functional (12) along the trajectory of the system (2) is

$$\dot{V} = S^T \dot{S} = S^T B^T P \dot{x}(t)$$
$$= \sum_{i=1}^{n} h_i(z(t))\, S^T B^T P[A_i x(t) + B_i u(t) + f_i(x,t)]$$

Substituting (12) into the above equation, yields

$$\dot{V} = \sum_{i=1}^{n} h_i(z(t))\, S^T B^T P[A_i x(t) + B_i u(t) + f_i(x,t)]$$
$$= \sum_{i=1}^{n} h_i(z(t))\, [S^T B^T PB\bar{f}_i(x,t) + S^T B^T PBu_n]$$

Considering (6)-(9) and (3), $\dot{V}$ can be expressed as

$$\dot{V} \leq \sum_{i=1}^{n} h_i(z(t))\, \|S^T\|[\|B^T PB_i\| \cdot \delta_f(t)] + S^T B^T Bu_n$$
$$= \sum_{i=1}^{n} h_i\left(z(t)\right) \varepsilon_0 S^T sgn(S)$$
$$= \varepsilon_0 \|S\|$$
$$\leq 0$$

This proved that the trajectories of the system will converge into sliding plane within a finite time. Therefore the proof is completed.

The subsequent part is to derive a robust switching control so that trajectories of the system will remains in the sliding plane once in reached and maintaining the stability of the system even in the presence of disturbance. Thus the following is obtained.

Theorem 2: As stated by Tanaka and Sano (1996), the fuzzy model in (2) is said to be asymptotically stable when there exist a common positive definite matrix P which satisfies the following Lyapunov inequalities:

$$G_{ii}^T P + PG_{ii}^T < 0 \tag{13}$$

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij}+G_{ji}}{2}\right) < 0 \quad \text{for i < j} \tag{14}$$

where $G_{ij} = A_i - B_i F_j$

For each $i, j \in \{1,2 \cdots r\}$ except for pairs of $\forall t \; h_i(z(t))h_j(z(t)) \neq 0$ where r is the number of if-then rules. Using linear matrix inequalities (LMl's) transformations as proposed by Tanaka and Sano (1994), yields the exponential stability condition. Then transforms into LMIs term by multiplication of P^{-1}. Denoting $X = P^{-1}$ and $M_i = F_i X$ yields:

$$X A_i^T + A_i X - B_i M_i - M_i^T B_i^T < 0 \quad \text{i = 1,…,r} \tag{15}$$

$$X(A_i^T + A_j^T) + (A_i + A_j)X - (B_i M_j + B_j M_i) - (B_i M_j + B_j M_i +)^T < 0 \quad i < j \leq r \tag{16}$$

Numerical Example:

Consider the control of Duffing-forced oscillation system.

$$\begin{aligned} \dot{x}_1(t) &= x_2 \\ \dot{x}_2(t) &= -0.1x_2 - x_1^3 + 12\cos(t) + u(t) \end{aligned} \tag{17}$$

The system is chaotic without control. This chaotic system is constructed by linearized the nonlinear terms to a set of operation point at phase plane in the form of T-S fuzzy model. Therefore the T-S fuzzy model rules are obtained as follows:

Model Rule 1: IF x_2 is M_1 , THEN
$\dot{x}(t) = A_1 x(t) + B_1[u(t) + f_1(x,t)]$

Model Rule 2: IF x_2 is M_2 , THEN
$\dot{x}(t) = A_2 x(t) + B_2[u(t) + f_2(x,t)]$

Where

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -12 & -0.1 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 1 \\ -48 & -0.1 \end{bmatrix}, \; B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

The chosen membership functions for $x_1(t)$ are as illustrates in Figure 1.

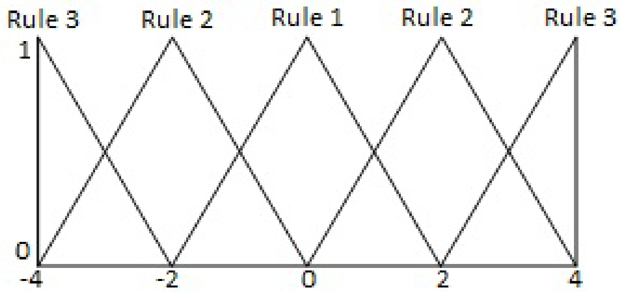

Fig. 1: Membership function for state x_1.

By using LMI toolbox, solving the LMIs (15)-(16) there exists a feasible solution with the symmetric positive definite P as:

$$P = \begin{bmatrix} 119.4856 & 9.3986 \\ 9.3986 & 1.6626 \end{bmatrix}$$

Based on the two theorems presented, the given uncertain nonlinear system is stably robust and therefore asymptotically stability is guaranteed. The simulation results with given initial $x(0) = [2\;2]^T$ are shown in Figure 1-3.

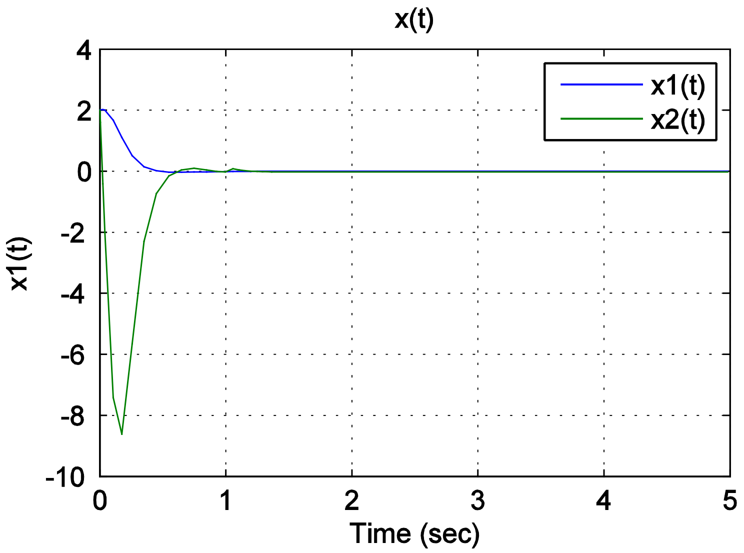

Fig. 2: System state under proposed SMC.

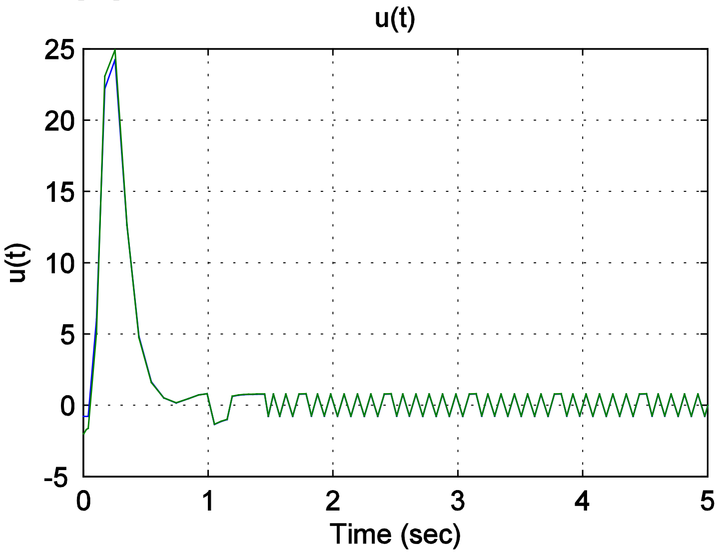

Fig. 3: The proposed SMC.

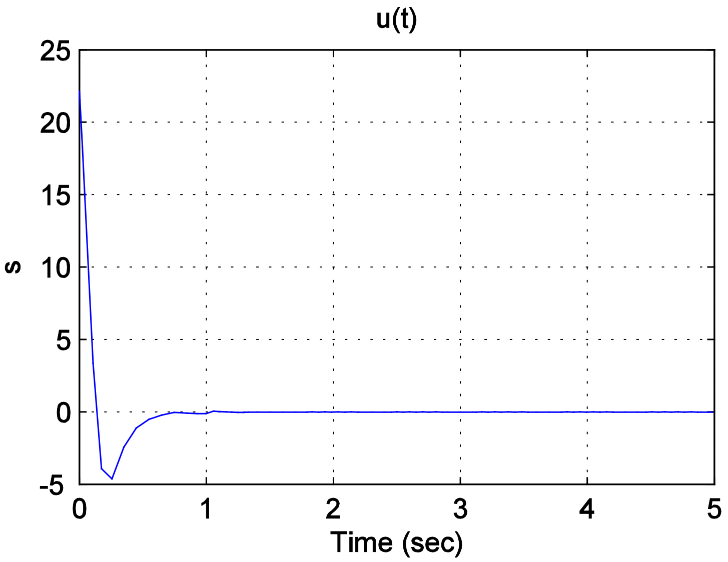

Fig. 4: The sliding mode of system.

Conclusion:

In this paper we present the studied of fuzzy sliding mode control for a class of nonlinear systems via Takagi Sugeno fuzzy. The approach design is conceptually simple thus reduce the conservatism and computational efforts even to complex nonlinear system. Furthermore, the analysis for system stability and controller design approach is formulated into Linear Matrix Inequality (LMI) terms. Finally an example used to illustrate numerically the feasibility and effectiveness of this control scheme.

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