Australian Journal of Basic and Applied Sciences, 7(7): 991-998, 2013
ISSN 1991-8178

Modeling Monthly Inflation Rate Volatility, using Generalized Autoregressive Conditionally Heteroscedastic (GARCH) models: Evidence from Nigeria

Osabuohien-Irabor Osarumwense and Edokpa Idemudia Waziri

Mathematics/Statistics Department Ambrose Alli University, Ekpoma, Nigeria

Abstract: This paper describe an empirical study of modeling financial time series data with application to inflation rate data for Nigeria. The theory of univariate non-linear time series analysis is explored and applied to the inflation data spanning from January, 1995 to December, 2011. The diagnostic checking has shown that the fitted model (GARCH(1,0) + ARMA(1,0)) is appropriate. A two-year (24 months) forecast from January 2012 to December 2013 was made. This empirical results have more general implications for small scale macroeconomics and will also be helpful for policy makers and citizens of the Federal Republic of Nigeria.

Key words: Autocorrelation, Forecast, ARMA, GARCH Model, Inflation

INTRODUCTION

Inflation measure the relative changes in the prices of commodities and service. This economic indicator has direct effect on the state of the economy. It influences the interest rate on saving and the rate we pay on our morgages. In other to devise better policies to control the inflation rates, it is necessary to know the pattern of inflation in the country. It is an important economic indicator that the government, politician, economist and other stake holders especially those in African continent including Nigeria of argument when debating about the state of the economy. In recent years, inflation has become one of the major economic challenges facing most countries in the world especially Nigeria. Inflation is a major focus of economic policy world wide as describe by David (2001). Inflation data are usually very volatile and this dynamics can better be studied using a stochastics modelling approach that capture the time dependant structure embedded in the time series inflation data. The Autoregressive Conditional Heteroscedasticity (ARCH) Models, with its extension to Generalised Autoregressive Conditional Heteroscedasticity (GARCH) Models as introduced by Engle and Bollerslev, (1982) and (1986) respectively, accommodates the dynamics on the conditional Heteroscedasticity (the changing variance nature of the data). Heteroscedasticity affects the accuracy of forecast confidence limit and thus has to be handled properly by constructing appropriate non-constant variance models (Amos, 2009). Inflation as describe by Webster's (2000), is the persistent increase in the level of consumer prices or persistent decline in the purchasing power of money. Inflation can also be expressed as a situation when the demand for goods and services exceeds their supply in the economy (Hall, 1982). In reality, inflation means that your money cannot buy asa much as what it could have bought yesterday. Inflation can be caused by either too much money in circulation in the country or too few goods offered for sale.

A lot of empirical researches have been carried out in the area of forecasting inflation rate using the popular Autoregressive Integrated Moving Average (ARIMA) model, popularised by Box and Jenkins (1976), some of them are; Junttila (2001), applied the Box and Jenkins (1976) approached to model and forecast the Finnish Inflation. Pufnik and Kunovac (2006), applied similar approach to forecast short term inflation in croatia. However, modelling and forecasting inflation rate is better done by the approach described by Engle and Bollerslev, (1982) and (1986) respectively because of it ability to tackle the dynamic nature of the data.

The most common way of measuring inflation is the Consumer Price Index (CPI) over monthly(as in the case of Nigeria), quarterly, or yearly. In Nigeria the CPI is calculated by the Central Bank of Nigera.

The Autoregressive (AR), Moving Average (MA) and the combination of the two, which is the Autoregressive Moving Average (ARIMA) models are useful in modeling general time series. These models remove the trends and smooth the series. However, these models and the traditional economics models have the assumption of a constant one-period variance or homoscedasticity for the error terms. This assumption is not appropriate in modeling financial market variables such as stock price indices and currency exchange rate. Financial market variables have characteristics which general time series models and some econometric models have failed to consider, these are;

1. The unconditional distribution of financial time series such as stock price return, Exchange Rate return etc has heavier tail than the normal normal distribution.
2. Values of the return do not correlate much but the square of the returns are highly correlated.
3. The changes in returns tend to cluster.

The Generalized Autoregressive conditional Heteroscedasticity (GARCH) model of Bollerslev (1986) has gained in popularity because of its ability to address these issue.

Corresponding Author: Osabuohien-Irabor Osarumwense, Mathematics/Statistics Department Ambrose Alli University, Ekpoma, Nigeria
E-mail: osabuohien247@gmail.com

The objectives of this study using the concept of the the Generalized Autoregressive conditional Heteroscedasticity (GARCH) together with the ARMA models is specifically to;

1. Present a brief concept of the GARCH model
2. Obtain a better model in fitting a secondary data of Nigeria monthly Inflation rate (CPI).
3. Using the fitted model to make forecst.
4. Use of the R statistical package in runing the GARCH + ARMA models in fitting the secondary data.

MATERIALS AND METHODS

The data employed in this study comprise of 204 monthly obsevations from January, 1995 to December, 2011, which was taken from the Central Bank of Nigeria statistical Bulletin. The R software, an open souce (GPL), interactive statistical environment modeled after S and S-plus was used in plotting the graphs and statistical analysis of the data set. The GARCH (p,q) model is a generalization of GARCH (1,1) with p as the autoregressive lag p and q as the Moving Average lag q. Formally a process $\{r_t\}$ is GARCH (p,q) if;

$$r_t = \sigma_t \varepsilon_t. \quad (1)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i r_{t-1}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2. \quad (2)$$

$$\sum_{i=1}^{\text{Max}} (\alpha_i + \beta_j) < 1 \quad i, j \in Z^+ \quad (3)$$

and for AR(1) process,

$$Y_t = \emptyset Y_{t-1} + \varepsilon_t \quad (4)$$

where $\{\varepsilon_t\}$ is a Gaussian white noise, with restrictions $\alpha_0 > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$ for $i = 1, 2, 3, \ldots\ldots\ldots\ldots, q$ and $j = 1, 2, 3, \ldots\ldots\ldots\ldots p$ being imposed in order to have the conditional variance remaining positive. GARCH (0,0) model is a simple white noise. Similar to the ARCH(q) model, the conditional mean of $\{r_t\}$ is zero, that is $E(r_t|H_t)$ implies the series $\{r_t\}$ is a martingale difference and observing $\{r_t\}$ is uncorrelated (Gourieroux, et al., 1997). The choice to include ARMA Model is to widen the higher amount of uncertainty i.e higher persistence Volatility (Rupper, 2011).

Estimation of the GARCH Model:

In GARCH (1,1), model estimation, initial values of both the squared return and past conditional variance are needed in estimating the parameters of the model. As suggested by Bollerslev, (1986) and Tsay, (2002), the unconditional variance in (2) or the past sample variance of the returns for the past variance may be used as initial value. Therefore, assuming $r_1, \ldots\ldots, r_q$ and $\sigma_1^2, \ldots\ldots\ldots, \sigma_p^2$ are known, then the conditional maximum likelihood estimates can be obtained by maximizing the conditional log-likelihood given by

$$\ell = \ell og f\left(r_{q+1}, \ldots\ldots, r_t, \sigma_{p+1}^2, \ldots\ldots. \sigma_t^2 \middle| \theta, r_1, \ldots\ldots r_q, \sigma_1^2, \ldots\ldots, \sigma_p^2\right). \quad (5)$$

$$= -\frac{1}{2} \sum_{t=m+1}^{T} \ell og(2\pi\sigma_t^2) - \frac{1}{2} \sum_{t=m+1}^{T} \left\{\frac{r_t^2}{\sigma_t^2}\right\}. \quad (6)$$

where,

$\theta = \left(\alpha_0, \alpha_1, \ldots\ldots, \alpha_p; \beta_1, \ldots\ldots, \beta_q\right)$ and $m = \max(p, q)$.

Model Checking:

Goodness of fit of the GARCH model is based on residuals and is more specifically on the standardized residuals (Talkle,2003). The residual are assumed to be independently and identically distributed following either a normal or standardized t distribution (Tsay, 2002) and (Gourieroux, 2001). Plots of the residuals such as the histogram, the normal probability plot and the time plot of the residual can be used. If the model fits the data well, the histogram of the residual should be approximately symmetric. The normal probability plot should be straight line while the time plot should exhibit random variation. The ACF and the PACF of the standardized residual are used for checking the adequacy of the conditional variance model. However, model checking of Garch(1,0) + Arma (1,0) is done simultaneously using the R Software.

Model Selection Criteria:

Selection criteria assess whether a fitted model offers an optimal balance between the goodness-of-fit and parsimony. This will help identify candidate models that are either too simplistic to accommodate the data or unnecessarily complex. Selecting the appropriate GARCH and ARMA models were fitted together using the fGarch package. The most comon model selection criteria such as the Akaike Information Criterion (*AIC*), the Bayesian Information Criterion (*BIC*), the Schwarz information criterion (*SIC*) and the Hannan –Quinn Criterion (HQ), were used as bases for selection criteria. These selection criteria are defined and calculated as;

$$\text{AIC} = -2\ell\text{og}\,(\text{L}) + 2(\text{m}) \quad (7)$$

$$BIC = -2\ell og(L) + m\ell ogn \quad (8)$$

$$HQ = -2\ell\text{og}\,(\text{L}) + 2\text{m}\ell\text{og}(\ell\text{ogn}) \quad (9)$$

$$SIC = -2\ell og(\text{L}) + (\text{m} + m\ell ogn) \quad (10)$$

where, n and m are number of observations (sample size) and parameter in the model respectively and $\ell og(L)$ is the ℓoglikelihood. The desirable model is one that minimizes the AIC, the BIC, the HQ and the SIC.

RESULTS AND DISCUSSION

The data employed in this study comprise of 204 monthly observations of CPI monthly inflation rate. The descriptive Statistics (see Table 1) for the monthly CPI rate and return, shows that a "stdev" (Standard deviation) of 29.133 from the series is high with a general mean of 58.550. Skewness and Kurtosis represent the nature of departure from Normality. Positive or Negative skewness indicate asymmetry in the series, therefore skewness of 0.792 implys that the distribution is positively skewed with long right tail and a deviation from Normality. A Kurtosis of -0.541 suggest flatness of the distribution. The series has a minimum inflation (CPI) rate value of 20.600 and a maximum value of 128.200 giving a CPI rate range of 107.600. The Jarque-Bera (JB) Statistics of 23.930 with two(2) degree of freedom confirms that the Null hypothesis (H_O) of Normality is rejected at a p-value of $6.362e^{-06}$. On the other hand, looking at the skewness, Kurtosis and the Jarque-Bera test Statistics, the return series of the Inflation rate also show serious signs of violation of the principle of Normality. Fig. 1 compriss plots of the CPI monthly inflation rate, Returns and Square Return series for the period January 1995 through December 2011. It shows a visual display of the changing variance and showing no tendency to return to it mean as seen in the monthly Inflation Rate (Top left of Fig 1). Therefore, both the inflation rate series and its return has all financial characteristics.

Table 1: Descriptive Statistics for Inflation Rate Series

	CPI Data	Return
nobs	204.000000	203.000000
NAs	0.00000000	0.00000000
Minimum	20.6000000	-24.0179740
Maximum	128.200000	12.1010760
1. Quartile	33.7000000	0.12657000
3. Quartile	75.1250000	2.00435000
Mean	58.5504900	0.90064100
Median	48.3000000	0.87848300
Sum	11944.3000	182.830047
SE Mean	2.03978100	0.19261600
LCL Mean	54.5286150	0.52084400
UCL Mean	62.5723660	1.28043700
Variance	848.784483	7.53152300
Stdev	29.1339060	2.74436200
Skewness	0.79227100	-3.26435000
Kurtosis	-0.54126200	33.5488340
Jarque-Bera	23.9303000	10092.9627

The inflation series we have study is non-stationary, therefore, we convert the CPI by logarithm transformation as shown in Top right of Fig 1. Applying a transformation to address nonconstant variance is regarded as a "first step" (Tebbs, 2011). The return appear to be quite stable overtime after "taking" the first different of the log CPI producing a process still with high persistence Volatility. This behavour of the inflation series Returns is in line with most financial theories and models which usually assume the price of Return to be stationary process. The Square of the Returns is also shown in botton right of Fig 1. Plots of the Return Autocorrelation (ACF) and Partial Correlation (PACF) function are shown in Fig 2. Top right and bottom left, provide evidence of serial correlation among observations. This result indicate that the variance of return is conditional on its past history and may change overtime.

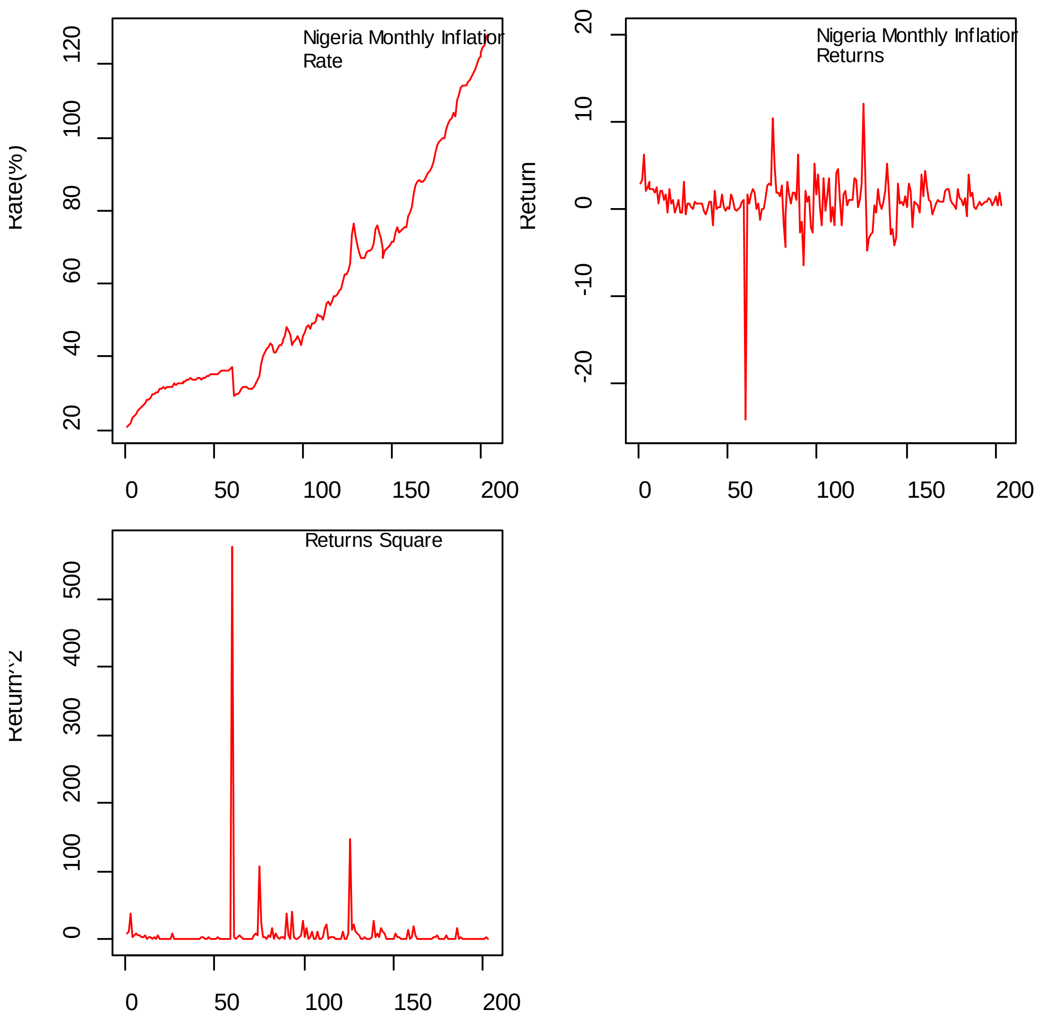

Fig. 1: Plots of the Nigeria Monthly Inflation Rate, Returns, and Square Returns

In this study, the Return of the Inflation rate series is calculated as;

$$r_t = \ell og\left(\frac{X_t}{X_{t-1}}\right) X\ 100\% = (\ell og X_t - \ell og X_{t-1})\ X\ 100\%$$

$$= \ell og\left(1 + \frac{X_t - X_{t-1}}{X_{t-1}}\right) X\ 100\%. \quad (11)$$

where;

r_t *is the Inflation rate series for any time, t*

X_t *is the Inflation rate value at time, t, (Monthly)*

X_{t-1} *is the Inflation rate value at time, t − 1 (Previous Month)*

The strategy and analysis for selecting the appropriate model from the fourteen (14) competing Garch and Arima models fitted together, is based on four(4) different models selection criteria which are shown in Table 2 for detailed analysis. The idea is to have a parsimonious model that capture as much variation in the data as possible. The model put on “bold” (Garch (1,0) + Arma(1,0)) is judged to be the most appropriate, having the smallest values for the four(4) different selection criteria and fourteen competing Garch and Arma model. The larger the values of the criterion, the unfavourable the model become.

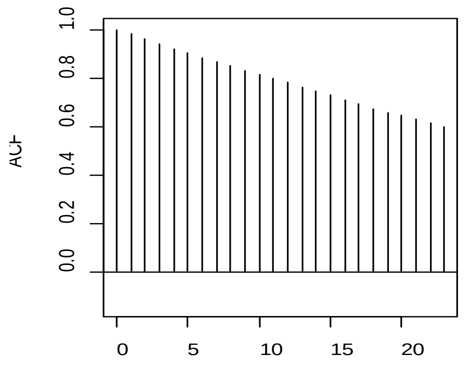

Fig. 2: Graphical Display CPI ACF, Returns ACF and PACF, Returns Square ACF and the Return Square

Table 2: Comparison of Selected GARCH Models

GARCH Models And ARMA Models	Akaike Criterion(*AIC*)	Bayesian Criterion(*BIC*)	Schwarz Criterion(*SIC*)	Hannan-Quin Criterion(HQIC)
Garch(1, 0) + Arma(0,0)	4.881593	4.930556	4.881164	4.901401
Garch(1, 0) + Arma(0,1)	4.861051	4.926336	4.860294	4.887463
Garch(1, 0) + Arma(1,0)	4.858177	4.923462	4.857420	4.884589
Garch(1, 0) + Arma(1,1)	4.868009	4.949615	4.866834	4.901023
Garch(1, 1) + Arma(0,0)	4.891445	4.956730	4.890688	4.917857
Garch(1, 1) + Arma(1,0)	4.872121	4.953728	4.870947	4.905136
Garch(1, 1) + Arma(0,1)	4.873903	4.955509	4.872729	4.906918
Garch(1, 1) + Arma(1,1)	4.881622	4.979549	4.879941	4.921240
Garch(2, 1) + Arma(0,1)	4.883676	4.981604	4.881995	4.923294
Garch(2, 1) + Arma(1,0)	4.881889	4.979816	4.880207	4.921506
Garch(1, 2) + Arma(0,1)	4.867598	4.965525	4.865916	4.907215
Garch(1, 2) + Arma(1,0)	4.881889	4.979816	4.880207	4.921506
Garch(2, 2) + Arma(0,1)	4.893529	5.007777	4.891255	4.954060
Garch(2, 2) + Arma(1,1)	4.901237	5.031807	4.898285	4.954060

Source: Researcher's Calculation

The coefficients of the output for the model estimation, where the conditional distribution are norm and the standard Error from the Error analysis based on Hessian respectively are given as;

Coefficient(s):

```
mu        ar1       omega     alpha1
0.67539   0.21698   7.13335   0.01954
```

Error Analysis:

```
Estimate    Std. Error   t value   Pr(>|t|)
mu        0.67539    0.21020    3.213    0.00131 **
ar1       0.21698    0.09033    2.402    0.01630 *
omega     7.13335    0.72257    9.872    < 2e-16 ***
alpha1    0.01954    0.02800    0.698    0.48520
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The chosen models seems to fit the data well as it obey the necessary condition for a model to be stationary and fitted as shown in the output of the coefficients and Error analysis. Where the sum of alpha and Beta are less than 1. If the sum is greater than 1, then the prediction of volatility are explosive. With the corresponding fit statistics (also shown in Table 2);

Information Criterion Statistics:

AIC	BIC	SIC	HQIC
4.858177	4.923462	4.857420	4.884589

The output of the standard Residuals has shown that the residuals are Random, independent and identically distributed. The Ljung-Box Test for Q(10), Q(15), Q(20), and Q(15), Q(20) for the squared standardized residual shows that there are some higher order dependence in the volatility while the histogram of residual and the probability plot (Q-Q) as shown in Fig: 3, confirmed the normality of the model.

Standardised Residuals Tests:

			Statistic	p-Value
Jarque-Bera Test	R	Chi^2	12214.07	0
Shapiro-Wilk Test	R	W	0.699108	0
Ljung-Box Test	R	Q(10)	4.090126	0.9431897
Ljung-Box Test	R	Q(15)	7.984142	0.9244169
Ljung-Box Test	R	Q(20)	15.58703	0.7418913
Ljung-Box Test	R^2	Q(10)	0.3572108	0.9999987
Ljung-Box Test	R^2	Q(15)	3.825207	0.9982557
Ljung-Box Test	R^2	Q(20)	3.967688	0.9999565
LM Arch Test	R	TR^2	0.4277478	0.9999999

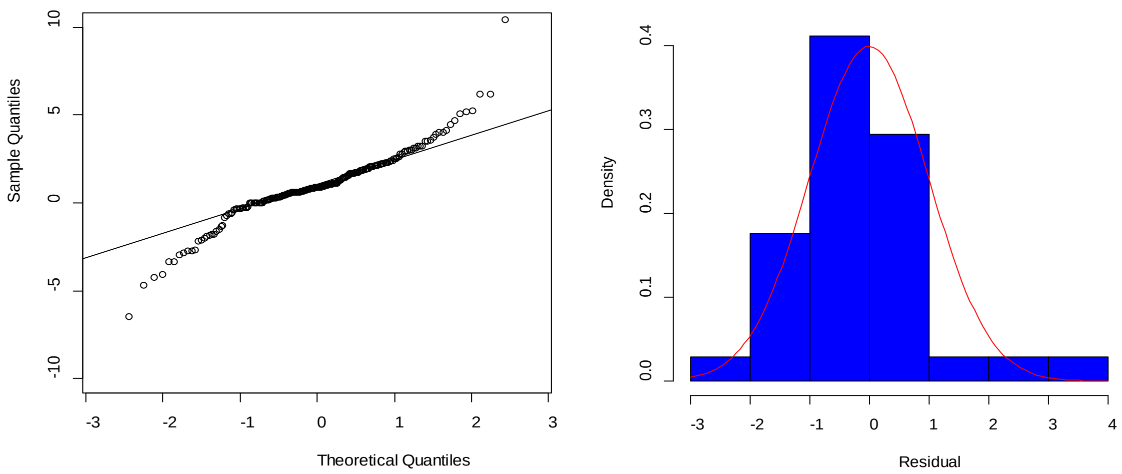

Fig. 3: Normal Probability Plot and Histogram for the fitted (GARCH + ARMA) Models

The “mean forecast” is the predicted values 1, 2, 3, …..etc time periods in the future and the “Standard deviation” is the predicted standard deviation of residuals from the mean forecast as shown in Table 3. A 95% confidence interval for future observation is also given. The standard deviation complies with the assumption homoscedasticity (Constant Variance) for the error terms

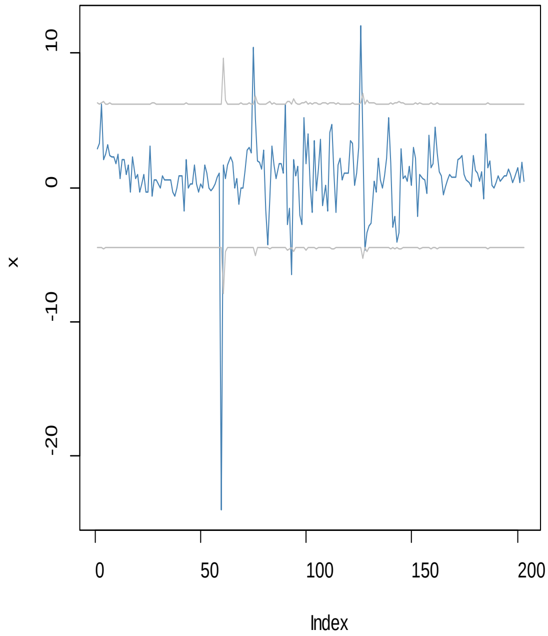

Fig. 4: Plots of Prediction with confidence Interval and series with 2 conditional SD Superimposed

Table 3: Prediction Output for Mean Forecast, Mean Error and Standard Deviation

Months	Mean Forecast	Mean Error	Std Deviation	Months	Mean Forecast	Mean Error	Std Deviation
1	0.777179	2.67223	2.67223	13	0.862546	2.76314	2.69731
2	0.844023	2.75845	2.69682	14	0.862546	2.76314	2.69731
3	0.858527	2.76291	2.69730	15	0.862546	2.76314	2.69731
4	0.861674	2.76313	2.69731	16	0.862546	2.76314	2.69731
5	0.862357	2.76314	2.69731	17	0.862546	2.76314	2.69731
6	0.862505	2.76314	2.69731	18	0.862546	2.76314	2.69731
7	0.862537	2.76314	2.69731	19	0.862546	2.76314	2.69731
8	0.862544	2.76314	2.69731	20	0.862546	2.76314	2.69731
9	0.862545	2.76314	2.69731	21	0.862546	2.76314	2.69731
10	0.862546	2.76314	2.69731	22	0.862546	2.76314	2.69731
11	0.862546	2.76314	2.69731	23	0.862546	2.76314	2.69731
12	0.862546	2.76314	2.69731	24	0.862546	2.76314	2.69731

$$r_t = 0.6753 + \varepsilon_t \quad (12)$$
$$\sigma_t^2 = 7.1334 + 0.0195 r_{t-1}^2 - 0.2169 Y_{t-1} \quad (13)$$

Again, Since $\widehat{\emptyset} = 0.2169$ and $\hat{\alpha} = 0.0195$ is statistically significant as shown in (13), implying that this is a small amount of positive autocorrelation.

Conclusion:

In this study, we were able to model and predict twenty-four (24) months Nigeria's inflation rate (Jan 2012 to Dec. 2013) confidence interval as put in (13) and shown in Fig: 4. The forecasted values will affect the quality of the policiec implemented based on the forecast and the model used. From Fig 4, it is clear that there will not be any major high volatility persistence as in the case of the year 1999, 2001, and 2004 as seen in the two(2) conditional standard deviation superimposed

REFERENCES

Amos, C., 2010. Time Series Modelling with Application to South African Inflation Data, A thesis submitted to University of Kwazulu Natal for the degree of Masters of Science in Statistics.

Bollerslev, T., 1986. Generalized autoregressive conditional heteroscedasticity, Journal of Econometrics, 31: 307-327.

Box, Gep and Jenkins, 1976. Time series Analysis, forecasting and control, Holden – Day Inc. USA, pp: 574.

Engle, R.F., 1982. Autoregressive Conditional Heteroskedasticity with Estimates of variance of U.K Inflation. Econometrica, 987-1008.

Gourieroux, C., 2001. Financial Econometrics, Problems, models and methods. Princeton series in finance, New Jersey, USA.

Hall, R., 1982. Inflation, Causes and Effects, Chicago University Press, Chicago.

Junttila, J., 2001. Structural breaks, ARIMA model and Finnish inflation forecasts. Int. J. Forecasting, 17: 203-230.

Pufnik, A. and D. Kunovac, 2006. Short-Term Forecasting of Inflation in Croatia with Seasonal ARIMA Processes. Working Paper, W-16, Croatia National Bank, pp: 1-6

Rupper, D., 2011. Statistics and Data Analysis for Financial Engineering. Springer Texts in Statistics, DOI 10.1007/978-1-4419-7787_18.

Talke, I.S., 2003. Modelling volatility in time series data, MSc thesis, University of Kwa-Zulu Natal.

Tebbs M. Joshua, 2011. Forecasting and Time Series, University of South Carolina, USA.

Tsay, R.S., 2002. Analysis of Financial Time Series". ISBN 0-471-41544-8.

Webster, D., 2000. Webster`s New Universal Unabridged Dictionary.Barnes and Noble Books, New York.