

## Carrier Frequency Offset Estimation in Uplink OFDMA System for Contiguous and Non-Contiguous Subcarrier Allocation Techniques

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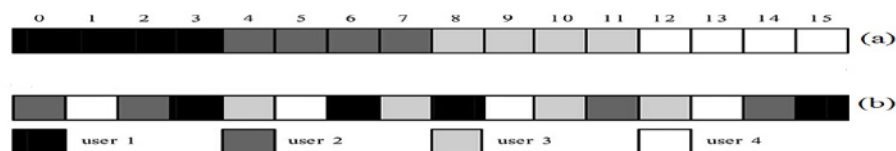
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**Abstract:** Orthogonal Frequency Division Multiplexing (OFDMA) is promising upcoming technology to be used in physical layer of Wi-Max and LTE. Carrier frequency offset (CFO) estimation in uplink of OFDMA is challenging task because each user experiences different CFO. The presence of CFO results into intercarrier interference (ICI), destroying orthogonality among subcarriers. At present the available solutions for CFO estimation are limited to particular subcarrier assignment technique. This paper investigates CFO estimation performance for two different subcarrier allocation techniques in uplink of OFDMA under slow fading and fast fading environment. Sub-band carrier assignment scheme is used for contiguous subcarrier allocation which estimates CFO using Moose principle by taking advantage of repetitive structure in training sequence. Generalized carrier assignment scheme is used for non-contiguous subcarrier allocation which estimates CFO using 1-D Maximum Likelihood method which is derived from basic Maximum likelihood in order to reduce complexity. Simulation results show the performance of mean square error and bit error rate of both techniques for an uplink OFDMA system employed in mobile Wi-Max.

**Key words:** Orthogonal frequency division multiplexing (OFDMA), Generalized carrier assignment (GCAS), Sub-band carrier assignment (SCAS), Carrier frequency offset (CFO), Cyclic prefix (CP).

### INTRODUCTION

Orthogonal Frequency Division Multiple Access (OFDMA) has attracted much attention in last few years, as it is to be deployed in fourth generation wireless communication. Wi-Max uses OFDMA in uplink as well as in downlink whereas LTE uses only in downlink (Tao jiang *et al.*, 2010). In OFDMA available subcarriers are distributed among users for simultaneous transmission. However; similar to OFDM it suffers from two major drawbacks that it is highly sensitive to timing offset (TO) and carrier frequency offset (CFO). Timing offset results in interblock interference (IBI). The problem of TO can be solved by considering quasi-synchronous scenario in which length of cyclic prefix (CP) sufficiently long so as to accommodate the channel delay spread and timing offsets (Tao jiang *et al.*, 2010). The problem of CFO arises due to mismatch of oscillator frequencies at transmitter and receiver and Doppler shift which causes loss of orthogonality among subcarriers causing inter-carrier interference (ICI) because of which orthogonality among subcarriers gets lost. The problem of CFO estimation in uplink of OFDMA is more challenging than downlink because in downlink CFO will be unique offset between transmitter and receiver so only one CFO needs to be handled at a time and estimation techniques in OFDM can be directly employed in it. In Uplink multiple users transmit to base station simultaneously, each having different CFO which makes CFO estimation at base station a multi-parameter estimation problem.



**Fig. 1:** (a) Sub-band carrier assignment (b) Generalized carrier assignment

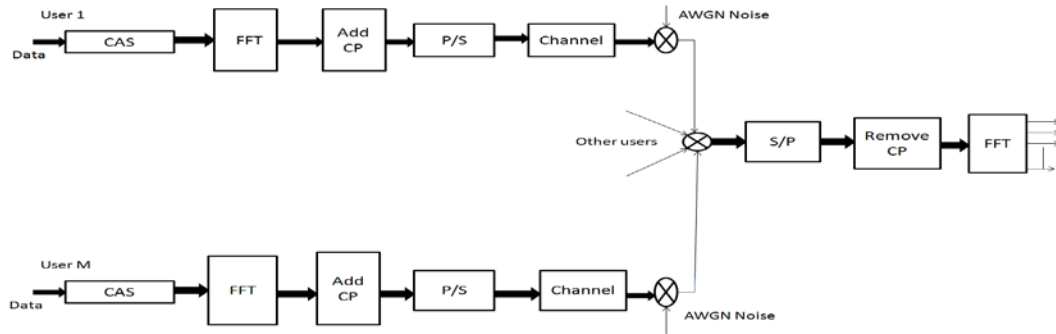
Fig. 1 shows the different subcarrier assignment techniques based on the allocation of available subcarriers among users. In sub-band carrier assignment scheme group of contiguous/adjacent subcarriers are assigned to each user. The generalized subcarrier assignment scheme sub-carriers are assigned non-contiguously/randomly among users (Man-on-pun *et al.*, 2005).

The methods used for CFO estimation are closely related to which subcarrier assignment scheme is employed. Various methods have been used previously for different carrier assignment schemes. In this paper, for CFO estimation in generalized assignmentscheme adopting exact maximum likelihood (ML) is turned out to be complex as it demands for search over multidimensional space (Zhongjun Wang *et al.*, 2009) and we have used iterative technique which reduces multidimensional search to sequence of 1-D searches. For CFO estimation in sub-band carrier assignment, Moose principle based estimator was primarily used in OFDM for single user. This method finds out CFO using phase difference between repetitive training sequence in user's frame (Luca Sanguinetti, Michele Morelli, 2010). For both methods, each user transmits a training block at the beginning of uplink frame.

In this paper we investigate how subcarrier allocation scheme affects the CFO estimation and BER performance in uplink of OFDMA under slow fading and fast fading environment.

The rest of the paper has following sections which gives description about system model used, methods employed for CFO estimation for two sub-carrier allocation techniques, simulation results and conclusions.

**System Model:**



**Fig. 2:** Baseband OFDMA Uplink Model

The baseband signal model of uplink OFDMA system is shown in fig. 2 Let M users are communicating with same base station and total available subcarriers be N. After taking N-point inverse discrete Fourier transform (IDFT) and adding  $N_g$  point CP and the resulting transmitted time domain symbol can be written as (Man-on-pun *et al.*, 2005)

$$x_m(k) = \sum_{n \in I_m} d_m(n) e^{j2\pi nk/N} ; -N_g \leq k \leq N - 1 \tag{1}$$

Where  $I_m$  is set of subcarriers assigned to considered user and  $d_m$  is symbol transmitted over  $n^{\text{th}}$  subcarrier.

Assume M users are simultaneously active in system and transmit the data to base station receiver. Each stream propagates through multipath channel with impulse response  $h_m = [h_m(0), h_m(1), \dots, h_m(L_m - 1)]^T$  and arrives at base station with timing offset  $m$  and frequency offset  $m$ . At receiver end, the received signal is the superposition of all signals from all users and it can be written as

$$r(k) = \sum_{m=1}^M r_m(k) + v(k) \tag{2}$$

The  $r_m(k)$  is the signal from  $m^{\text{th}}$  user and it can be written as

$$r_m(k) = \left\{ e^{j2\pi m k/N} \sum_{l=0}^{L_m-1} h_m(l) x_m(k - m - l) \right\} + v(k) \tag{3}$$

Where  $v(k)$  represents complex valued AWGN noise.

Now the base station must compute  $m$  and  $m$  for each active user. These estimates are used to restore the orthogonality among subcarriers. Quasi synchronous scenario is considered to mitigate the effect of user's timing offset. For achieving quasi synchronous scenario, we select the length of cyclic prefix in such a way it has to accommodate both channel delay spread and timing offsets. Then the received samples can be written as (Man-on-pun *et al.*, 2005)

$$r_m(k) = \left\{ e^{j2\pi m k/N} \sum_{l=0}^{L-1} h'_m(l) x_m(k - l) \right\} + v(k) \tag{4}$$

Where;  $h'_m = [h'_m(0), h'_m(1), \dots, h'_m(L-1)]^T$  is the extended channel vector with entries  $h'_m(l) = h_m(l-m)$ ;  $0 \leq l \leq L-1$  and length  $L = \max(L_m + m)$ .

Thus in practice, quasi-synchronous system is equivalent to perfectly time synchronized system.

**Cfo Estimation Methods:**

Consider the received signal in (2) which represents the sum of independent components of different users added with common noise term. This transmitted uplink OFDMA signal will remain common for the following CFO estimation methods except subcarrier assignment technique.

*A. CFO Estimation Using 1-D Maximum Likelihood (ML)*

The OFDMA blocks are organized into frames and CFO of each user is estimated using training block located at the beginning of frame. In the following methods, we concentrate on training block in order to estimate the CFO. Time domain samples of pilot symbols denoted as  $p_m(n)$  ( $n \in I_m$ ) transmitted by  $m^{\text{th}}$  user as

$$b_m(k) = \frac{1}{\sqrt{N}} \sum_{n \in I_m} p_m(n) e^{j2\pi nk/N}; -N_g \leq k \leq N-1 \tag{5}$$

At base station, the CP removed and remaining samples are expressed as (Man-on-pun *et al.*, 2006)

$$r(k) = \sum_{m=1}^M e^{\frac{j2\pi\epsilon_m k}{N}} \sum_{l=0}^{L-1} h'_m(l) b_m(k-l) + v(k); 0 \leq k \leq N-1 \tag{6}$$

Collecting received samples into vector  $r$  and can be written as

$$r = \sum_{m=1}^M r_m + v \tag{7}$$

Here  $r_m = D(m)B_m h'_m$  and  $D(m) = \text{diag}\{1, e^{j2\pi\epsilon_m/N}, \dots, e^{j2\pi(N-1)\epsilon_m/N}\}$  is diagonal matrix consisting CFO experienced by each sample and  $B_m$  is matrix with known entries  $[B_m]_{k,l} = b_m(k-l)$  for  $0 \leq k \leq N-1$  and  $0 \leq l \leq L-1$

Now by adopting exact ML approach, log-likelihood function can be written as (Sameer S.M., R.V. Rajakumar, 2008)

$$(\tilde{h}) = -N \ln(\pi\sigma^2) - |r - A(\tilde{h})\tilde{h}|^2 \tag{8}$$

Where  $A(\tilde{h}) = [D(\tilde{c}_1)B_1, D(\tilde{c}_2)B_2, \dots, A(\tilde{c}_M)B_M]$

Then the estimate of CFO ( $\tilde{h}$ ) can be obtained by maximizing (8)

$$\tilde{h} = \arg.\max\{|r - P r|^2\} \tag{9}$$

Where  $P = A(\tilde{h})[A^H(\tilde{h})A(\tilde{h})]^{-1}A^H(\tilde{h})$

But the maximization of (8) requires complex grid search over multi-dimension and complex matrix inverse procedures. To overcome alternating projection approach (Man-on-pun *et al.*, 2006), the multi-dimensional optimization is broken into simple 1-D searches so the CFOs are estimated sequentially instead of jointly. It consists of cycles and steps. A cycle is made up of  $k$  steps and each step updates the CFO of single user by keeping constant CFO for other users (Zhongjun Wang *et al.*, 2009).

We have the estimated CFO as

$$\tilde{h} = \arg.\max\{|P_{CB} r|^2\} \tag{10}$$

Where  $P_{CB} = C_B[C_B^H C_B]^{-1}C_B^H$  (Man-on-pun *et al.*, 2005) is the residual projection onto columns space of  $C_B = (I - P_B)C$  and  $P_B = B[B^H B]^{-1}B^H$  is projection of  $P$  onto columns space of  $B(\tilde{h})$ .  $B$  and  $C$  are formed by splitting  $A(\tilde{h})$  such that  $C = [A(\tilde{c}_1), A(\tilde{c}_2), \dots, A(\tilde{c}_M)]A(\tilde{c}_1)$ .

*B. CFO Estimation Using Moose Principle*

The received uplink signal model in (2) is considered and the transmission is organized into frames. Each uplink frame is preceded by at least two identical training blocks denoted as 2P. The pilot symbols are composed of repetitive parts (W. Aziz *et al.*, 2012). Here we find the CFO of individual user using Moose principle which states that the CFO of each user can be estimated by measuring phase shift between transmitted pilot tones (Luca Sanguinetti, Michele Morelli, 2010). (4) represents the mixture of signals and can be written as (Man-on-pun *et al.*, 2005)

$$r(k) = \sum_{m=1}^M z_m(k) e^{j2\pi\epsilon_m k/N} + w(k) \tag{11}$$

with

$$z_m(k) = \frac{1}{\sqrt{N}} \sum_{n \in I_m} H_m(n) d_m(n) e^{\frac{j2\pi nk}{N}} \quad (12)$$

and

$$H_m = \sum_{l=0}^{L-1} h'_m(l) e^{j2\pi nl/N} \quad (13)$$

In frequency domain

$$R(k) = \sum_{m=1}^M Z_m \otimes D(m) + v \quad (14)$$

Where  $\otimes$  denotes N-point circular convolution.

In order to separate the signals for different users in frequency domain, filters are used at the output of FFT. Each active user's signal can be obtained by putting zero to all entries of  $R$  in (13) that do not correspond to the subcarriers of considered user. Thus each user's separated signal is given as  $X_m = P_m R$  where  $P_m$  is diagonal matrix with entries

$$[P_m]_{n,n} = \begin{cases} 1; & \text{if } n \in I_m \\ 0; & \text{else} \end{cases} \quad (15)$$

Thus in practice  $P_m$  acts as band-pass filter. From (13) we may write

$$X_m = Z_m \otimes D(m) + v \quad (16)$$

Thus we can estimate CFO of each user using Moose principle. Only the frames of  $j^{\text{th}}$  user out of available  $M$  ( $j = 1, 2, \dots, M$ ) users is considered and denote it as  $Y_j(p, n)$ . Then the DFT output over  $n^{\text{th}}$  subcarrier of  $p^{\text{th}}$  training block where  $p \in 0, 1, \dots, P-1$  is given as

$$Y_j(p, n) = e^{j2\pi p j N_T / N} X_p(n) + v(k) \quad (17)$$

We are interested in the estimation of CFO of  $j^{\text{th}}$  user ( $j$ ). The resulting scheme operates in frequency domain and provides CFO estimate by measuring phase shift between corresponding DFT output over adjacent training blocks and it is given as

$$Y(p+1, n) Y_j^*(p, n) = |X_j(n)|^2 e^{j2\pi j N_T / N} \quad (18)$$

Approximate value of CFO estimate is obtained by

$$j = \frac{N}{2\pi N_T} \arg \left\{ \sum_{p=0}^{P-2} Y_j(p+1, n) Y_j^*(p, n) \right\} \quad (19)$$

Mean square error (MSE) can be computed by

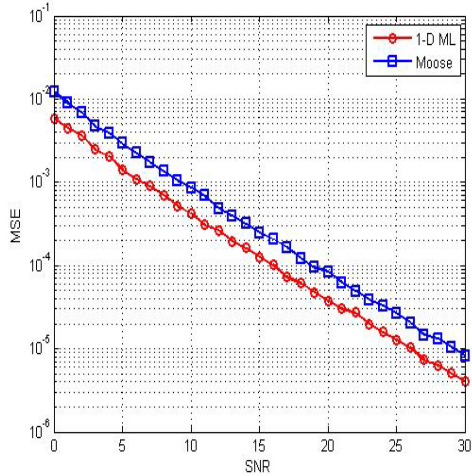
$$MSE = \frac{1}{M} \sum_{j=1}^M E \left\{ \left( \hat{(\cdot)} - (\cdot) \right)^2 \right\} \quad (20)$$

### Simulation Results And Discussions:

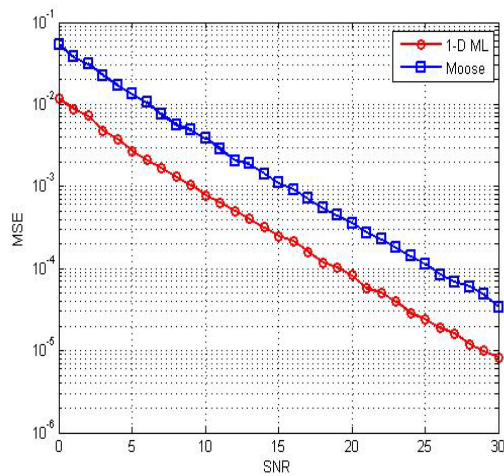
The simulation parameters shown in table I are taken according to IEEE 802.16m standard which is for mobile Wi-Max. The performance of CFO estimators has been studied through computer simulations using MATLAB. For both methods we considered quasi-synchronous scenario. All the simulations are performed under the Vehicular-A and pedestrian-A channel. Pedestrian-A is taken to model slow fading and vehicular-A is used to model fast fading channel environment. The channel tap coefficients are taken from ITU-R recommendations (ITU-R Recommendation M.1225, 1997). Each user is assigned 128 subcarriers. The transmitted symbols belong to QPSK constellation and length of cyclic prefix  $N_g = 32$ . GCAS is used for 1-D ML and SCAS is used for Moose principle based estimator. For both the methods, we estimate the CFO using training sequence present at the start of each frame. The CFO's of four users are considered as  $[0.11, -0.16, 0.21, -0.27]$  for simulation.

**Table I:** Simulation parameters

Parameter	Value
Bandwidth	5 MHz
Subcarrier Spacing	10.94 KHz
FFT size	512
No. Of Active Users	4 and 2
Subcarrier allocation schemes	Sub-band and Generalized
CFO range	[-0.5,+0.5]



**Fig. 3:** MSE Vs SNR for 4 users in Pedestrian-A channel



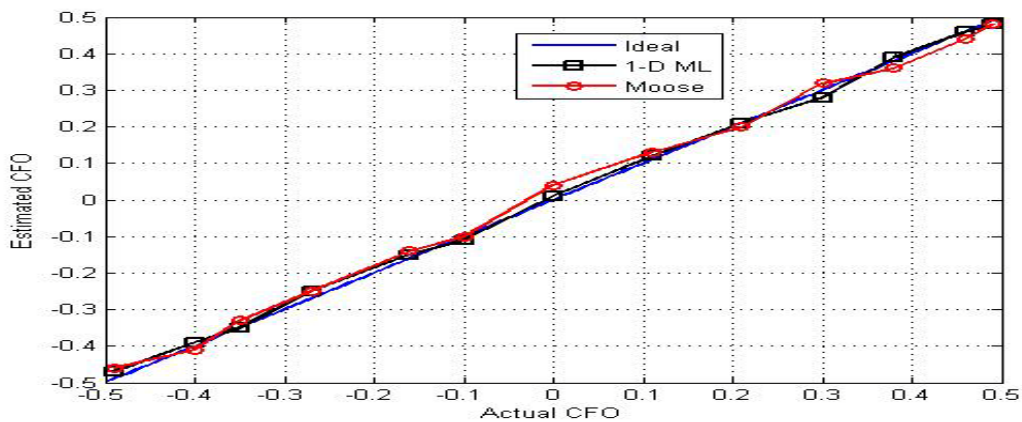
**Fig. 4:** MSE Vs SNR for 4 users in vehicular-A channel

Fig. 3 and 4 gives MSE Vs SNR performance for both methods for four active users in different channel conditions. The simulation results show that both methods gives satisfactory results in different channel conditions with slightly less accurate estimation accuracy in vehicular-A channel. For both schemes, performance in Pedestrian-A channel is comparable with each other because of less fading experienced by users whereas in vehicular-A large fading is experienced by users. The MSE performance is better for 1-D ML employing GCAS than Moose principle based estimator employing SCAS. Fig. 5 and table II depicts actual versus estimated CFO's for both methods in pedestrian-A channel at SNR of 15dB. It can be seen that estimation accuracy of 1-D ML is slightly better and it can also be observed from its MSE performance. Both methods perform within scope of practical implementations as normal requirement of estimation. Accuracy of the estimation is approximately 2% to 4% of subcarrier spacing.

**Table II:** Actual Vs Estimated CFO numerical values

Actual CFO	0.11	-0.16	0.21	-0.27
1-D ML method	0.109	-0.16	0.215	-0.265
Moose principle	0.10	-0.14	0.20	-0.26

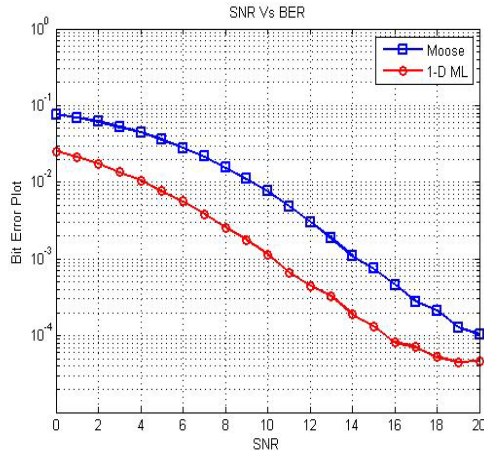
Fig. 6 and 7 show the performance in terms of bit error rate of two users for SNR ranges from 0 to 20dB under Vehicular-A channel and pedestrian-A channel. Perfect channel estimation means that the channel impulse response values are completely known to receiver has been assumed for analysing its performance. Each user's estimated CFO value has been compensated. Then the equalization is performed using zero forcing criterions which are achieved by dividing received signal with known channel frequency response. Finally, the received bits are compared with transmitted bits.



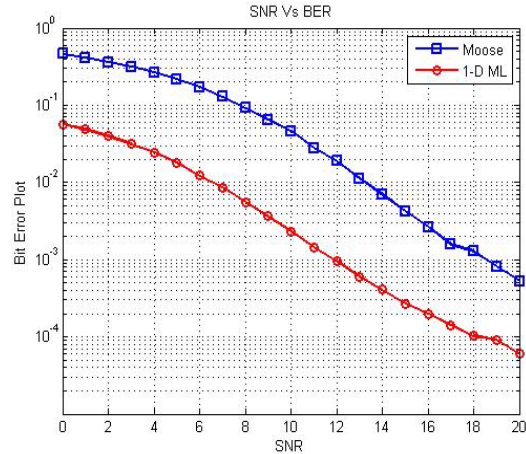
**Fig. 5:** Actual Vs Estimated CFO estimates

From fig 6 and 7, it is observed that BER is higher in vehicular-A than pedestrian-A. This is because of more fading environment encountered by a mobile user. It can be seen Moose principle based estimator employing sub-band carrier assignment scheme gives the results that are substantially worst because GCAS

used in 1-D ML provides system with some kind of frequency diversity. This means that the subcarriers allotted to user does not belong to a specific part of whole bandwidth but they were spread across the available bandwidth resulting an increases in slope of BER curve. This is obvious because grouping of subcarriers prevents possibility of exploiting channel diversity since a deep fade might hit substantial number of subcarriers. Thus GCAS leads to improvement of bit error rate performance as compared with SCAS. The BER performance of both schemes can be further improved by applying forward error control coding schemes.



**Fig. 6:** SNR Vs BER for 4 users in pedestrian-A channel



**Fig. 7:** SNR Vs BER for 4 users in vehicular-A channel

It can also be inferred that subcarrier assignment scheme affects the computational complexity. The task of user separation in SCAS is simpler as subcarriers are assigned contiguously. But in GCAS, the subcarriers are assigned randomly which makes user separation complex. Though Moose principle based estimator employing SCAS comes with comparatively less complexity with increased training overhead and reduced BER. The performance of Moose principle based estimator employing SCAS is better compared to 1-D ML employing GCAS in mobile environment. For both methods estimation accuracy can be increased by increasing the number of training blocks and averaging the frequency estimates over available blocks.

**Conclusion:**

In this paper, we discussed the requirement of future broadband wireless data communication based on two different subcarrier assignment techniques used for CFO estimation in OFDMA. MSE and BER estimation performances are observed under slow fading and fast fading channel conditions. It can be observed that subcarrier assignment scheme greatly affects the system performance accuracy. In case of slow fading environment, CFO estimation and BER performance of Moose principle based estimator employing SCAS and 1-D ML employing GCAS is comparable. But in fast fading environment GCAS outperforms the SCAS because of inherent frequency diversity exhibited by GCAS. From BER performance, it can be observed that a scheme providing better CFO estimation gives better BER due to smaller accumulated phase errors in CFO compensation. It can be concluded that GCAS is more robust in fast fading environment and is well suited for mobile user at the cost of increased system complexity. As OFDMA used in Wi-Max and LTE, systems employing GCAS with lesser training overhead will be of great of interest in practical implementation.

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