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Continua and Separation of the Components of Fuzzy Topographic Topological Mapping

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Abstract: The mathematical model Fuzzy Topographic Topological Mapping (FTTM) is used to solve neuromagnetic inverse problem during a seizure in order to determine the location of epileptic foci. Continuum and separation axioms are topological properties. In this paper, the properties of connectedness, compactness, and separations for the components of FTTM are presented.

Key words: Fuzzy Topographic Topological Mapping, connected, compact, continuum, separation.

INTRODUCTION

Fuzzy Topographic Topological Mapping (FTTM) is a mathematical model for determining the location of epileptic foci in epilepsy disorder patients. This model had been developed by Fuzzy Research Group (FRG) at UTM. In fact, FTTM is a mathematical model using topology and fuzzy structure for detecting the current sources of the magnetic fields (Tahir *et al*., 2005). These structures were shown to be homeomorphics (Yun, 2001). Furthermore, FTTM is given as a set of mathematical operations, namely, topological transformations with four components and connected by three different algorithms. There are FTTM1 and FTTM2, whereby FTTM1 is used to solve the inverse problem for determining single current source. In 2001, Yun proved topological equivalents between these components. FTTM2 is specifically designed to solve the inverse problem for determining multiple current sources.

The study of invariant topological properties under homeomorphism is a central point in aspect of topology. The main aim of this paper is to establish more properties of FTTM.

Preliminaries:

In this section, some related basic definitions of topological structures are reviewed. The notion of boundedness which is often based on the particular metric function is given as follows:

Definition 2.1 (Adams and Franzosa, 2008) Let (X, d) be a metric space. A subset E of X is called a ***bounded set*** if and only if, there exists a positive real number M such that $d(x, y) \leq M$, for every $x, y \in E$. When X itself is bounded under d, then d is a ***bounded metric***.

By using the following theorem, each topology induced by a metric space is also induced by a bounded metric space.

Theorem 2.2 (Adams and Franzosa, 2008) Let (X, d) be a metric space, and define a metric function $d'\colon X\times X\to\mathbb{R}^+$ as $d'= \min\{d(x,y),1\}$. Then, d' is a bounded metric that is induced the same topology as d.

The definitions of separation axioms are stated as follows:

Definitions 2.3 (Shick, 2007) Let (X,τ) be a topological space, then X is called a:

i) ***T_o - space*** if and only if, for any two distinct points in X, there is an open set in X which contains one of those points, but does not contain the other.

ii) ***T_1- space*** if and only if, for any two distinct points $x, y \in X$, there are two open subsets U and V of X, such that $x \in U$ while $y \notin U$ and $y \in V$ while $x \notin V$.

iii) ***T_2 - space (Hausdorff space)*** if and only if, for each two distinct points x and y of X, there are disjoint open subsets U and V of X, such that $x \in U$ and $y \in V$.

iv) ***regular space*** if and only if, for any closed subset F of X and for any point $x \in X/F$, there exist two disjoint open subsets U and V of X such that $x \in U$ and $F \subseteq V$.

v) ***T_3- space*** if and only if, X is a regular and T_1- space.

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vi) ***normal space*** if and only if, for any two disjoint closed subsets H and K of X, there exist two disjoint open subsets U and V of X such that $H \subseteq U$ and $K \subseteq V$.

vii) ***T_4- space*** if and only if, X is a normal and T_1- space.

viii) ***path connected*** if and only if, for each $x, y \in X$, there exists a continuous function $f:[0,1]\rightarrow X$ such that $f(0)=x$ and $f(1)=y$. A mapping f is called a ***path*** which connects x to y.

The notion of continuum is defined as follows:

Definition 2.4 (Whyburn and Duda, 1979) Let (X,τ) be a topological space, then X is called a ***continuum*** if and only if, X is connected and compact.

Fuzzy Tppographic Topological Mapping (FTTM):

The components in the first version of FTTM are illustrated in the following figure:

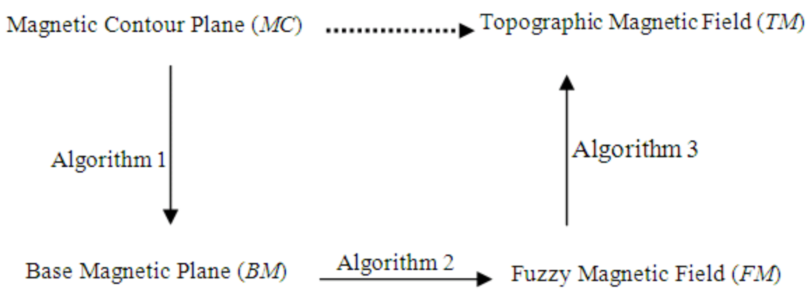

Fig. 1: FTTM Version 1 (Yun, 2006).

As mentioned by Yun (2006) that the first component in FTTM1 is magnetic contour plane (MC). In fact, MC is a plane at the level z = 0 above single current source, which contains a counter plotted according to magnetic field data (magnetic field readings in a parallel direction with z-axis, B_z at the level z = 0) generated by the single current source, such that:

$$MC = \{((x,y,0), B_{Z\,(x,y)}) : x, y \in \mathbb{R}, B_{Z\,(x,y)} \in [B_{Z\,min}, B_{Z\,max}]\}$$
$$= \{((x,y)_0, B_{Z\,(x,y)}) : x, y \in \mathbb{R}, B_{Z\,(x,y)} \in [B_{Z\,min}, B_{Z\,max}]\}, \quad (1)$$

and

$B_{Z\,min}$: the smallest magnetic field reading.

$B_{Z\,max}$: the largest magnetic field reading.

$$B_{Z\,(x,y)} = \frac{\mu_o I}{2\pi}\left[\frac{(y-y_p)}{(y-y_p)^2 + [h + |x-x_p| \tan(\phi\text{-}90^o)]^2}\right] \quad (2)$$

and

μ_o : is the permeability of free space and its value is $4\pi\times10^{-7}$ (meter. Tesla/ ampere).

I : is the magnitude of current in ampere.

$((x_p, y_p)_0, B_{Z\,(x_p,y_p)})$: is the element of MC which is exactly above the current source.

ϕ: is the angle between current source and z-axis.

h: is the distance between MC and the current source in meter.

Magnetic field readings values are either positive or negative based on the direction of the flowing current.

Figure 2 shows the direction of the current (I) and the height of a measuring point from I, such that $u = |x - x_p| \tan(\phi\text{-}90^o)$.

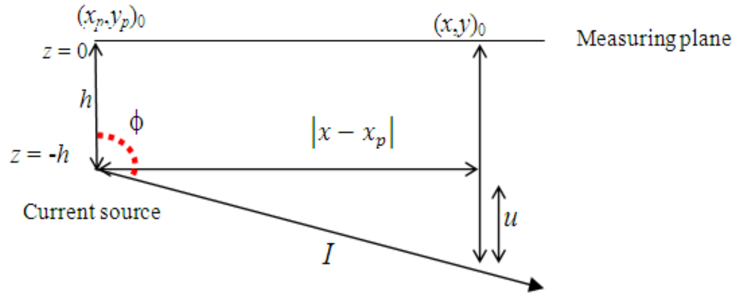

Fig. 2: The height of a measuring point from the flowing current (Yun, 2006).

Base magnetic plane (*BM*) is a base at the level $z = -h$, which contains a contour that is lowered vertically from *MC*, such that:

$$\begin{aligned} BM &= \left\{\left((x,y,-h), B_{Z\,(x,y)}\right): x,y \in \mathbb{R}, B_{Z\,(x,y)} \in [B_{Z\,min}, B_{Z\,max}]\right\} \\ &= \left\{\left((x,y)_{-h}, B_{Z\,(x,y)}\right): x,y \in \mathbb{R}, B_{Z\,(x,y)} \in [B_{Z\,min}, B_{Z\,max}]\right\}. \end{aligned} \tag{3}$$

Fuzzy set is applied on each of magnetic field readings of *BM* and

$$\begin{aligned} FM &= \left\{\left((x,y,-h), \mu_{B_{Z\,(x,y)}}\right): x,y,-h \in \mathbb{R}, \mu_{B_{Z\,(x,y)}} \in [0,1]\right\} \\ &= \left\{\left((x,y)_{-h}, \mu_{B_{Z\,(x,y)}}\right): x,y,-h \in \mathbb{R}, \mu_{B_{Z\,(x,y)}} \in [0,1]\right\}, \end{aligned} \tag{4}$$

such that:

$$\mu_{B_{Z\,(x,y)}} = \frac{|B_{Z\,(x,y)}| - MB_{Z\,min}}{B_{Z\,max} - B_{Z\,min}} \tag{5}$$

and $B_{Z\,(x,y)}$ as in (2).

The values of *z* are found by defuzzification of the fuzzified magnetic field readings,

$$TM = \left\{\left(x, y, z_{B_{Z\,(x,y)}}\right): x,y \in \mathbb{R}, z_{B_{Z\,(x,y)}} \in [-h, 0]\right\} \tag{6}$$

such that:

$$z_{B_{Z\,(x,y)}} = h\left(\mu_{B_{Z\,(x,y)}} - 1\right), \tag{7}$$

The homeomorphisms between the components of FTTM1 are proven in (Tahir *et al.*, 2005). The mappings for these homeomorphisms are defined as below (Yun, 2006):

i) $bm: MC \rightarrow BM$ such that :

$$bm\left((x,y)_0, B_{Z\,(x,y)}\right) = \left((x,y)_{-h}, B_{Z\,(x,y)}\right),\ \forall\left((x,y)_0, B_{Z\,(x,y)}\right) \in MC. \tag{8}$$

ii) $fm: BM \rightarrow FM$ such that:

$$fm\left((x,y)_{-h}, B_{Z\,(x,y)}\right) = \left((x,y)_{-h}, \mu_{B_{Z\,(x,y)}}\right),\ \forall\left((x,y)_{-h}, B_{Z\,(x,y)}\right) \in BM. \tag{9}$$

iii) $tm: BM \rightarrow FM$ such that :

$$tm\left((x,y)_{-h}, \mu_{B_{Z\,(x,y)}}\right) = (x,y,z),\ \forall\left((x,y)_{-h}, \mu_{B_{Z\,(x,y)}}\right) \in FM. \tag{10}$$

With $-h<0$, and $B_z \in B \subseteq \mathbb{R}$.

The components of FTTM2 are described in Figure 3.

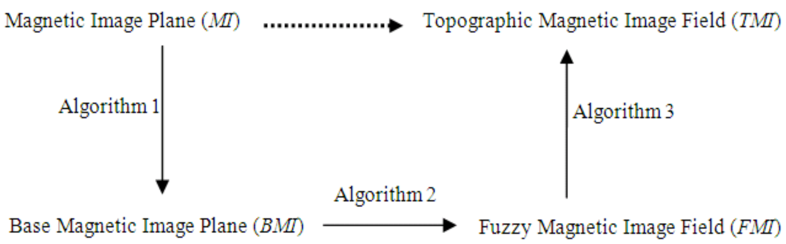

Fig. 3: FTTM Version 2 (Yun, 2006).

FTTM2 is reduced from the component MC of FTTM1 by analysing and transforming the magnetic fields data to image processing data. Figure 4 illustrates the mathematical structures for the components of FTTM2, whereas $M_I : [B_{Z\,min}, B_{Z\,max}] \rightarrow [0,255]$ is defined by:

$$M_{I\,(x,y)} = M_I\left(B_{Z\,(x,y)}\right) = \frac{255\left(|B_{Z\,(x,y)}| - MB_{Z\,min}\right)}{MB_{Z\,max} - MB_{Z\,min}}\ , \tag{11}$$

for each $B_{Z\,(x,y)} \in [B_{Z\,min}, B_{Z\,max}]$, such that:

$MB_{Z\,min}$ is the smallest possible magnitude of magnetic fields reading, $MB_{Z\,max}$ is the largest possible magnitude of magnetic fields reading, $B_{Z\,(x,y)}$ is the initial magnetic fields reading, and $M_I\left(B_{Z\,(x,y)}\right)$ is the transformed magnetic field value. A mapping $\mu_{M_I}: [0,255] \rightarrow [0,1]$ is defined as follows:

$$\mu_{M_{I(x,y)}} = \mu_{M_I}\left(M_{I\,(x,y)}\right) = \frac{M_{I\,(x,y)}}{255}\ , \tag{12}$$

for each $M_{I\,(x,y)} \in [0,255]$. Furthermore, $z: [0,1] \rightarrow [-h, 0]$ is defined as:

$$z_{M_{I\,(x,y)}} = z(\mu_{M_{I(x,y)}}) = h(\mu_{M_{I(x,y)}} - 1) \quad , \tag{13}$$

for each $\mu_{M_{I(x,y)}} \in [0,1]$ and $-h < 0$ is a constant (Yun, 2006).

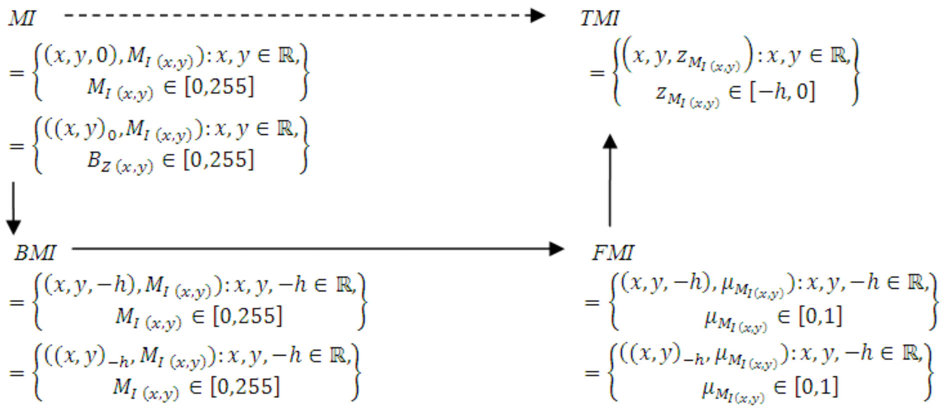

Fig. 4: The components of FTTM 2 (Yun, 2006).

FTTM1 as well as FTTM2 are specially designed to have equivalent topological structure between its components, that is, a homeomorphism between each component of FTTM1 and its corresponding components of FTTM2 (see Figure 5) (Ahmad *et al*., 2010) exists.

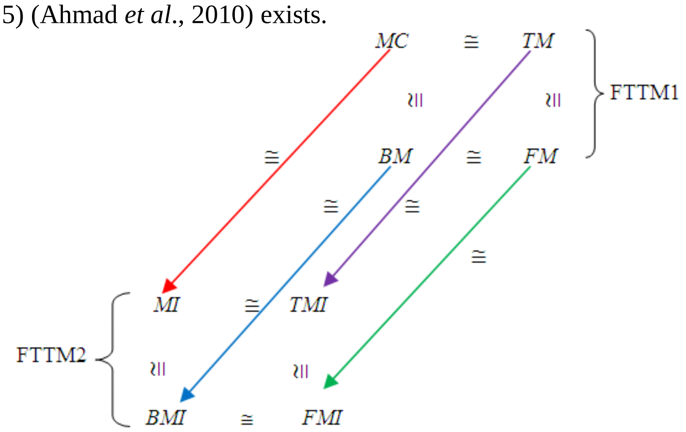

Fig. 5: Homeomorphisms between FTTM1 and FTTM2 (Ahmad *et al*., 2010).

In general, according to Ahmad *et al*. (2010) that a sequence of FTTM will exist when there are n elements of FTTM. A sequence of n FTTM is defined as FTTM1, FTTM2, FTTM3,…, FTTMn such that $MC_i \cong MC_{i+1}$, $BM_i \cong BM_{i+1}$, $FM_i \cong FM_{i+1}$, $TM_i \cong TM_{i+1}$.

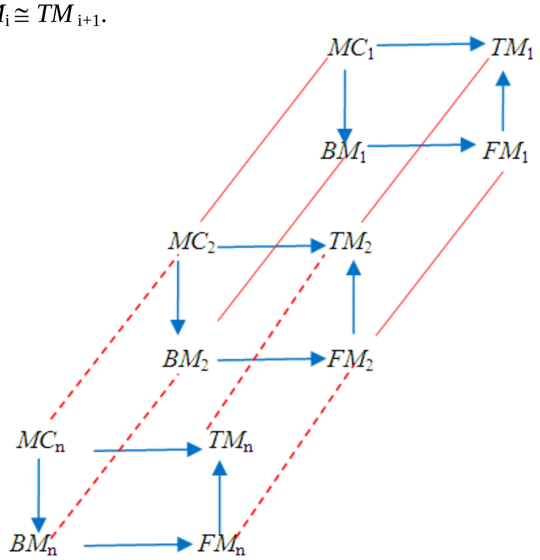

Fig. 6: Sequence of FTTM (Ahmad *et al*., 2010).

Continua of Fttm:

In this section, continuum of the component *MC* of FTTM1 is presented. The result will be utilizes to other components of FTTM1.

Theorem 4.1 The component *MC* in FTTM1 is a connected space.

Proof

Let a, b $\in MC$, such that $a = \left((x_1, y_1)_0, B_{Z(x_1,y_1)}\right)$ and $b = \left((x_2, y_2)_0, B_{Z(x_2,y_2)}\right)$, where x_1, x_2, y_1, $y_2 \in \mathbb{R}$ and $B_{Z(x_i,y_i)} \in [B_{Z\,min}, B_{Z\,max}]$, for i=1, 2. Let f be a function between the points a and b given by $f(t) = a(1-t)+bt$, $\forall\ t \in [0,1]$.

Thus

$$f(t) = (1-t)\left((x_1, y_1)_0, B_{Z(x_1,y_1)}\right) + t\left((x_2, y_2)_0, B_{Z(x_2,y_2)}\right)$$
$$= \left((1-t)(x_1, y_1)_0, (1-t)B_{Z(x_1,y_1)}\right) + \left(t(x_2, y_2)_0, tB_{Z(x_2,y_2)}\right)$$
$$= \left(((1-t)x_1, (1-t)y_1)_0, (1-t)B_{Z(x_1,y_1)}\right) + \left((t\,x_2, t\,y_2)_0, t\,B_{Z(x_2,y_2)}\right)$$
$$= \left(((1-t)x_1 + t\,x_2, (1-t)y_1 + t\,y_2)_0, (1-t)B_{Z(x_1,y_1)} + t\,B_{Z(x_2,y_2)}\right).$$

By using the polar coordinate (r,φ), such that $r>0$ and $\varphi \in (-\pi, \pi]$, that $r = ((1-t)x_1 + t\,x_2, (1-t)y_1 + t\,y_2)_0$, and $\varphi = (1-t)B_{Z(x_1,y_1)} + t\,B_{Z(x_2,y_2)}$. Consider $a = r\cos\varphi$ and $b = r\sin\varphi$, so

$a = ((1-t)x_1 + t\,x_2, (1-t)y_1 + t\,y_2)_0 \cos((1-t)B_{Z(x_1,y_1)} + t\,B_{Z(x_2,y_2)})$, and

$b = ((1-t)x_1 + t\,x_2, (1-t)y_1 + t\,y_2)_0 \sin((1-t)B_{Z(x_1,y_1)} + t\,B_{Z(x_2,y_2)})$.

Then

$$a^2 + b^2 = [((1-t)x_1 + t\,x_2, (1-t)y_1 + t\,y_2)_0 \cos((1-t)B_{Z(x_1,y_1)} + t\,B_{Z(x_2,y_2)})]^2 + [((1-t)x_1 + t\,x_2, (1-t)y_1 + t\,y_2)_0 \sin((1-t)B_{Z(x_1,y_1)} + t\,B_{Z(x_2,y_2)})]^2 .$$

$$= [\{((1-t)x_1 + t\,x_2, (1-t)y_1 + t\,y_2)_0\}^2 \cos^2((1-t)B_{Z(x_1,y_1)} + t\,B_{Z(x_2,y_2)})] + [\{((1-t)x_1 + t\,x_2, (1-t)y_1 + t\,y_2)_0\}^2 \sin^2((1-t)B_{Z(x_1,y_1)} + t\,B_{Z(x_2,y_2)})].$$
$$= \{((1-t)x_1 + t\,x_2, (1-t)y_1 + t\,y_2)_0\}^2 [\cos^2((1-t)B_{Z(x_1,y_1)} + t\,B_{Z(x_2,y_2)}) + \sin^2((1-t)B_{Z(x_1,y_1)} + t\,B_{Z(x_2,y_2)})].$$
$$= \{((1-t)x_1 + t\,x_2, (1-t)y_1 + t\,y_2)_0\}^2 = r^2 \in MC \text{ since } r>0 \text{ and } \sqrt{a^2+b^2} = r.$$

Because of the points a and b are arbitrary, thus the path between any two distinct points in *MC* is also in *MC*. Hence, *MC* is path connected. Because every path connected space is connected (Shick, 2007). Therefore, *MC* is a connected space.

The following corollary deals with the connectedness for the other components of FTTM1.

Corollary 4.2 The components *BM*, *FM*, and *TM* of FTTM1 are connected spaces.

Proof

The mappings *bm*, *fm*, and *tm* are homeomorphisms (Yun, 2001). Since *bm* is a continuous mapping from *MC* onto *BM* and by using theorem 4.1, *BM* is a connected space. FM is also a connected space due to *fm* and connectedness of *BM*. By using the same reason, *TM* is also connected.

The compactness of the component *MC* of FTTM1 is presented in the following theorem.

Theorem 4.4 The component *MC* of FTTM1 is a compact space.

Proof

This theorem can be established by Heine –Borel theorem. So to prove the boundedness for *MC*, since $MC \subseteq \mathbb{R}^2$, so the Euclidean metric space d on $\mathbb{R}^2$ induced a metric space d_{MC} on *MC*. In fact, the topological structure of *MC* described by Ahmad *et al*. (2005) as:

$$\tau_{MC} = \{\theta_i \cap MC : \theta_i \text{ is an open subset of } \mathbb{R}^2, i \in I, I \text{ is an index set}\}, \tag{14}$$

since $\theta_i = \bigcup_{a \in \theta_i} B_d(a, r_a)$. Thus

$$\tau_{MC} = \{B_d(a,r_a) \cap MC : a \in \mathbb{R}^2, r_a \in \mathbb{R}^+\}. \tag{15}$$

Since $MC \subseteq \mathbb{R}^2$,

$$\tau_{MC} = \{B_{d_{MC}}(a, r_a) : a \in MC, r_a \in \mathbb{R}^+\}. \tag{16}$$

Figure 8 illustrates an example for description the open set in *MC.*

Clearly (MC, τ_{MC}) is a topological space induced by the metric space (MC, d_{MC}). Define d'_{MC}: $MC \times MC \rightarrow \mathbb{R}^+$ as another metric function on *MC by*:

$$d'_{MC} = \min\{d_{MC}(a,b), 1\} \tag{17}$$

(MC, d'_{MC}) is a bounded metric space by theorem 2.2, since $d'_{MC}(a,b) \leq 1$, for all a, $b \in MC$. So, *MC* is a bounded set due to Definition 2.1.

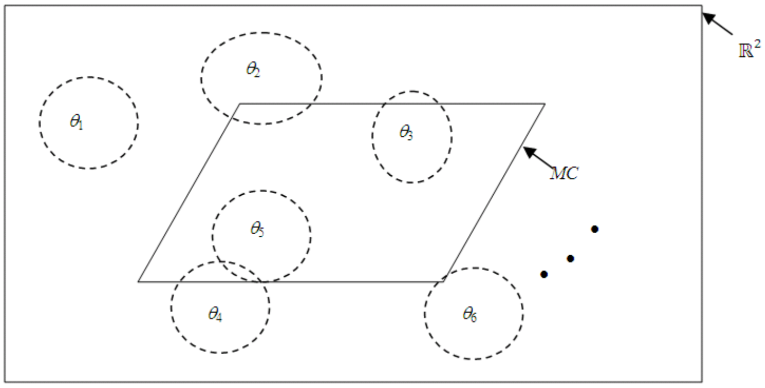

Fig. 7: (MC, τ_{MC}) in $\mathbb{R}^2$

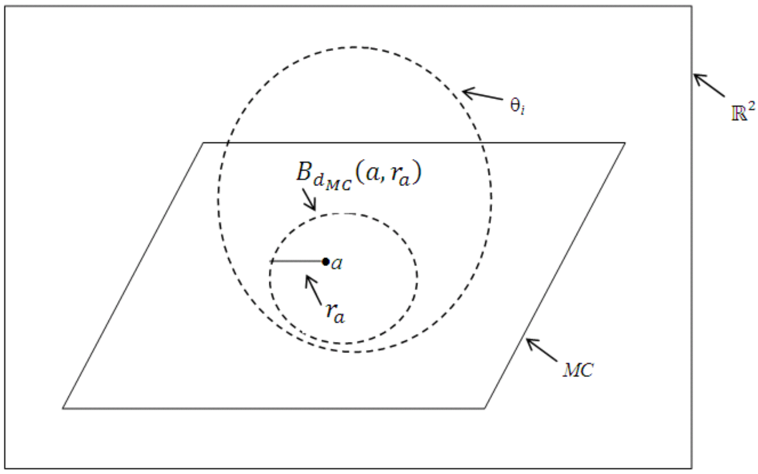

Fig. 8: The open set in *MC.*

To show that *MC* is closed, Pick $a \in MC'$(viz. the limit points of *MC*). Hence, for each open neighbourhood $B_{d_{MC}}(a, r_a)$ of a that $[B_{d_{MC}}(a, r_a)/\{a\}] \cap MC \neq \phi$, that is $B_{d_{MC}}(a, r_a) \cap MC \neq \phi$. However, there is $B_{d_{MC}}(a, r_a) \ni B_{d_{MC}}(a, r_a) \subseteq MC$ by (16). Because $a \in B_{d_{MC}}(a, r_a)$, which implies that $a \in MC$. Hence, $MC' \subseteq MC$. Thus, *MC* is a closed set, since each set containing its limit points is a closed set (Shick, 2007). Heine- Borel theorem states that every bounded and closed subset of $\mathbb{R}^n$ is compact. Therefore *MC* is compact.

Compactness of other components in FTTM1 is presented as follows.

Corollary 4.5 The components *BM*, *FM*, and *TM* are compact spaces.

Proof

The mappings *bm*, *fm*, and *tm* between the components of FTTM1 are homeomorphisms. *bm* is a continuous function from *MC* onto *BM*, therefore *BM* is a compact space. *FM* is also a compact space due to the surjective continuous function of *fm.* The compactness for the component *TM* is established with the same argument

Using Theorem 4.1, Corollary 4.2, Theorem 4.4, and Corollary 4.5, the following result is established.

Corollary 4.6 The components *MC*, *BM*, *FM*, and *TM* of FTTM1 are continua.

The continua for the components of other versions of FTTM are presented by the following theorem.

Theorem 4.7 Each component in FTTM*i* is continuum, for *i*=1, 2, 3, …, $n \in N$.

Proof

This theorem can be proven by mathematical induction as follows.

i) Each FTTM1 component is continua by Corollary 4.6.

ii) Suppose that each component in FTTMn is continuum, namely MC_n, BM_n, FM_n, and TM_n.

iii) Observe that $MC_i \cong MC_{i+1}$, $BM_i \cong BM_{i+1}$, $FM_i \cong FM_{i+1}$, $TM_i \cong TM_{i+1}$ as in (Ahmad *et al.*, 2010). This implies that $MC_n \cong MC_{n+1}$, $BM_n \cong BM_{n+1}$, $FM_n \cong FM_{n+1}$, $TM_n \cong TM_{n+1}$. By the homeomorphisms between the corresponding components, the components MC_{n+1}, BM_{n+1}, FM_{n+1}, and TM_{n+1} are connected and compact. Hence, they are continua.

From parts (i), (ii), and (iii), the components of FTTM*i* are continua, for *i*=1,2,3, …, $n \in N$.

Separation of Fttm:

Firstly, the component *MC* has the property T_i, for *i*=1, 2,3,4, as shown bellow.

Theorem 5.1 The component *MC* in FTTM1 is a T_i-space, for *i*=1, 2, 3, 4.

Proof

Basri (2009) proved that the component *MC* of FTTM1 is a Hausdorff space. So, *MC* will be a T_1-space due to every Hausdorff space is a T_1–space. Consequently, *MC* is a T_o-space. In addition, each compact and Hausdorff space is normal and regular. Thus, *MC* is normal and regular, whereas the compactness of *MC* is obtained from Theorem 4.4, Definition 2.3 parts (v) and (vii) reveal that *MC* is a T_3 and T_4–spaces.

The following result is immediate established from Theorem 5.1 and from the invariant of the separation axioms under the homeomorphisms between the components of FTTM1.

Corollary 5.2 The components of FTTM1 have separations T_i, for *i*=0, 1, 2, 3, 4.

The following theorem generalizes separations of FTTM components.

Theorem 5.4 Each component in FTTM*i* is satisfied the separations T_o, T_1, T_2, T_3, and T_4, for *i*=1, 2, 3, …, $n \in N$..

Proof

This theorem can be established by a mathematical induction.

i) Each FTTM1 component is continuum by Theorem 5.1 and Corollary 5.2.

ii) Suppose that the components MC_n, BM_n, FM_n, and TM_n of FTTM*n* satisfy the separations axioms, namely T_o, T_1, T_2, T_3, and T_4.

iii) Based on Ahmad *et al*. (2010), $MC_i \cong MC_{i+1}$, $BM_i \cong BM_{i+1}$, $FM_i \cong FM_{i+1}$, $TM_i \cong TM_{i+1}$, which imply that $MC_n \cong MC_{n+1}$, $BM_n \cong BM_{n+1}$, $FM_n \cong FM_{n+1}$, $TM_n \cong TM_{n+1}$. Due to the homeomorphisms between the corresponding components of FTTM*n* and FTTM*n*+1, MC_{n+1}, BM_{n+1}, FM_{n+1}, and TM_{n+1} satisfy the separations axioms T_o, T_1, T_2, T_3, and T_4.

From parts (i), (ii), and (iii), each component of FTTM satisfies T_i, for *i*= 0, 1, 2, 3, 4.

Conclusion:

In this paper, the component *MC* of FTTM1 is proven a continuum which is meant a non separable one piece. MC is used to preserve properties to other components of FTTM. This result enhances that MC is important to generate other topological properties for other components of FTTM. By utilizing continuum of MC, the properties of connectedness, compactness, and separations T_o, T_1, T_2, T_3, and T_4 for FTTM are preserved.

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