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Upside - Down β—Numbers

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Abstract: For any partition $\mu = (\mu_1, \mu_2, \ldots, \mu_n)$of anon – negative integer number r there exist a diagram (A)of β — numbers; which introduced by James in 1978.In this work, we introduced a new diagram by "*upside – down*" diagram (A) denoted (A′) or (A″).

Key words: Partition theory, β — numbers.

INTRODUCTION

Let r bea non – negative integer. Apartition $\mu = (\mu_1, \mu_2, \ldots, \mu_n)$ofris a sequence of non – negative integers suchthat $|\mu| = \sum_{j=1}^{n} \mu_j$ = r and$\mu_j \geq \mu_{j+1}$,∀ j ≥ 1. For example,μ=(6 ,4 , 3 , 3 , 3 ,1) is a partition of r = 20. β — numbersis defined by; see (James, G., 1978):
"*Fix*μ*is a partition of r choose an integer b greater than or equal to the number of parts of* μ*and define*$\beta_i = \mu_i + b - i$*, 1 ≤i≤b. The set*$\{\beta_1, \beta_2, \ldots, \beta_b\}$*is said to be a set of* β — *numbers for* μ."
From the above example, if we take b = 7, then β — numbers are {12 , 9 , 7 , 6 , 5 , 2 , 0}.

Now, let e be a partition integer number greater than or equal to 2 we can represent β — numbers by the diagram (A).

run.1	run.2	…	run.e
0	1	…	e - 1
e	e + 1	…	2e - 1
2e	2e + 1	…	3e - 1
.	.	.	.
.	.	.	.
.	.	.	.

diag.(A)

Where every β will be represented by a bead which takes its location in diag. (A).
Returning to above example will see

e = 2	b = 7
0 1	• –
2 3	• –
4 5	– •
6 7	• •
8 9	– •

and

e = 3	b = 7
0 1 2	• – •
3 4 5	–– •
6 7 8	• • –

This subject as it has a connection representation theory of Iwahori – Hecke algebras, see (Fayers, M., 2007). Also any partition μ of r called W – regular;w ≥ 2, if there does not exist j ≥ 1such that$\mu_j = \mu_{j+w-1} > 0$, and$\mu$is w – restricted if $\mu_j - \mu_{j+1} > w$, ∀ j.

The Intersection of β — NUMBERS Inmaindia Grams:

Mahmood in (Mahmood, A. S., 2011) introduced the definition of main diagram (s) of (A) andthe idea of the intersection of these main diagrams. In this section we repeat the principalsresults, as following:

Since the value of b ≥ n, then we deal with an infinite numbers of values of b. Here we want to mention that these values has a special diagram (A) for it, but there is a repeated part of this diagram with other values where a "*Down – shifted*" or "*Up – shifted*", occurs when we take the following:

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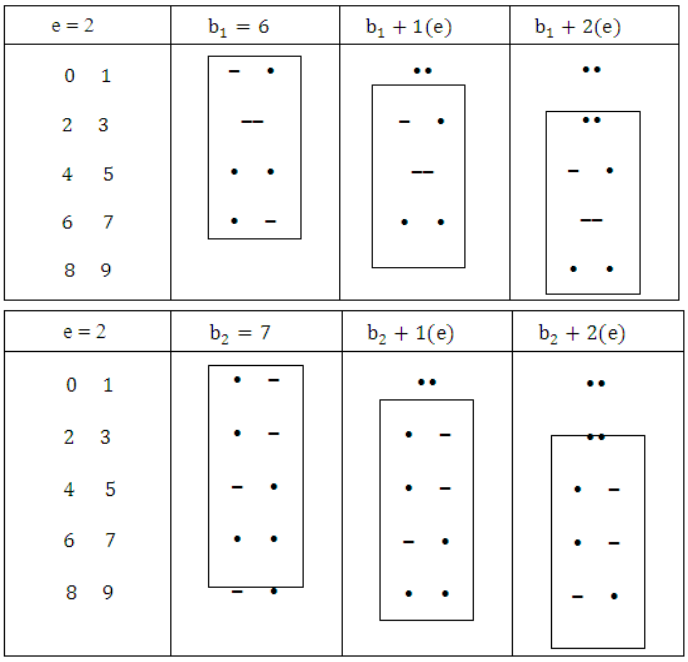
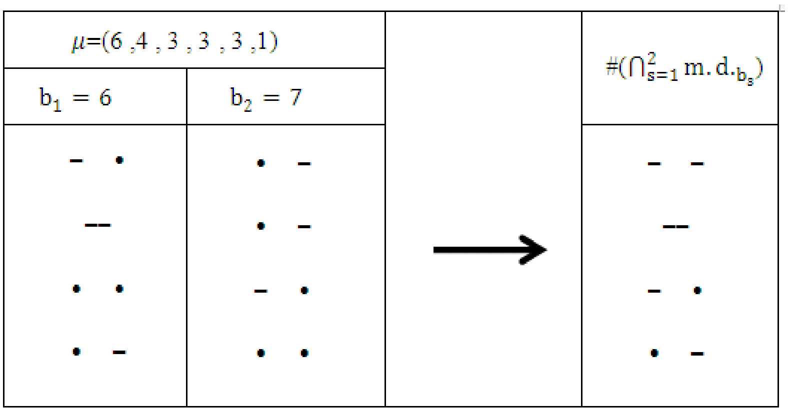
$(b_1 \text{ if } b = n), (b_2 \text{ if } b = n+1), \quad \ldots \quad \text{and} \quad (b_e \text{ if } b = n + (e-1)).$

Definition(2.1): (Mahmood, A. S., 2011) The value $b_1, b_2, \ldots, b_e$ are called the guides of any diagram (A) of $\beta-$numbers.
By the above example, the guides values are $b_1 = 6$ and $b_2 = 7$ if e = 2, when μ=(6 ,4 , 3 , 3 , 3 ,1):



We define any diagram (A) that corresponds any b guides as a main diagram.

Theorem (2.4): (Mahmood, A.S., 2011) There is e of main diagrams for any partition μ of r.
The idea of the intersection of any main diagrams is defined by the following:
1. Let τ be the number of redundant part of the partition μ of r, then we have $\mu = (\mu_1, \mu_2, \ldots, \mu_n) = (\alpha_1^{\tau_1}, \alpha_2^{\tau_2}, \ldots, \alpha_m^{\tau_m})$ such that: $r=\sum_{j=1}^{n} \mu_j = \sum_{l=1}^{m} \alpha_l^{\tau_l}$.
2. We denote the intersection of main diagrams by $\#(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s})$.
3. The intersection result as a numerical value will be $\emptyset$ in the case of no existencefor any bead, orγ in the case that γ common beads exist in main diagrams.



The two principle theorems about the idea of the intersection of any main diagrams are:

Theorem (2.3): (Mahmood, A. S., 2011) For any $e \geq 2$ the following are holds:
1. $\#(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}) = \emptyset$ if $\tau_k = 1 \;\forall\, 1 \leq k \leq m$.
2. let Ω be thenumber of parts of α which satisfies the condition $\tau_k \geq e$ for some k, then $\#(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}) = (\sum_{t=1}^{\Omega} \tau_t - \Omega(e-1))$.

Theorem (2.4): (Mahmood, A. S., 2011)
1. Let μ be a partition of r be a w – regular, then:

$$\#\left(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}\right) = \begin{cases} \text{value} & \text{if } e < w, \\ \emptyset & \text{if } e \geq w. \end{cases}$$

2. Let μ be a partition of r be a h – restricted, then:

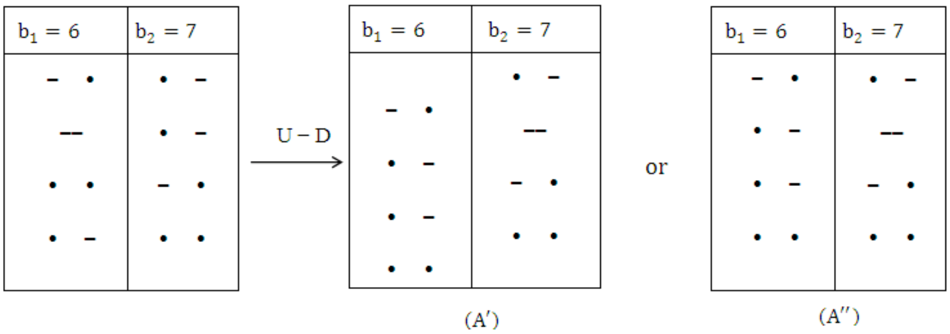
$$\#\left(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}\right) = \begin{cases} \text{value} & \text{if } e < h \quad or\ (e = h \text{ and } h < w), \\ \emptyset & \text{if } e > h \quad or\ (e = h \text{ and } h \geq w). \end{cases}$$

Also, Sarah M. Mahmood in (Mahmood, S.M.) gave the same subject by using a new technique which supporttheresults of Mahmoodin (Mahmood, A.S., 2011), formore details see the reference (Mahmood, S.M.).

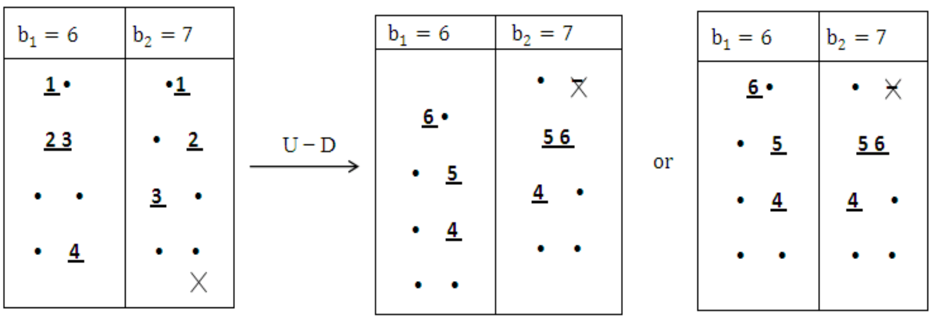
Upside – Down β — NUMBERS:

In this work, we introduce a new diagram depend the "*old diagram (A)*" with application of "*Upside – down*". The new diagram have another partition of the origin partition and if we using the idea of the intersection, the partitions of the beads is not the same (or is not the sum) in $\#\left(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}\right)$ in normal main diagrams.

From the above example, if μ=(6 ,4 , 3^3,1) and e = 2, then we have two (groups) of diagram (A′) to take the idea of upside – down without respect the new position of the rows, and (A′′) to take the idea of upside – down with respect the new conversion (i.e. is not sufficient the idea of upside– down but also with "*up– shifted*" diagram).



Now, if we use the old technique for finding any partition of any diagram in (A′) or (A′′), the value of partition is not equal to the origin partition? so, we delete any effect of (–) in (A) after the position of β_1, and we start with number 1 for the first (–) a(left to right) in any row exist of (A), and with number 2 for the second (–) and …,etc,and we stop with lest (–) before the position β_1 in (A). Now, to apply (upside – down) about (A), the new version (A′) or (A′′) have the same partition of (A).



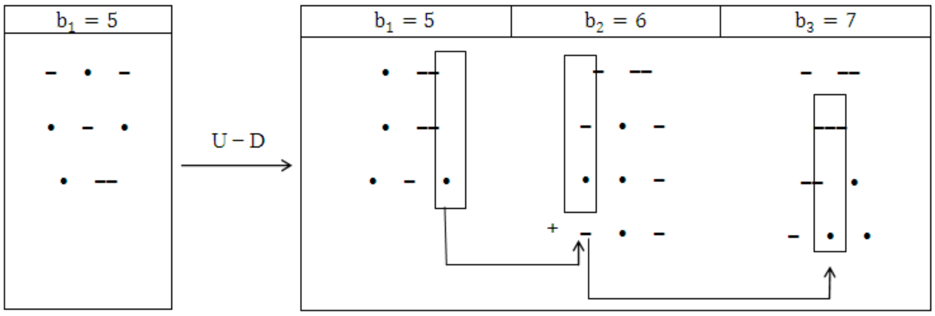
Remark:

The main diagram in the case $b_1 = n$, in(A′) or (A′′) play a main role to design all the main diagrams (A′) or (A′′) in ($b_2 = n + 1$), … , ($b_e = n + (e - 1)$), as following.

Rule (3.1) Sincethe main diagram (A′) or(A′′) in the caseb_1, we can find the successive b_2, ... and b_e, as follows:

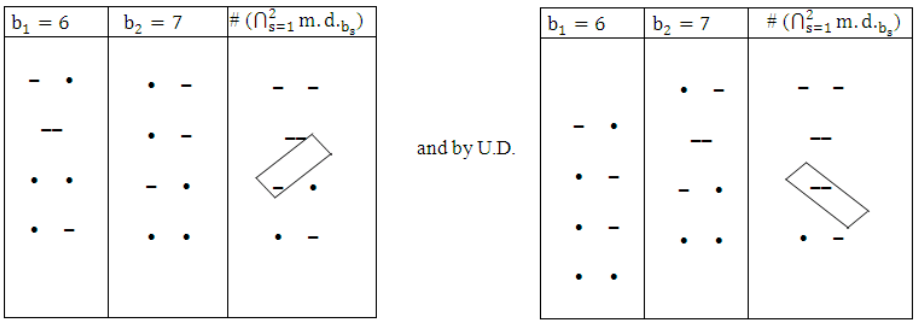
a.1^{st}column in the case b_1= n2^{nd} column in the case $b_2$and to add one(–)in up
3^{rd} column in the case b_3 and to add two (–) in up… lastcolumn in thecase $b_e$and to add (e – 1)

of (–) in up of main diagram (A′) or (A″).
b. 2^{nd}column in the caseb_1 3^{rd}column in the case b_2 and to add one(–) in up4^{th} column in the case$b_3$and to add two(–) in up…1^{st}column in the caseb_e and to add one (•) in down of main diagram (A′) or (A″).
e. Last column in the case b_1 1^{st}column in the case b_2 and to add one (•) indown 2^{nd} column in the case b_3 and to add one (–) in up …(e – 1) column in the caseb_e and to add (e – 2).
For example, let μ = (5 ,3 , 3 , 2 ,1) and e = 3, then



Theorem (3.2):

All the results in (Mahmood, A. S., 2011; Mahmood, S. M.) about the main diagrams (A) the same of (A′) but in position upside – down.



In (A″), the results if we apply the idea of the intersection is totally difference to the Theorem (3.2), and to explain this phenomenon we need to go back to Rule (3.1) and we have three zones:

1. Zone of Cancelation 1: (ZC_1):

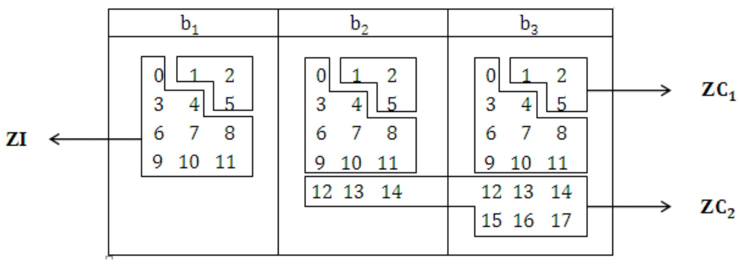
All the (–) adding in up after using Rule (3.1) are not effective in the intersection of main diagram (A″), and the position of this zone is: "*Lest (e – 1) position in*1^{st} *rows, last (e – 2) position in* 2^{nd} *rows, …, last position in (e – 1) rows in all (A″)* ".

2. Zone of Cancelation 2: (ZC_2):

After Rule (3.1), "*all last row in b_2, all two last rows in b_3, … ,last (e – 1) rows in b_e*" are not effect in the intersection of main diagrams (A″).

3. Zone of Influence: (ZI):

This zone is very important in the intersection of main diagram (A″).

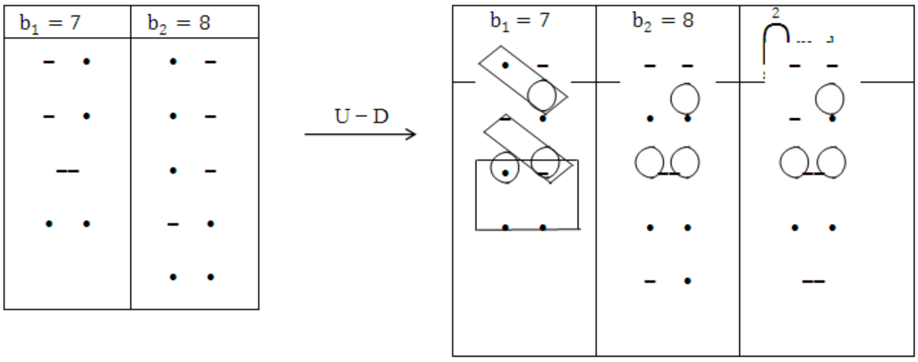


Theorem (3.3):

Let e = 2, the intersection of all main diagrams (A′′) is connection:

1. Two beads in each row in b_1 if the row before it have (• →(• or –)) respectively.
2. First bead in each row in b_1 if
   i. In the same row exist another bead and the row before it have (–→ (–or •))
   ii. The row before the row of bead is the first row and it have two beads.
3. Second bead in each row if the all position it of main diagram in up have beads.

For example, let μ = (6^2, 4^3, 2 ,1) and the



Theorem (3.4): The intersection of main diagrams (A′′) is connection the following:

1. For anye ≥ y where y = 3. All the position have beads in any row in b_1(out ZC_1)if all the (y – 2) rows before it have beads in all positions and the existentsof bead in position number twoin row before all (y – 2) − rows.
2. Foranye ≥ ywhere y = 3.Thelast row in ZC_1 have beads in all positions, thisfirst (e – 1) beads in the result of the intersection if all the rows before it have beadsin all positions.

Theorem (3.5):

For anye ≥ ywhere y = 3. If any row in b_1 have (• – • • … •), then the intersection of maindiagrams(A′′) is connection only the first bead a left if all (y – 3) rows before it have beads and the row before (y – 2) rows have (– • • … •) respectively.

Theorem (3.6):

Forany e, the intersection of main diagrams (A′′) is connection the first and last bead in row with (• – – … – •) if the all positionofitmain and secondly diagonals in up have beads.

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