Australian Journal of Basic and Applied Sciences, 8(9) June 2014, Pages: 493-503

AENSI Journals

Australian Journal of Basic and Applied Sciences

ISSN:1991-8178

Journal home page: www.ajbasweb.com

Rational Cubic Spline for Positivity Preserving Interpolation

Samsul Ariffin Abdul Karim

Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 31750 Tronoh, Perak Darul Ridzuan, Malaysia.

ARTICLE INFO

Article history:
Received 2 March 2014
Received in revised form
13 May 2014
Accepted 28 May 2014
Available online 23 June 2014

Keywords:
Rational cubic spline, cubic Ball, positivity, smoothness

ABSTRACT

Background: A rational cubic spline scheme is developed with cubic spline as numerator and cubic Ball function as denominator. The two parameters, in the description of the rational interpolant, have been constrained to preserve the shape of the data. The positivity-preserving properties of this rational interpolant, to a given data set are shown. The degree of smoothness C^1 is attained (first order of parametric continuity). **Objective:** Preserving the positive data by using new rational cubic spline and produces the positivee interpolating curves. **Results:** The sufficient condition for the rational cubic interpolant to be positive are derived and from the numerical results the propose rational cubic spline interpolant gives comparable results. **Conclusion:** The sufficient condition for positivity constraints were restricted on two shape parameters v_i and w_i to assure the positivity of the data will be preserved completely. The solution to the shape preserving spline is always exists. We conclude that, the developed scheme work well and is comparable to the existing schemes. It also provides good alternative to the existing rational spline for shape preserving interpolation problem.

To Cite This Article: Samsul Ariffin Abdul Karim, Rational Cubic Spline for Positivity Preserving Interpolation. ***Aust. J. Basic & Appl. Sci.,*** *8(9): 493-503, 2014*

INTRODUCTION

Spline method has been widely being used in the fields of sciences and engineering such as in the car bodies design, wings of airplane, scientific visualization, computer graphics and 3D modeling (human body etc.). Besides, scientific visualization are important in computer graphics for the purpose of simulation and visualization the given data. One of the main requirement to visualize the data is that the method (in our case, it is rational function) must be preserved certain properties of the data sets such as positivity, monotonicity and convexity. Other desirable properties are the degree of smoothness of the interpolant, the approximation order, the locality or globally of the interpolant and the fairness of the resulting curves/surfaces(Goodman, 2002). Positivity preserving may read as follows: given a finite set of positive data (either strictly positive or not), construct a positive rational interpolant that matches all the given data sets with certain degree of smoothness attained (in this paper, the degree of smoothness is C^1). Positivity preserving always arise in our daily life. For example, stability of radioactive substance and chemical reaction, solvability of solute in solvent and the population statistics are always positive (Hussain *et al.*, 2010). The volume-pressure curve should have a nonnegative derivative value (Greiner, 1991). The monthly rainfall amounts, resistance offered by an electric circuit and levels of gas discharge in certain chemical reactions are always having positive values (Sarfraz *et al.*, 2012). Thus positivity preserving is unavoidable and it is a requirement that the interpolant (rational or polynomial) must preserve the positivity of the interpolating data. The cubic spline with C^2 continuity will be not able to preserve the positivity on entire given intervals, due to the existing of unwanted wiggles that will completely destroys the characteristics of the original data. Various splines schemes for positivity preserving can be found in Greiner (1991), Sarfraz *et al*. (2012), Sarfraz *et al*. (2001), Sarfraz (2002), Hussain and Sarfraz (2008), Butt and Brodlie (1993), Abbas *et al*. (2012), Sarfraz *et al*. (2010), Sarfraz *et al*. (2005), Asim and Brodlie (2003) and Karim (2013a). The spline scheme can be either polynomial spline or rational spline.

Meanwhile, Sarfraz *et al*. (2012) have introduced the rational spline interpolant with cubic numerator and cubic denominator for shape preserving interpolation with four shape parameters. The sufficient conditions for positivity, monotonicity and convexity preserving have been derived. Motivated by their work, in this paper, the author proposed a new rational cubic spline interpolant with two shape parameters. The sufficient conditions for

Corresponding Author: Samsul Ariffin Abdul Karim, Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, 31750 Tronoh, Perak Darul Ridzuan, Malaysia.
Tel: +605-3687692 Fax: +605-3655905

Australian Journal of Basic and Applied Sciences, 8(9) June 2014, Pages: 493-503

the rational cubic spline to be positive on entire given intervals will be derived. This will ensure the existence of positive rational cubic interpolating curves along the whole interval.

The main scientific contribution this paper is as follows:

(i) In this paper the rational cubic with two parameters has been used for positivity preserving while Sarfraz *et al*. (2012) the rational cubic spline with 4 parameters has been used for positivity preserving.
(ii) The sufficient condition for positivity is different from the sufficient condition for positivity in Sarfraz *et al*. (2012).
(iii) Numerical comparison between the proposed schemes with rational cubic Ball interpolation in Karim (2013a) also has been given. Our rational cubic interpolant also gives satisfactory results.
(iv) No extra knots are needed in order to preservers the positivity of the data, while in Butt and Brodlie (1993) and Asim and Brodlie (2003) the positivity is preserves by inserting extra knots between any two knots in the interval in which the negativity are found.
(v) The work in this paper focusing on positivity preserving while in Shukri *et al*. (2012) the cubic Bézier curve were used for data constrained interpolation subject to circle, an ellipse and straight line. Furthermore the work in this paper can be extended to constrained data modeling.

The remainder of the paper is organized as follows. Section 2 introduces the rational cubic spline interpolant with two shape parameters, Section 3 discusses the determination of first derivatives and Section 4 discusses the proposed rational interpolant for positivity preservation together with algorithm for computer implementation. All numerical results together with the comparison between the proposed rational spline with rational cubic Ball by Karim (2013a) will be discussed in Section 5. A summary and conclusions are given in Section 6.

Rational Cubic Spline Interpolant:

The cubic Ball polynomial basis originally proposed by (Ball, 1974). In this section, the new rational cubic spline will be discussed in details. It started with the definition of the rational cubic spline interpolant:

Given that the data points $\{ (x_i, f_i), i = 0,1,...,n \}$, $x_0 < x_1 < ... < x_n$. Let $h_i = x_{i+1} - x_i$, $\Delta_i = (f_{i+1} - f_i)/h_i$ and $\theta = (x - x_i)/h_i$ where $\theta \in [0,1]$. For $x \in [x_i, x_{i+1}], i = 0,1,2,...,n-1$

$$s(x) = s(x_i + h_i\theta) \equiv S_i(\theta) = \frac{P_i(\theta)}{Q_i(\theta)}, \tag{1}$$

where

$$P_i(\theta) = A_0(1-\theta)^3 + A_1\theta(1-\theta)^2 + A_2\theta^2(1-\theta) + A_3\theta^3$$

$$Q_i(\theta) = (1-\theta)^2 + v_i\theta(1-\theta)^2 + w_i\theta^2(1-\theta) + \theta^2$$

The following interpolatory properties will ensure the rational cubic spline interpolant defined by (1) satisfy C^1 continuity:

$$s(x_i) = f_i, \quad s(x_{i+1}) = f_{i+1},$$
$$s^{(1)}(x_i) = d_i, \quad s^{(1)}(x_{i+1}) = d_{i+1}. \tag{2}$$

Thus the unknowns $A_i, i = 0,1,2,3$ can be shown to have the following values:

$A_0 = f_i$, $A_1 = (1+v_i) f_i + h_i d_i, A_2 = (1+w_i) f_{i+1} - h_i d_i, A_3 = f_{i+1}$, where $s^{(1)}(x)$ denotes derivative with respect to x and d_i denotes the derivative value which is given at the knots $x_i, i = 0,1,2,...,n$. The data dependent sufficient conditions on the parameters v_i and w_i will be developed to produce the positive interpolating curves that will preserves the positivity of the data on the entire interval $[x_i, x_{i+1}], i = 0,1,2,...,n-1$. Some main observations for the rational cubic spline interpolant defined by (1) can be described as follows:

- When $v_i = w_i = 2$, the rational cubic spline interpolant (1) reduced to standard cubic Hermite spline given as follows:

$$s(x) = (1-\theta)^2(1+2\theta) f_i + \theta^2(3-2\theta) f_{i+1} + \theta(1-\theta)^2 d_i - \theta^2(1-\theta) d_{i+1}.$$

- The rational cubic spline in (1) can be written as follows:

$$s(x)=(1-\theta)f_i+\theta f_{i+1}+\frac{h_i(1-\theta)\theta\left[(d_i-\Delta_i)(1-\theta)+\theta(\Delta_i-d_{i+1})+(1-\theta)\theta\Delta_i(w_i-v_i)\right]}{Q_i(\theta)}.$$

- Thus it is clearly that when $v_i, w_i \to \infty$, then the rational cubic spline in (1) converges to the straight line $s(x)=(1-\theta)f_i+\theta f_{i+1}$.

The shape parameters $v_i, w_i, \quad i=0,1,2,...,n-1$ can be utilized in order to modify the shape of the interpolating curve.

Determination of Derivatives:

In most applications, derivative parameters d_i are not given and cannot be obtained analytically and it can be estimated by using some methods. There exist many numerical methods to approximate the first derivative values. Usually, arithmetic mean method, geometric mean method and harmonic mean method could be used (Delbourgo and Gregory , 1985a; Sarfraz *et al*., 1997). In this paper, the arithmetic mean method (AMM) will be used to estimate the value of the first derivative. This method is easy to use and it is suitable for the shape preserving interpolation for positive data. Below the details on how the method works:

At the end points x_0 and x_n

$$d_0=\Delta_0+(\Delta_0-\Delta_1)\left(\frac{h_0}{h_0+h_1}\right). \quad (3)$$

$$d_n=\Delta_{n-1}+(\Delta_{n-1}-\Delta_{n-2})\left(\frac{h_{n-1}}{h_{n-1}+h_{n-2}}\right). \quad (4)$$

At the interior points, $x_i, i=1,2,...,n-1$, the values of d_i are given as

$$d_i=\frac{h_{i-1}\Delta_i+h_i\Delta_{i-1}}{h_{i-1}+h_i} \quad (5)$$

Positivity-Preserving Using Rational Cubic Spline Interpolant:

The proposed rational cubic spline interpolant (cubic spline as numerator and cubic Ball as denominator) in Section 2, does not always preserves the positivity of the data along the whole interval. The ordinary cubic spline interpolation also does not guarantee to fully preserve the positivity of the positive data sets. Our rational cubic spline and the usual cubic splines may give some negative values that may destroy the geometric shape of the given data. These shape violations can be seen clearly from Fig. 1 and Fig. 2 respectively. The user may manipulate the values of the shape parameters and by trial and error basis in order to preserves the positivity of the data. But this approach may take longer time than what we may expect. Indeed, this really time consuming and obviously this is not a practical task to do by the inexperience user.

Here, we follow the same idea as in Sarfraz (2002) and Karim (2013a). The automated choice of the shape parameters v_i and w_i will be derived to produce the sufficient conditions for positivity of the rational interpolant defined by (1).

For simplicity of the presentation, let us assume strictly positive set of data $(x_i, f_i), i=0,1,...,n$ are given, such that

$$x_0<x_1<...<x_n \quad (6)$$

and

$$f_0>0, f_1>0,..., f_n>0 \text{ or } f_i>0, i=0,1,...,n. \quad (7)$$

Now, the sufficient conditions for positivity of piecewise rational cubic spline with C^1 continuity will be developed. The main idea is that, in order to preserve the positivity of $s(x)$, the suitable values of shape parameters v_i and w_i in each corresponding interval must be chosen and assigned properly. For all $u_i, w_i>0$, the denominator $Q_i(\theta)>0, i=0,1,...,n-1$, therefore the positivity of rational cubic interpolant in (1) depend only on the positivity of cubic spline polynomial $P_i(\theta), i=0,1,...,n-1.$ Thus the problem reduces to the determination of appropriate value of v_i and w_i on each interval $[x_i, x_{i+1}], i=0,1,...,n-1$ that will ensure the polynomial $P_i(\theta)$ are positive everywhere. Now, $P_i(\theta)$ can be re-written as follows:

$$P_i(\theta)=B_i\theta^3+C_i\theta^2+D_i\theta+E_i,$$

where

$B_i = v_i f_i - w_i f_{i+1} + h_i d_i + h_i d_{i+1},$
$C_i = (1+w_i) f_{i+1} + (1-2v_i) f_i - 2h_i d_i - h_i d_{i+1},$
$D_i = (v_i - 2) f_i + h_i d_i,$
$E_i = f_i.$

Now by extending the results from Schmidt and Hess (1988), the sufficient condition for the cubic polynomial to be positive is given in Theorem 1 below. The proposition may read as follows:
Theorem 1 (Positivity of Cubic polynomial)

For the strict inequality positive data in (7), $P_i(\theta) > 0$ if and only if

$$\left(P_i'(0), P_i'(1)\right) \in R_1 \cup R_2 \tag{8}$$

where

$$R_1 = \left\{(a,b) : a > \frac{-3f_i}{h_i}, b < \frac{3f_{i+1}}{h_i}\right\}, \tag{9}$$

$$R_2 = \left\{\begin{array}{l}(a,b) : 36 f_i f_{i+1}\left(a^2 + b^2 + ab - 3\Delta_i (a+b) + 3\Delta_i^2\right) \\ \quad + 3\left(f_{i+1}a - f_i b\right)\left(2h_i ab - 3f_{i+1}a + 3f_i b\right) \\ \quad + 4h_i\left(f_{i+1}a^3 - f_i b^3\right) - h_i^2 a^2 b^2 > 0\end{array}\right\} \tag{10}$$

Now, by differentiating $P_i(\theta)$ with respect to θ, the results are

$$P_i'(0) = \frac{-3f_i + f_i(1+v_i) + h_i d_i}{h_i}.$$

and

$$P_i'(1) = \frac{-f_{i+1}(1+w_i) + h_i d_{i+1} + 3f_{i+1}}{h_i}$$

Now from Proposition 1, it can be deduced that $\left(P_i'(0), P_i'(1)\right) \in R_1$ if

$$P_i'(0) = \frac{-3f_i + f_i(1+v_i) + h_i d_i}{h_i} > \frac{-3f_i}{h_i} \tag{11}$$

and

$$P_i'(1) = \frac{-f_{i+1}(1+w_i) + h_i d_{i+1} + 3f_{i+1}}{h_i} < \frac{3f_{i+1}}{h_i} \tag{12}$$

The inequality (11) leads to the following relation on the free parameter v_i :

$$v_i > -\frac{h_i d_i}{f_i} - 1. \tag{13}$$

The inequality (12) leads to the following restriction on the free parameter w_i :

$$w_i > \frac{h_i d_{i+1}}{f_{i+1}} - 1. \tag{14}$$

Furthermore, $\left(P_i'(0), P_i'(1)\right) \in R_2$ if

$$\phi(a,b) = \left\{\begin{array}{l}(a,b) : 36 f_i f_{i+1}\left(a^2 + b^2 + ab - 3\Delta_i (a+b) + 3\Delta_i^2\right) \\ \quad + 3\left(f_{i+1}a - f_i b\right)\left(2h_i ab - 3f_{i+1}a + 3f_i b\right) \\ \quad + 4h_i\left(f_{i+1}a^3 - f_i b^3\right) - h_i^2 a^2 b^2 \ge 0\end{array}\right\}. \tag{15}$$

where
$a = P_i'(0), b = P_i'(1).$

The constraints on the free parameters can be derived either from Equations (9) or (10). But equation (10) involves a lot of computation, thus we will develop the data dependent constraints for positivity preserving by using Equation (9) because it is more economy and less computation as compared with the sufficient conditions given in (10) and (15).

Now, Equation (13) and (14) can be summarized as Theorem 2 below.

Theorem 2. For a strictly positive data, the rational cubic spline interpolant with two parameters v_i and w_i (defined over the interval $\left[x_0, x_n\right]$ is positive if in each subinterval $\left[x_i, x_{i+1}\right], i = 0,1,...,n-1$ the following sufficient conditions are satisfied:

$$v_i = l_i + Max\left\{0, \frac{-h_i d_i}{f_i} - 1\right\}, w_i = \lambda_i + Max\left\{0, \frac{h_i d_{i+1}}{f_{i+1}} - 1\right\}. \quad (16)$$

where $l_i, \lambda_i > 0.$

Remarks 1: The constraints (16) can be further modified for shape control and shape preserving interpolation. Let

$$E_i = Max\left\{0, \frac{-h_i d_i}{f_i} - 1\right\}, C_i = Max\left\{0, \frac{h_i d_{i+1}}{f_{i+1}} - 1\right\}.$$

Following the same idea from Sarfraz *et al*. (2001) and Karim (2013a), by choosing $r_i, q_i \geq 1$, then (16) can be rewritten as follows:

$$v_i = \left(1 + E_i\right) r_i, w_i = \left(1 + C_i\right) q_i \quad (17)$$

Eq. (17) provides freedom of choices of the shape parameters v_i and w_i also the parameter r_i and q_i. Another choices of shape parameters is as follows:

$$v_i = w_i = 1 + Max\left\{E_i r_i, C_i q_i\right\} \quad (18)$$

For implementation, the user may use either conditions (16) or (17) or (18).

Remarks 2: The sufficient condition in (16) or (17) is the same as sufficient condition for the positivity by using rational cubic Ball interpolant (cubic numerator and cubic denominator) as proposed by Karim (2013a). The main different is that the rational cubic spline in this paper utilized cubic spline as numerator where in Karim (2013a) the cubic Ball has been used both as numerator and denominator. Furthermore the resultaing positive interpolating curves will totally be different.

An algorithm to generate C^1 positivity-preserving curves using the results in Theorem 2 is given as follows.

Algorithm for positivity-preserving

1. Input the data points $\left\{x_i, f_i\right\}_{i=0}^{n}$.
2. For $i = 0,1,\ldots,n$, estimate d_i using arithmetic mean method (AMM).
3. For $i = 0,1,\ldots,n-1$

- Calculate the shape parameter v_i, w_i using (16) or (17) or (18) with suitable choices of $l_i, \lambda_i > 0.$
- Calculate the inner control ordinates A_1 and A_2.

4. For $i = 0,1,\ldots,n-1$

Construct the piecewise positive interpolating curves using (1).

Repeat Step 1 until Step 4 for each tested data sets.

RESULTS AND DISCUSSION

In order to illustrate the shape preserving interpolation by using the proposed rational cubic spline interpolation (cubic spline as numerator and cubic Ball as denominator), two sets of data taken from Sarfraz *et al*. (2005) and Asim and Brodlie (2003) were used.

Table 1. A Positive data from Sarfraz *et al*. (2005)

x_i	2	3	7	8	9	13	14
f_i	10	2	3	7	2	3	10
d_i	-9.65	-6.35	3.25	0	-3.95	5.65	8.35

Table 2. A Positive data from Asim and Brodlie (2003)

x_i	0	1
f_i	1	1.5
d_i	3.1	15

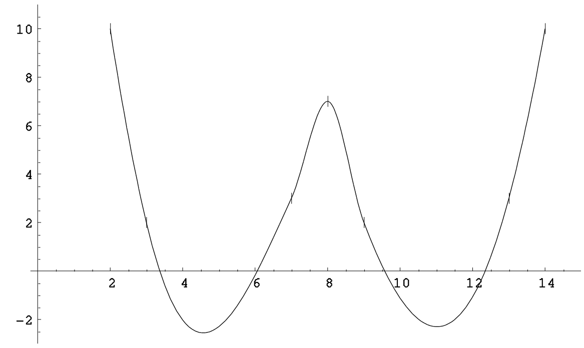

Fig. 1: Default Cubic Spline Polynomial $(v_i = w_i = 2)$ for data in Table 1.

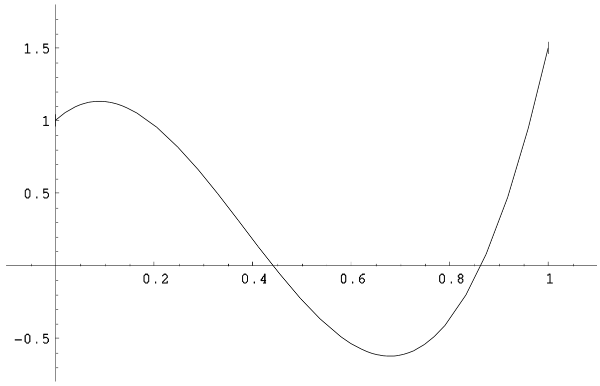

Fig. 2: Default Cubic Spline Polynomial $(v_i = w_i = 2)$ for data in Table 2.

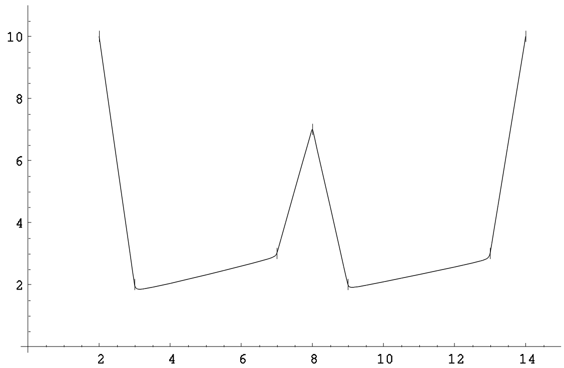

Fig. 3: Shape preserving interpolation using Eq. (16) with $(l_i = \lambda_i = 100)$ for data in Table 1. The rational cubic interpolant approach to straight line.

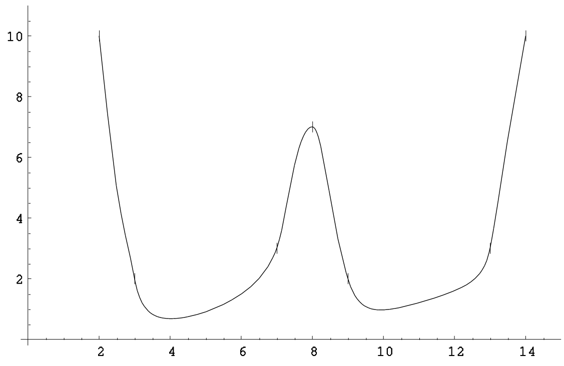

Fig. 4: Shape preserving interpolation using Eq. (17) with $(r_i = 1, q_i = 2)$ for data in Table 1.

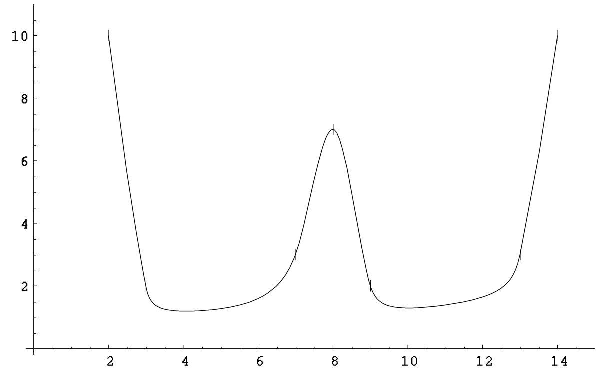

Fig. 5: Shape preserving interpolation using Eq. (17) with $(r_i = q_i = 2)$ for data in Table 1.

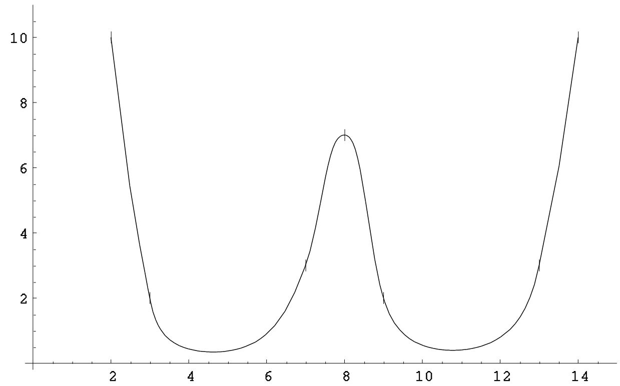

Fig. 6: Shape preserving interpolation using Eq. (16) with $(l_i = \lambda_i = 0.5)$ for data in Table 1.

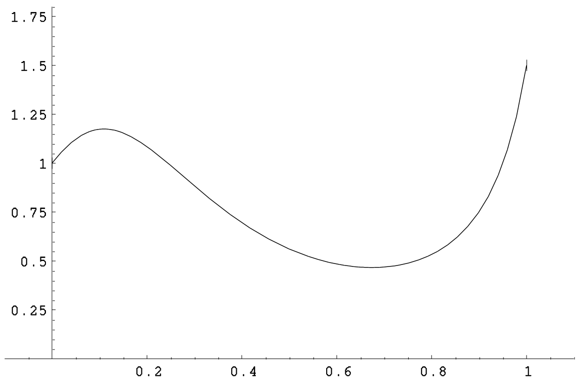

Fig. 7: Shape preserving interpolation using Eq. (16) with $(l_i = \lambda_i = 0.5)$ for data in Table 2.

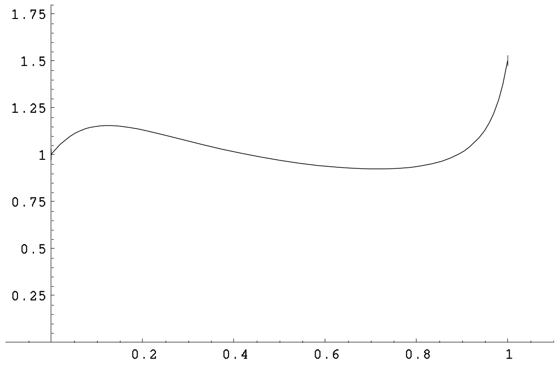

Fig. 8: Shape preserving interpolation using Eq. (17) with $(r_i = 5, q_i = 2)$ for data in Table 2.

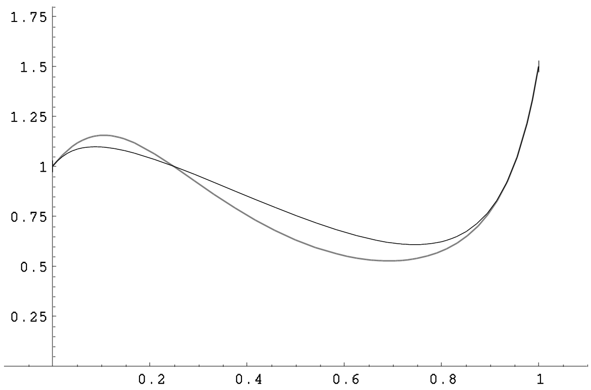

Fig. 9: Shape preserving interpolation using Eq. (16) with $(r_i = 2, q_i = 1)$ - gray and $(r_i = 10, q_i = 1)$ - normal for data in Table 2.

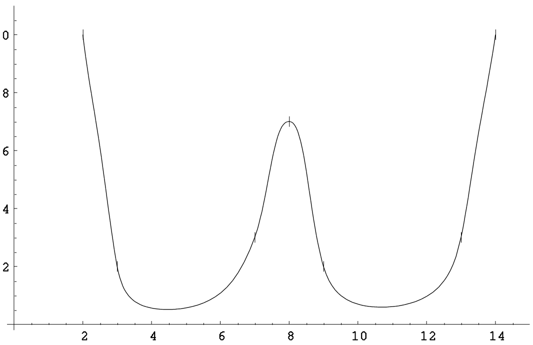

Fig. 10: Shape preserving interpolation using Karim (2013a) with $(l_i = \lambda_i = 0.5)$ for data in Table 1.

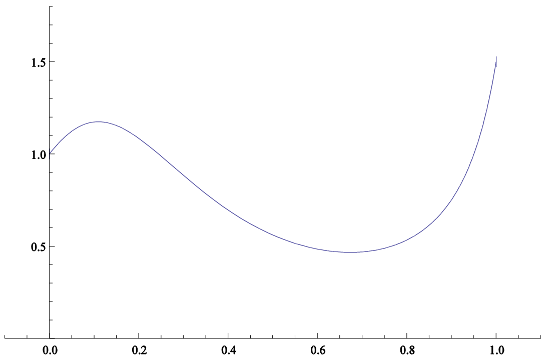

Fig. 11: Shape preserving interpolation using Karim (2013a) with $(l_i = \lambda_i = 0.5)$ for data in Table 2.

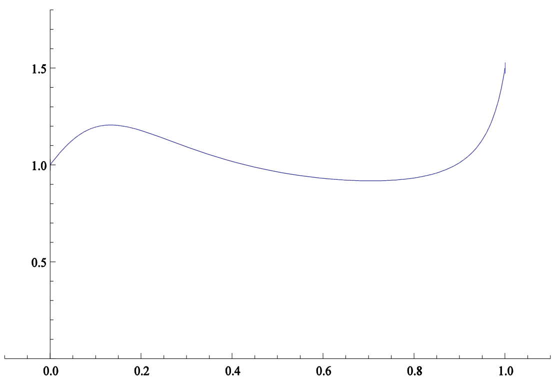

Fig. 12: Shape preserving interpolation using Karim (2013a) with $(r_i = 1, q_i = 2)$ for data in Table 2.

Australian Journal of Basic and Applied Sciences, 8(9) June 2014, Pages: 493-503

Figure 1 and Figure 2 shows the default cubic spline interpolation for data in Table 1 and Table 2 respectively. Figure 3 until Figure 6 shows the shape preserving interpolation using proposed scheme for data in Table 1 and Figure 7 until Figure 9 show the shape preserving for data in Table 2. Meanwhile Fig. 10, Fig. 11 and Fig. 12, show the shape preserving interpolation using rational cubic Ball function proposed by Karim (2013a). From Fig. 3, Fig. 4, Fig. 5, Fig. 6, Fig. 7 and Fig. 8, it can be seen clearly that the proposed rational cubic spline interpolant gives satisfactory outputs. Furthermore the numerical results are comparable with the work of Karim (2013a). Two parameters in the description of the rational cubic spline interpolant have been constrained to produced the sufficients condition for positivity. This sufficient conditions are easy to obtained and simple to used without the needs any extra calculation to find the suitable values each of the shape parameters. This is where the proposed rational cubic spline interpolant is useful and gives good alternatives to the other shape preserving interpolation methods. By through inspection, Figure 6 and Figure 10, Figure 7, Figure 8, Figure 11 and Figure 12, it can be seen clearly that the resulting positive interpolating curves between the proposed rational cubic spline and rational cubic Ball (Karim, 2013a) are different. For more details on shape preserving interpolation and approximation, the readers can referred to Delbourgo and Gregory (1985b), Dougherty *et al*. (1989), Duan *et al*. (1999, 2003), Foley (1988), Fristch and Carlson (1980), Fristch and Butland (1984), Gregory (1986), Kvasv (2000), Karim (2013b) and the references cited therein.

For the completeness of this paper, we give the sufficient condition for positivity by using rational cubic spline as proposed by Sarfraz *et al*. (2012).

Theorem 3 (Sarfraz *et al*., 2012). The piecewise rational cubic spline with 4 shape parameters will preserved the positivity of the data (7), if the following sufficient condition are satisfied:

$$\alpha_i > 1.5, \delta_i > 1.5,$$
$$v_i = l_i + Max\left\{0, \frac{-\alpha_i h_i d_i}{f_i}\right\}, w_i = m_i + Max\left\{0, \frac{\delta_i h_i d_{i+1}}{f_{i+1}}\right\}. \tag{19}$$

where $l_i, m_i > 0$.

Final Remark: We would like to stress, the proposed rational cubic spline was based from the work of Sarfraz *et al*. (2012). In the original formulation of Sarfraz *et al*. (2012), the rational cubic spline have 4 shape parameter namely α_i, δ_i, v_i and w_i meanwhile our proposed rational cubic spline have only 2 shape parameters v_i, w_i (the values of α_i, δ_i are set equal to 1). In Sarfraz *et al*. (2012) the condition: $\alpha_i > 1.5, \delta_i > 1.5$ were required in order to ensure the sufficient conditions for positivity is satisfied. This is one of the different between the proposed rational cubic spline and the Sarfraz *et al*. (2012).

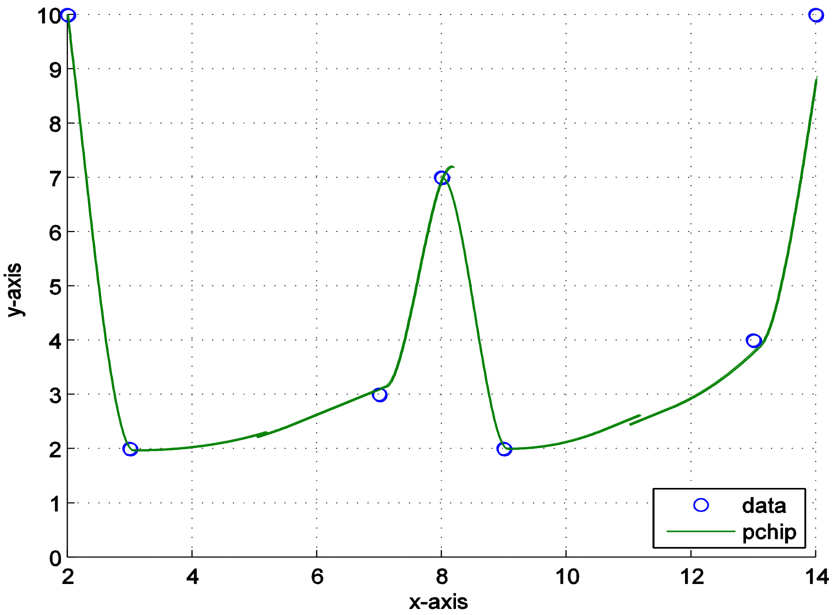

Fig. 13: Shape preserving interpolation using PCHIP for data in Table 1.

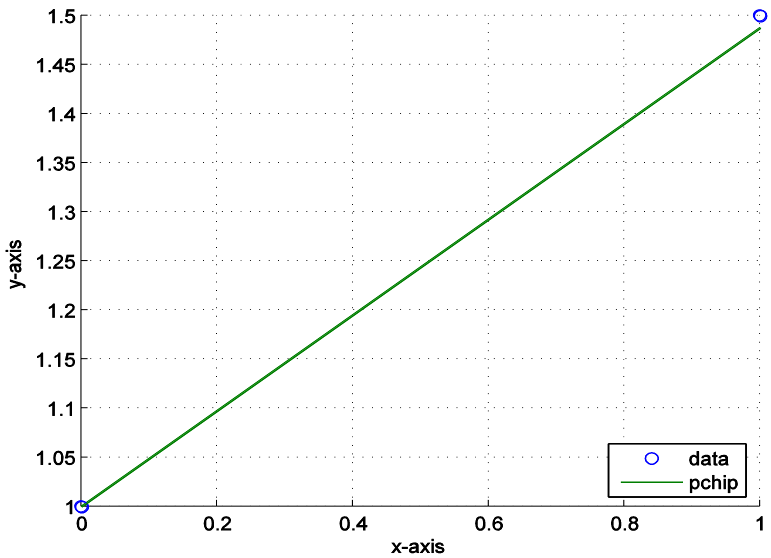

Fig. 14: Shape preserving interpolation using PCHIP for data in Table 2.

Figure 13 and Figure 14 shows the shape preserving by using Fristch and Carlson (1980) methods that well documented in Matlab programming under the name of PCHIP. The interpolating curves for data in Table 2 reproduced the straight line by using PCHIP.

Conclusions:

The work in this paper is concerned about the positivity-preserving for scalar data sets by using rational cubic spline interpolant (cubic spline as numerator and cubic Ball function as denominator). The constraints were restricted on two parameters v_i and w_i to assure the positivity of the data will be preserved completely. The solution to the shape preserving spline is always exists. The sufficient condition for the positivity of the rational cubic spline also has been derived. This will ensure the positive rational cubic interpolating function will be always exists. It is worth to mention that the rational spline scheme in this paper is analogous to the work of Sarfraz *et al.* (2012). One of main differences between our rational interpolant and Sarfraz *et al.* [4] is that, there is no restriction to the parameters that exists in Sarfraz *et al.* (2012). We conclude that, the developed scheme work well and is comparable to the existing schemes. Furthermore it also provides good alternative to the existing rational spline for shape preserving interpolation problem. Finally, the author is in the final process to complete the works for monotonicity and convexity preserving for curves and surface interpolation by using the proposed rational cubic spline.

ACKNOWLEDGMENT

The author would like to acknowledge Universiti Teknologi PETRONAS for the financial support received in the form of a research grant: Short Term Internal Research Funding (STIRF) No. 35/2012 and Mathematica Software.

REFERENCES

Abbas, M., A.A. Majid, M. Awang, A. Hj. and J.M. Ali, 2012. Shape preserving positive surface data visualization by spline functions. Applied Mathematical Sciences, 6(6): 291-307.

Asim, M.R. and K.W. Brodlie, 2003. Curve drawing subject to positivity and more general constraints. Computer & Graphics 27: 469-485.

Ball, A.A., 1974. CONSURF Part I: Introduction to Conic Lifting Title. Computer Aided Design, 6(4): 243-249.

Butt, S. and K.W. Brodlie, 1993. Preserving positivity using piecewise cubic interpolation”, Computer and Graphics, 17(1): 55-64.

Delbourgo, R. and J.A. Gregory, 1985a. The Determination of Derivative Parameters for a Monotonic Rational Quadratic Interpolant. IMA Journal of Numerical Analysis, 5: 397-406.

Delbourgo, R. and J.A. Gregory, 1985b. Shape preserving piecewise rational interpolation. SIAM J. Sci. and Statist. Comput, 6: 967-976.

Dougherty, R.L., A. Edelman and J.M. Hyman, 1989. Nonnegativity, Monotonicity, or Convexity-Preserving Cubic and Quintic Hermite Interpolation. Mathematics of Computation, 52(186): 471-494.

Duan, Q., K. Djidjeli, W.G. Price and E.H. Twizell, 2003. Constrained control and approximation properties of a rational interpolating curve. Information Sciences, 152: 181-194.

Duan, Q., X. Wang and F. Cheng, 1999. Constrained interpolation using rational cubic spline curve with linear denominators. Korean J. Computer and Appl. Math, 1: 203-215.

Foley, T.A., 1988. A shape preserving interpolant with tension controls. CAGD 5: 105-118.

Fritsch, F.N. and J. Butland, 1984. A method for constructing local monotone piecewise cubic interpolants. SIAM J. Sci. and Statist. Comput, 5: 300-304.

Fritsch, F.N. and R.E. Carlson, 1980. Monotone piecewise cubic interpolation. SIAM J. Numer. Anal, 17: 238-246.

Goodman, T.N.T., 2002. Shape preserving interpolation by curves. In J. Levesley, I.J. Anderson, J.C. Mason (Eds.), Algorithms for Approximation IV, University of Huddersfield Proceedings, Huddersfield, pp. 24-35.

Gregory, J.A., 1986. Shape Preserving Interpolation. Computer Aided Design, 18(1): 53-57.

Greiner, H., 1991. A Survey on Univariate Data Interpolation and Approximation by Splines of Given Shape. Mathl. Comput. Modelling, 15(10): 97-106.

Hussain, M.Z., M. Sarfraz and M. Hussain, 2010. Scientific Data Visualization with Shape Preserving C^1 Rational Cubic Interpolation. European Journal of Pure and Applied Mathematics, 3(2):194-212.

Hussain, M.Z. and M. Sarfraz, 2008. Positivity-Preserving interpolation of positive data by rational cubics. Journal of Computational and Applied Mathematics, 218: 446-45.

Sarfraz, M., M.A. Mulhem, and F. Ashraf, 1997. Preserving Monotonic Shape of the Data Using Piecewise Rational Cubic Functions", Computer & Graphics, 21(1): 5-14.

Karim, S.A.A., 2013a. Rational cubic Ball function for positivity preserving. Far East Journal of Mathematical Sciences (FJMS), 82(2): 193-207.

Karim, S.A.A., 2013b. GC^1 Monotonicity Preserving using Ball Cubic Interpolation. Australian Journal of Basic and Applied Sciences, 7(2): 780-790.

Kvasov, B.I., 2000. Methods of Shape-Preserving Spline Approximation. World Scientific, Singapore.

Sarfraz, M., M.Z. Hussain and M. Hussain, 2012. Shape-preserving curve interpolation. International Journal of Computer Mathematics, 89(1): 35-53.

Sarfraz, M., S. Butt and M.Z. Hussain, 2001. Visualization of shaped data by a rational cubic spline interpolation. Computers & Graphics, 25: 833-845.

Sarfraz, M., 2002. Visualization of positive and convex data by a rational cubic spline interpolation. Information Sciences 146(1-4): 239-254.

Sarfraz, M., M.Z. Hussain, and A. Nisar, 2010. Positive data modeling using spline function. Applied Mathematics and Computation, 216: 2036-2049.

Sarfraz, M., M.Z. Hussain and F.S. Chaudary, 2005. Shape Preserving Cubic Spline for Data Visualization", Computer Graphics and CAD/CAM 01, pp: 185-193.

Schmidt, J.W. and W. Hess, 1988. Positivity of cubic polynomials on intervals and positive spline interpolation. BIT, 28: 340-352.

Shukri, F.N.A., M. Abbas, U. Bashir, M.N.Hj. Awang, E. Jamal and J.M. Ali, 2012. Cubic Bézier Constrained Curve Interpolation, J.Basic. Appl. Sci. Res., 2(4): 3682-3692.