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Analytical Solution for Magnetohydrodynamic (MHD) Flow of a Viscoelastic Fluid Over a Stretching Sheet Filled with Nanoparticles

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Abstract: This work deals with the problem of steady two-dimensional magnetohydrodynamic (MHD) flow of a viscoelastic fluid near a stretching sheet in the presence of nanoparticles. The fundamental equations of the boundary layer are transformed into ordinary differential equations, which are then solved analytically using the Homotopy Asymptotic Method (HAM). Subsequently, effects of important parameters involved on local Nusselt number, local Sherwood number and skin-friction coefficient are presented and discussed in detail. The results show that by increasing the values of the local Reynolds number and the electric parameter, the value of the skin friction coefficient is decreased.

Key words: MHD flow; viscoelastic fluid; stretching sheet; Homotopy Analysis Method

INTRODUCTION

The problem of boundary layer flow over a stretching sheet has long been studied, as it is a representative model problem for numerous engineering applications such as in the polymer processing of a chemical engineering plant, metallurgy for the metal processing glass fiber, paper production, liquid films in condensation process and filament extrusion from a dye. Due to the high applicability of this problem in such industrial phenomena, it has attracted the attentions of many researchers. Perhaps, Sakiadis (1971) was pioneer in formulating this problem to study a steady two-dimensional boundary layer flow due to stretching sheet. Many investigators have extended the work of Sakiadis (1971) in order to study heat and mass transfer under various physical situations (e.g., McLeod and Rajagopal 1987; Chiam 1996).

However, all the above-mentioned researches did not consider the situations, where hydro-magnetic effects arose. The study of hydrodynamic flow and heat transfer over a stretching sheet may find its applications in sheet extrusion in order to make flat plastic sheets. In doing so, it is important to investigate cooling and heat transfer for improvement of the final products. The conventional fluids such as water and air are amongst the most widely used fluids as the cooling medium. However, the rate of heat exchange achievable by above fluids is realized to be unsuitable for certain sheet materials. Thus, in recent years it has been proposed to alter flow kinematics that it leads to a slower rate of solidification, as compared with water. Among the techniques to control flow kinematics, the idea of using magnetic fields appears to be the most attractive one both because of its ease of implementation and also because of its non-intrusive nature. In view of this, the study of MHD flow of Newtonian/non-Newtonian flow over a stretching sheet was carried out by many researchers (Yazdi *et al*. 2011; Hayat and Sajid 2007; Babaelahi *et al*. 2010; Alizadeh-Pahlavan *et al*. 2009; Nandeppanavar *et al*. 2011; Farzaneh-Gord *et al*. 2010). Analytical solutions have been done for boundary layer flow over a stretching sheet. Rana and Bhargava (2012) studied the problem of steady laminar boundary fluid flow, which results from the non-linear stretching of a flat surface in a nanofluid. Javed *et al*. (2010) studied heat transfer of a viscous fluid over a non-linear shrinking sheet in the presence of magnetic field, where they have obtained dual solutions for the exact and numerical solutions in the shrinking sheet problem. Hydrodynamic nano-boundary layer flow over permeable stretching surface by employing Homotopy Analysis Method (HAM) and Boundary Value Problem solver (BVP) was studied by Van Gorder *et al*. (2010). Kechil and Hashim (2008) studied the boundary-layer equation of flow over a nonlinearly stretching sheet in a magnetic field with chemical reaction. Fang *et al*. (2009) investigated hydrodynamic boundary layer of slip MHD viscous flow over a stretching sheet. They have concluded that wall drag force increases with magnetic parameter. Ishak *et al*. (2009) studied numerically steady two-dimensional MHD stagnation point flow towards a stretching sheet with variable surface temperature. They found that the heat transfer rate at the surface increases with the magnetic parameter, when the free stream velocity exceeds the stretching velocity. Recently, Nadeem *et al*. (2010) presented HAM solution for unsteady boundary layer flow in the region of the stagnation point towards a stretching sheet. More recently, Rasekh *et al*. (2013) investigated the problem of the boundary layer flow and heat transfer of a nanofluid over a stretching circular cylinder in the presence of non-uniform heat source/sink.

There have been many theoretical models developed to describe MHD flow towards a stretching sheet. However, to the best of our knowledge, no investigation has been made yet to analyze the MHD flow of a

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viscoelastic fluid over a stretching sheet filled with nanofluids. The highly non-linear momentum equation, to gather with the heat and mass transfer equations are solved analytically using the Homotopy Analysis Method (HAM) proposed by Liao (2003).

Governing Equations:

Let us consider the steady-state 2D incompressible electrically conducting viscoelastic Maxwell fluid flow over a stretching sheet in presence of nanoparticles (see Fig. 1). It is assumed that a uniform magnetic field of strength B_0 is applied in the negative y-direction normal to the sheet. The induced magnetic field due to the motion of the electrically conducting fluid and the pressure gradient are neglected. The equations governing transport of heat, momentum and mass can be written as (Abel *et al*., 2008),

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 v}{\partial y^2}-k_0\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right)+\frac{\sigma}{\rho}\left(E_0B_0-B_0{}^2u\right), \tag{2}$$

$$\begin{aligned}u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}&=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{(uB_0-E_0)^2\sigma}{\rho c_p}\\&+\tau\left\{D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]\right\},\end{aligned} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial x^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

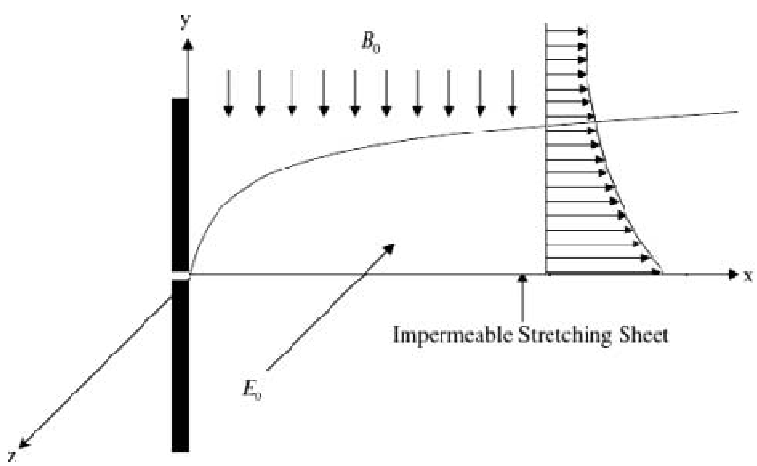

Fig. 1: Geometry of problem under investigation.

where (u, v) are the velocity components in (x, y) directions, ρ_f is the fluid density of base fluid, υ is the kinematic viscosity, T is the temperature, C is the nanoparticle volume fraction, α is the thermal diffusivity of the base fluid, $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ is the ratio of nanoparticle heat capacity and the base fluid heat capacity, D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient.
The boundary conditions for this problem will be:

$$y=0,\ T=T_w,\ u=U_w(x), v=0, C=C_w, \tag{5}$$

$$y\to\infty,\ T=T_\infty,\ u=0, C=C_\infty, \tag{6}$$

We introduce the following dimensionless quantities:

$$\eta=\left(\frac{b}{\upsilon}\right)^{1/2} y,\quad u=bxf'(\eta),\quad v=-\sqrt{b\upsilon}\,f(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \tag{7}$$

Here *f(η)* is the dimensionless stream function, *θ(η)* is dimensionless temperature and ϕ*(η)* is dimensionless concentration.
After transformation we have:

$$f'''+ff''-f'^2-k_1^*(2f'f'''-ff''''-f''^2)+Ha^2(E_1-f')=0, \tag{8}$$

$$\theta''+\Pr(f\theta'-2\theta f'')+\Pr Ec\left(f''^2-Ha^2\left(f'^2+E_1^2-2E_1f'\right)\right)+N_b\theta'\phi'+N_t\theta'^2=0, \tag{9}$$

$$\phi''+Lef\phi'+\frac{N_t}{N_b}\phi^2=0, \tag{10}$$

The associated boundary conditions are,

$$f(0)=0,\ f'(0)=1,\ \theta(0)=1,\ \phi(0)=1, \tag{11}$$

$$f'(\infty)=0,\ \theta(\infty)=0,\ \phi(\infty)=0, \tag{12}$$

where $k_1^*=\frac{k_0 b}{\gamma}$ is the dimensionless viscoelastic parameter, $Ha=\sqrt{\frac{\sigma}{\rho b}}B_0$ is Hartmann number, $E_1=\frac{E_0}{B_0bx}$ is the local electric parameter, and the prime stands for differentiation with respect to η. The four parameters are defined by:

$$\Pr=\frac{\upsilon}{\alpha}, Le=\frac{\upsilon}{D_B}, N_b=\frac{(\rho c)_p D_B(\phi_w-\phi_\infty)}{(\rho c)_f\upsilon}, N_t=\frac{(\rho c)_p D_T(T_w-T_\infty)}{(\rho c)_f T_\infty\upsilon} \tag{13}$$

The reduced Nusselt number Nu_r and the reduced Sherwood number Sh_r may found in terms of the dimensionless temperature at the sheet surface, $\theta'(0)$ and the dimensionless concentration at the sheet surface, $\phi'(0)$, respectively i.e.

$$Nur=\mathrm{Re}_x^{-1/2}\,Nu=-\theta'(0), \tag{14}$$

$$Shr=\mathrm{Re}_x^{-1/2}\,Sh=-\phi'(0), \tag{15}$$

The dimensionless local skin-friction coefficient C_f, is expressed as

$$\begin{aligned} C_f &= -\frac{\left(\upsilon\frac{\partial u}{\partial y}-k_0[u\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^2 u}{\partial y^2}-2\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}]\right)}{(bx)^2}, \text{at } y=0 \\ &= -\frac{1}{\sqrt{\mathrm{Re}_x}}f''(0)(1-3k_1^*) \end{aligned} \tag{16}$$

Where $\mathrm{Re}_x=\frac{bx^2}{\gamma}$ is the local Reynolds number.

Solution using Homotopy Analysis Method (HAM):

In this section, we will apply HAM to solve Eqs. (8–10), subjected to boundary conditions Eqs. (11–12). In doing so, we consider the initial guess approximation for $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ as:

$$L_f(f) = f''' + f'',\ L_\theta(\theta) = \theta'' + \theta',\ L_\phi(\phi) = \phi'' + \phi', \tag{17}$$

And as the auxiliary linear operator which has the property:

$$L_f(c_1 + c_2\eta + c_3 e^{-\eta}) = L_\theta(c_4 + c_5 e^{-\eta}) = L_\phi(c_6 + c_7 e^{-\eta}) = 0 \tag{18}$$

where c_1 to c_7 are constants. Let us choose the initial guesses using the auxiliary linear operators and boundary conditions in the following form,

$$f_0(\eta) = 1 - e^{-\eta}, \quad \theta_0(\eta) = 1, \quad \phi_0(\eta) = 1, \tag{19}$$

Then, the following equations will be constructed.

Zeroth-Order Deformation Equations:

$$(1-p)L_f[f(\eta;p) - f_0(\eta)] = p\hbar_f H_f(\eta) N_f[f(\eta;p)] \tag{20}$$

$$(1-p)L_\theta[\theta(\eta;p) - \theta_0(\eta)] = p\hbar_\theta H_\theta(\eta) N_\theta[\theta(\eta;p)] \tag{21}$$

$$(1-p)L_\phi[\phi(\eta;p) - \phi_0(\eta)] = p\hbar_\phi H_\phi(\eta) N_\phi[\phi(\eta;p)] \tag{22}$$

$$\begin{aligned} &f(0;p) = 0,\ f'(0;p) = 1,\ f'(\infty;p) = 0, \\ &\theta(0;p) = 0,\ \ \theta(\infty;p) = 0, \\ &\theta(0;p) = 0,\ \ \theta(\infty;p) = 0, \end{aligned} \tag{23}$$

$$\begin{aligned} N_f[f(\eta;p),\theta(\eta;p),\phi(\eta;p)] &= \frac{\partial^3 f(\eta;p)}{\partial\eta^3} + f(\eta;p)\frac{\partial^2 f(\eta;p)}{\partial\eta^2} - \left(\frac{\partial f(\eta;p)}{\partial\eta}\right)^2 \\ &- k_1^*\left(2\frac{\partial f(\eta;p)}{\partial\eta}\frac{\partial^2 f(\eta;p)}{\partial\eta^2} - f(\eta;p)\frac{\partial^3 f(\eta;p)}{\partial\eta^3} - \left(\frac{\partial^2 f(\eta;p)}{\partial\eta^2}\right)^2\right)\theta(\eta;p) + Ha^2\left(E_1 - \frac{\partial f(\eta;p)}{\partial\eta}\right) = 0 \end{aligned} \tag{24}$$

$$\begin{aligned} N_\theta[f(\eta;p),\theta(\eta;p),\phi(\eta;p)] &= \frac{\partial^2\theta(\eta;p)}{\partial\eta^2} + \Pr\left(f(\eta;p)\frac{\partial\theta(\eta;p)}{\partial\eta} - 2\theta(\eta;p)\frac{\partial^2 f(\eta;p)}{\partial\eta^2}\right) \\ &+ \Pr Ec\left(\left(\frac{\partial^2 f(\eta;p)}{\partial\eta^2}\right)^2 - Ha^2\left(\left(\frac{\partial f(\eta;p)}{\partial\eta}\right)^2 + E_1^2 - 2E_1\frac{\partial f(\eta;p)}{\partial\eta}\right)\right) + \\ &N_b\frac{\partial\theta(\eta;p)}{\partial\eta}\frac{\partial\phi(\eta;p)}{\partial\eta} + N_t\left(\frac{\partial\theta(\eta;p)}{\partial\eta}\right)^2 = 0 \end{aligned} \tag{25}$$

$$N_\phi[f(\eta;p),\theta(\eta;p),\phi(\eta;p)] = \frac{\partial^2\phi(\eta;p)}{\partial\eta^2} + le\left(f(\eta;p)\frac{\partial\phi(\eta;p)}{\partial\eta}\right) + \frac{N_t}{N_b}\left(\frac{\partial^2\theta(\eta;p)}{\partial\eta^2}\right) = 0 \tag{26}$$

For p = 0 and p = 1 we get,

$$\begin{aligned} &f(\eta;0)=f_0(\eta),\ \ f(\eta;p)=f(\eta),\\ &\theta(\eta;0)=\theta_0(\eta),\ \ \theta(\eta;1)=\theta(\eta),\\ &\phi(\eta;0)=\phi_0(\eta),\ \ \phi(\eta;1)=\phi(\eta), \end{aligned} \tag{27}$$

where $p \in [0,1]$ is an embedding parameter and $f(\eta; p)$, $\theta(\eta; p)$ and $\phi(\eta; p)$ are real functions of η and p. Let $\hbar_f$, $\hbar_\theta$ and $\hbar_\phi$ denote the nonzero auxiliary parameters, and $H_f(\eta)$, $H_\theta(\eta)$ and $H_\phi(\eta)$ the nonzero auxiliary functions, respectively. By Taylor's theorem and using equations (25) and (26), $f(\eta; p)$, $\theta(\eta; p)$ and $\phi(\eta; p)$ can be expanded in a power series of p as follows,

$$\begin{aligned} &f(\eta;p)=f_0(\eta)+\sum_{j-1}^{\infty} f_j(\eta)p^j\\ &f_j(\eta)=\frac{1}{j!}\frac{\partial^j[f(\eta;p)]}{\partial p^j} \end{aligned} \tag{28}$$

$$\begin{aligned} &\theta(\eta;p)=\theta_0(\eta)+\sum_{j-1}^{\infty} \theta_j(\eta)p^j\\ &\theta_j(\eta)=\frac{1}{j!}\frac{\partial^j[\theta(\eta;p)]}{\partial p^j} \end{aligned} \tag{29}$$

$$\begin{aligned} &\phi(\eta;p)=\phi_0(\eta)+\sum_{j-1}^{\infty} \phi_j(\eta)p^j\\ &\phi_j(\eta)=\frac{1}{j!}\frac{\partial^j[\phi(\eta;p)]}{\partial p^j} \end{aligned} \tag{30}$$

Here we suppose $\hbar_f = \hbar_\theta = \hbar_\phi = \hbar$. where $\hbar$ is chosen in such a way that these three series are convergent at $p = 1$. Therefore we have through equations (28 - 30),

$$f(\eta)=f_0(\eta)+\sum_{j-1}^{\infty} f_j(\eta)p^j \tag{31}$$

$$\theta(\eta)=\theta_0(\eta)+\sum_{j-1}^{\infty} \theta_j(\eta)p^j \tag{32}$$

$$\phi(\eta)=\phi_0(\eta)+\sum_{j-1}^{\infty} \phi_j(\eta)p^j \tag{33}$$

Jth-Order Deformation Equations:

Differentiating the zero-order deformation Equations (20-22) and j times with respect to p then dividing by j! and setting $p = 0$, we obtain the jth-order deformation equations as,

$$L_f[f_j(\eta)-\chi_j f_{j-1}(\eta)]=\hbar_f H_f(\eta)R_j^f(\eta) \tag{34}$$

$$L_\theta[\theta_j(\eta)-\chi_j \theta_{j-1}(\eta)]=\hbar_\theta H_\theta(\eta)R_j^\theta(\eta) \tag{35}$$

$$L_\phi[\phi_j(\eta)-\chi_j \phi_{j-1}(\eta)]=\hbar_\phi H_\phi(\eta)R_j^\phi(\eta) \tag{36}$$

Subject to the boundary conditions,

$$\begin{aligned} &f_j(0)=f_j'(0)=f_j'(\infty)=0,\\ &\theta_j(0)=\theta_j(\infty)=0,\\ &\phi_j(0)=\phi_j(\infty)=0, \end{aligned} \tag{37}$$

For $j \geq 1$, where

$$R_j^f(\eta) = f'''_{j-1} - k_1^* \sum_{i=0}^{j-1}\left(2f'_{j-1-i}f''_i - f_{j-1-i}f'''_{i-j-1} - f''_{j-1-i}f'_i\right) + Ha^2\left(E_1 - f'_{j-1}\right), \tag{38}$$

$$R_j^\theta(\eta) = \theta''_{j-1} + 2E_1Ha^2 \Pr Ecf'_{j-1} + \sum_{i=0}^{j-1}\left(\begin{array}{l}\Pr\left(f_{j-1-i}\theta'_i - 2f'_i\theta_{j-1-i} - E_1Ha^2Ec\right) + \Pr Ec \\ \left(f''_{j-1-i}f''_i - Ha^2\left(f'_{j-1-i}f'_i\right)\right) + N_b\phi'_{j-1-i}\theta'_i + N_t\theta'_{j-1-i}\theta'_i\end{array}\right) \tag{39}$$

$$R_j^\phi(\eta) = \phi''_{j-1} + \frac{N_t}{N_b}\theta''_{j-1} + le\sum_{i=0}^{j-1}\left(f_{j-1-i}\phi'_i\right) \tag{40}$$

And

$$\chi_j = \begin{cases} 0, & j \leq 1 \\ 1, & j > 1 \end{cases} \tag{41}$$

Convergence of the HAM Solution:

As pointed by Liao (2003), the convergence rate of approximation for the HAM solution strongly depend on the value of auxiliary parameter $\hbar$. This has a great effect on the convergence region because the convergence region and rate of a series are chiefly determined by the base functions and its convergency is ensured. Fig. 2 clearly depict that the range, for admissible values of $\hbar$ is (-0.2< $\hbar$ <-1). Our calculations for this case clearly indicate that for whole region of η when $\hbar$=*-0.4*.

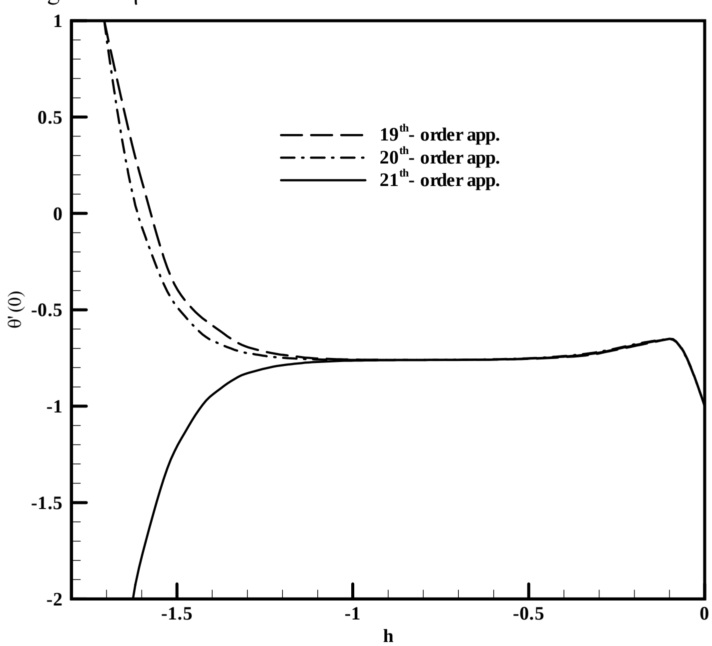

Fig. 2: The $\hbar$ curve for $k_1^* = E_1 = Ha = 0.2$, and $N_t = N_b = 0.1$.

RESULTS AND DISCUSSION

The system of Eqs. (8–10), along with boundary conditions of Eqs. (11,12), has been solved by using the HAM. The results are presented graphically through Figs. 3–10. The significance of parameter involved on the MHD flow and heat & mass transfer has been discussed.

Fig. 3 indicates the effect of *Nt* and *Nb* on reduced Nusselt number. It seems that for fixed thermophoresis parameter *Nt*, the reduced Nusselt number decreases sharply with the increase in Brownian motion, that as *Nb* is increased from 0.1 to 0.5. As the Brownian motion intensifies, it impacts a larger extent of the fluid, causing the thermal boundary layer to thicken, which in turn decreases the reduced Nusselt number. On the other hand, the reduced Nusselt number decreases as the thermophoresis diffusion penetrates deeper into the fluid and causes the thermal boundary layer to thicken. The effect of Prandtl number on heat transfer rate over the stretching plate has been illustrated in fig. 4. As seen, the higher Pr numbers result in an increase in reduced Nusselt number.

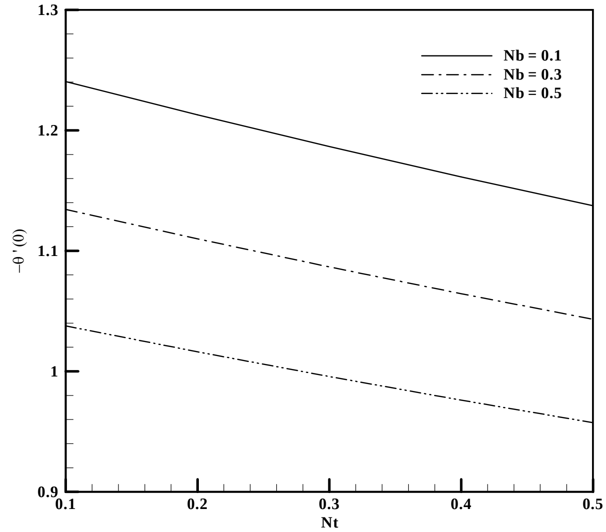

Fig. 3: Reduced local Nusselt number versus *Nt* for various *Nb for* $k_1^* = E_1 = Ha = 0.2$, and Pr=1.

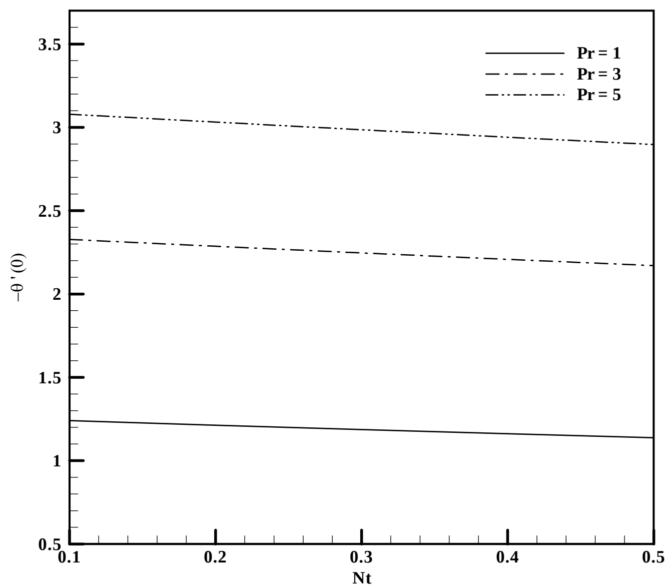

Fig. 4: Reduced local Nusselt number versus *Nt* for various *Pr for* $k_1^* = E_1 = Ha = 0.2$.

Variations of the concentration gradient magnitude at the sheet versus the thermophoresis parameter *Nt* for different values of *Nb* and *Le* have been depicted in Figs. 5–6, respectively. It can be interpreted that thermophoresis parameter deteriorates the rate of mass transfer, whereas by increasing the Brownian motion as well as Lewis number enhances it.

Fig. 7 plots the velocity profile for various values of the viscoelastic parameter (k_1^*). The effect of the viscoelastic parameter on velocity profile is to increase its value throughout the boundary layer. In fact, at any particular point in the boundary layer region, the downstream velocity will be minimum if the fluid is viscoelastic. Fig. 8 shows the effect of the Hartmann number (*Ha*) on the velocity profile. It can be realized that the Hartmann number decreases the boundary layer velocity throughout the boundary layer but significantly near the stretching sheet.

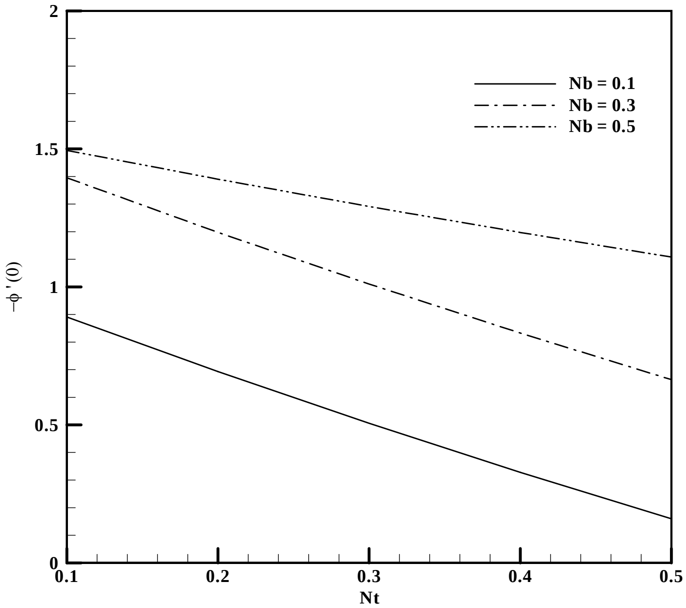

Fig. 5: Reduced local Sherwood number versus *Nt* for various *Nb for* $k_1^* = E_1 = Ha = 0.2$.

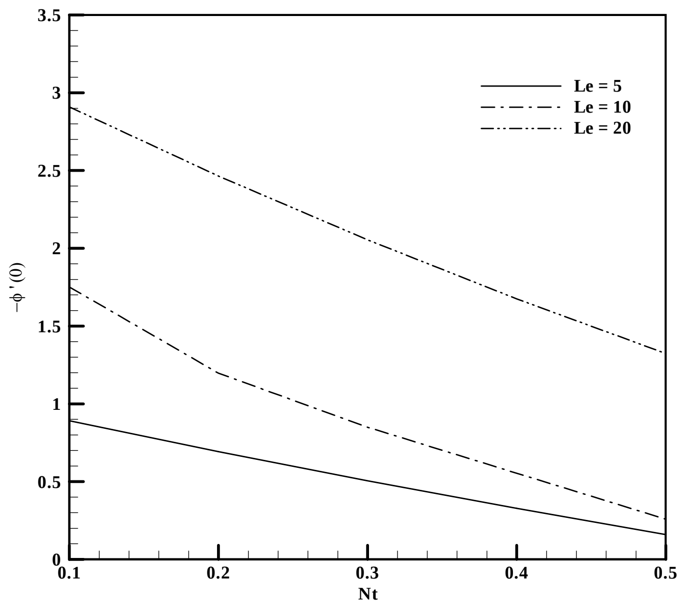

Fig. 6: Reduced local Sherwood number versus *Nt* for various *Le for* $k_1^* = E_1 = Ha = 0.2$.

Figs. 9a-9b represent the variation of the skin-friction parameter C_f with the variation of the viscoelastic parameter (k_1^*) for different values of the local Reynolds number Re_x and the local electric parameter (E_1). It can be seen that the skin-friction coefficient C_f decreases with the increase of the local Reynolds number Re_x and the local electric parameter E_1. The combined effect of increasing the values of local Re_x and the local electric parameter E_1 is to decrease the skin friction C_f largely. There will be a separation of boundary layer for the value of the viscoelastic parameter $k_1^* = 1/3$, which is independent of the values of the local Reynolds number Re_x, the Hartmann number (Ha), and the electric parameter (E_1).

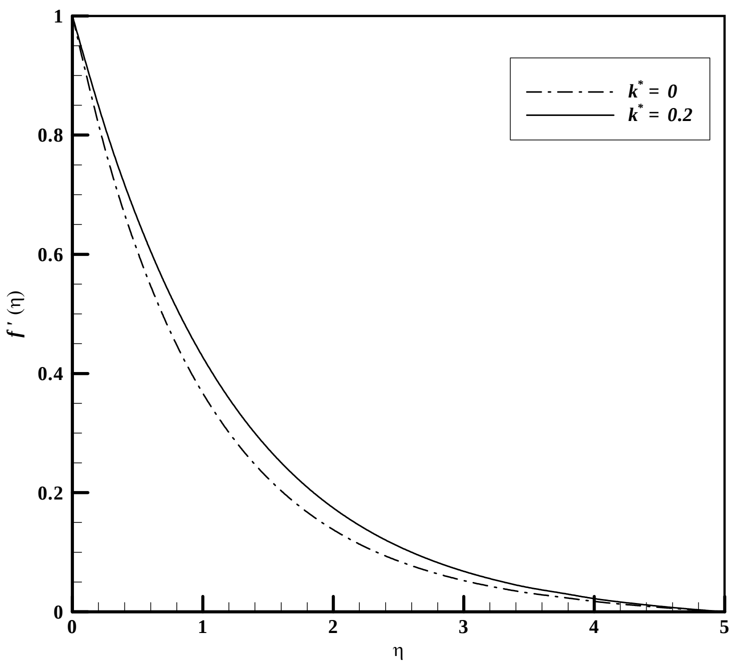

Fig. 7: Effects of the viscoelastic parameter on the velocity profile when $E_1 = Ha = 0.2$.

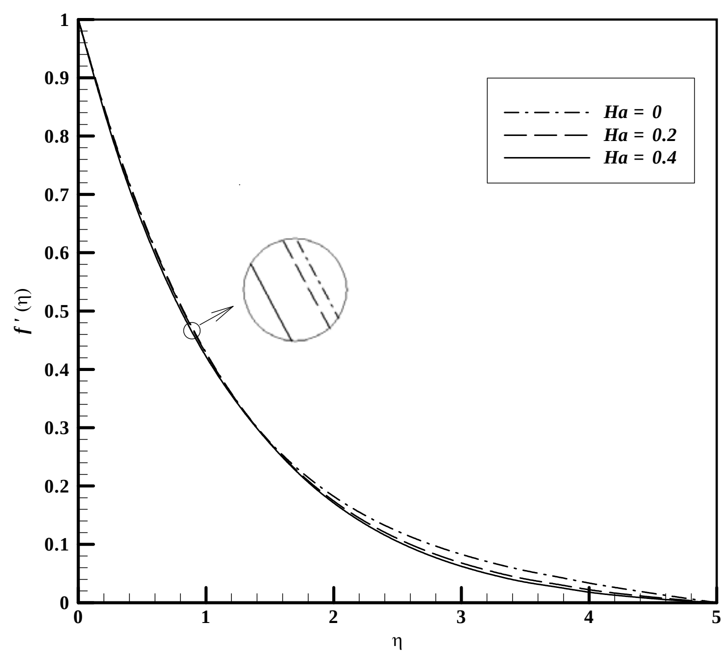

Fig. 8: Effects of the Hartmann parameter on the velocity profile when $E_1 = k_1^* = 0.2$.

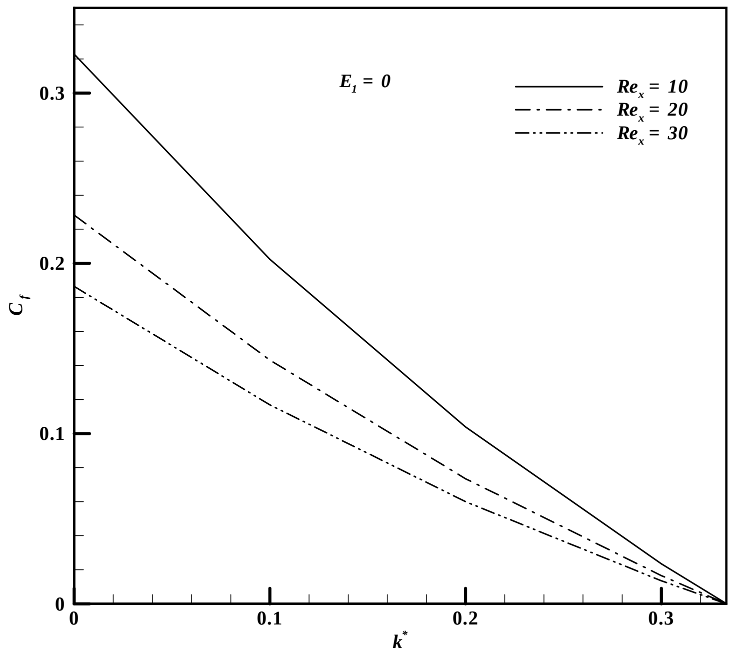

Fig. 9a: Local skin-friction parameter versus of the viscoelastic parameter for various Re_x when E_1=0.

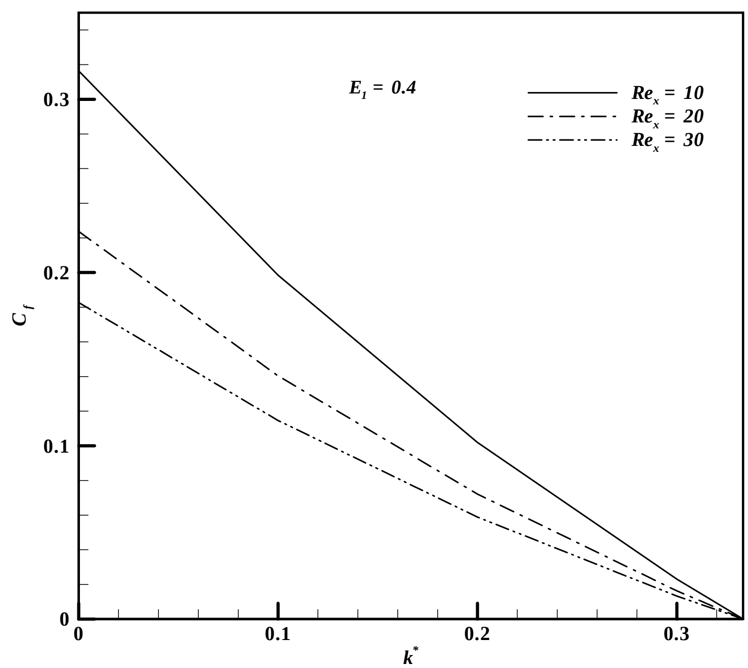

Fig. 9b: Local skin-friction parameter versus of the viscoelastic parameter for various Re_x when E_1=0.

Figs. 10a-10b demonstrate the variation of the skin-friction parameter C_f with the variation of the viscoelastic parameter (k_1^*) for different values of the local Reynolds number Re_x and the Hartmann number (*Ha*). It can be realized that the effect of the Hartmann number (*Ha*) is to increase the skin-friction coefficient C_f. The interaction of the velocity and the magnetic field, which is directed from upstream to downstream, leads to generate the Lorentz force and consequently the increase of the skin friction.

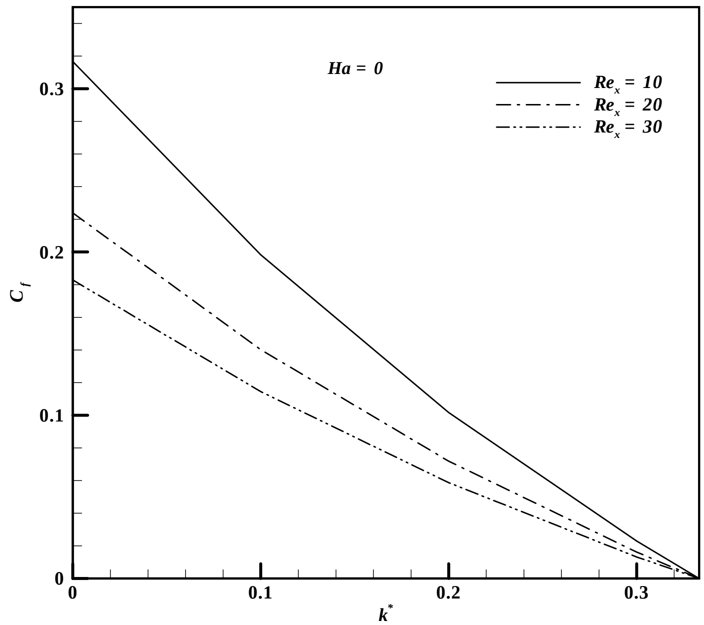

Fig. 10a: Local skin-friction parameter versus of the viscoelastic parameter for various Re_x when Ha=0.

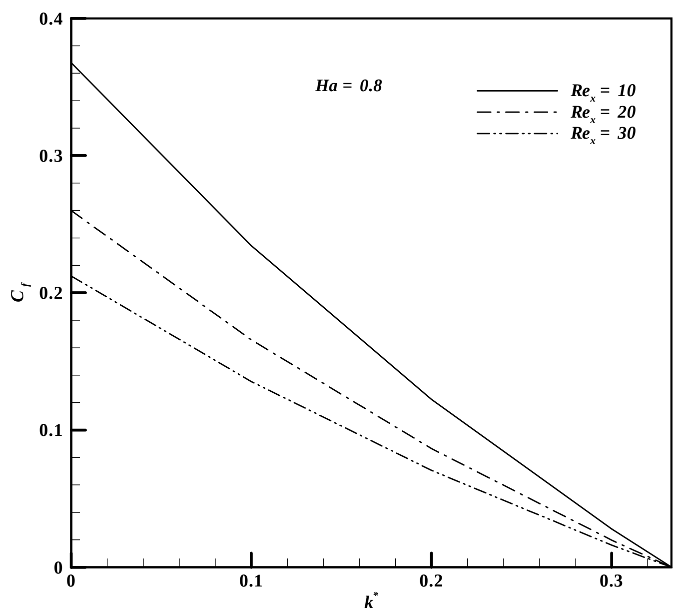

Fig. 10b: Local skin-friction parameter versus of the viscoelastic parameter for various Re_x when Ha=0.8.

Conclusions:

In this paper, the problem of the 2D steady-state incompressible viscoelastic boundary layer MHD flow and heat transfer over a stretching sheet have been studied in the presence of nanoparticles, electric and magnetic fields. The governing partial differential equations were converted into ordinary differential equations by using a suitable similarity transformation, which were solved analytically by employing HAM technique. The results suggest that:

- The rate of heat transfer at the sheet surface increases due to an increase in the Prandtl number. Conversely, there is a decrease in local Nusselt number as thermophoresis parameter and Brownian motion parameter of the nanoparticles increase.
- The rate of mass transfer decreases with the thermophoresis parameter. By contrast, it intensifies with the Brownian motion as well as Lewis number.

- The skin friction coefficient decreases with the local Reynolds number and the local electric parameter, whereas it is enhanced by increasing the Hartmann number.

The analytical solution presented here is potentially influential in controlling wall shear stress as well as the local Nusselt and Sherwood numbers. It is hoped that this work will serve as a vehicle for understanding more complex problems involving the various physical effects investigated in the present problem.

Nomenclature:

B_0 magnetic field
C concentration
C_f local skin-friction coefficient
c_p heat capacity
D_B Brownian diffusion coefficient
D_T thermophoretic diffusion coefficient,
E Eckert number
$E1$ local electromagnetic parameter
$E0$ electric field
f stream function
$\hbar$ non-zero auxiliary parameters in the HAM
Ha Hartmann number
HAM Homotopy analysis method
K thermal conductivity
Le Lewis number
k_0 elastic parameter
k_1^* viscoelastic parameter
N_b Brownian motion parameters
N_t thermophoresis parameters
Nu_x local Nusselt number
Pr Prandtl number
Re_x local Reynolds number
Sc Schmidt number
Sh_x local Sherwood number
T temperature
u,v dimensional velocity components along x and y axes
x,y dimensional Cartesian coordinates

Greek symbols

ν Kinematic viscosity
ρ density
θ dimensionless temperature
η similarity variable
σ electrical conductivity
ϕ dimensionless concentration
α thermal diffusivity
ε velocity ratio parameter
ψ stream function
σ electric conductivity
γ chemical reaction parameter

Subscripts

f fluid
p nanoparticles
w condition at wall
∞ condition at infinity

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