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On Classical Primary Subsemimodules of Semimodules over Commutative Semiring

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ABSTRACT

Let R be a commutative semiring and let M be a semimodule over a semiring R. A proper subsemimodule N of M is called a classical primary subsemimodule, if for any $a,b\in R$ and $m\in M, abm\in N$ implies that $am\in N$ or $b^n m\in N$, for some positive integer n. In this paper we study some basic properties of classical primary and primary subsemimodules of semimodule M over a commutative semiring R. Moreover, we investigate relationships between classical primary and primary subsemimodules of M. Finally, we obtain necessary and sufficient conditions of a classical primary to be a primary in M.

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INTRODUCTION

Throughout this paper a semiring will be defined as follows: A semiring is a set R together with two binary operations called addition "$+$" and multiplication "$\cdot$" such that $(R,+)$ is a commutative semigroup and $(R,\cdot)$ is semigroup; connecting the two algebraic structures are the distrubutive laws : $a(b+c) = ab+ac$ and $(a+b)c = ac+bc$ for all $a,\ b,\ c\in R.$ A subset A of a semiring R is called an ideal of R if for $a,b\in A$ and $r\in R,$ $a+b\in A, ar\in A$ and $ra\in A.$ A proper ideal P of R is called a primary ideal if $ab\in P,$ where $a,b\in R,$ implies that either $a\in P$ or $b^n\in P,$ for some positive integer $n.$ P is said to be quasi-primary if for all $a,b\in R, ab\in P$ implies that either $a^n\in P$ or $b^n\in P,$ for some positive integer $n.$ Clearly every primary is a quasi-primary. A semimodule M over a semiring R is a commutative monoid M with additive identity 0, together with a function $R\times M\to M,$ defined by $(r,m)\mapsto rm$ such that:

1. $r(m+n)=rm+rn$
2. $(r+s)m=rm+sm$
3. $(rs)m=r(sm)$
4. $r0=0=0m$
5. $1m=m$

for all $m,n\in M$ and $r,s\in R.$ Clearly every ring is a semiring and hence every module over a ring R is a left semimodule over a semiring $R.$ A nonempty subset N of a R-semimodule M is called subsemimodule of M if N is closed under addition and closed under scalar multiplication. A proper subsemimodule N of an R-semimodule M is said to be primary if $rm\in N,$ $r\in R, m\in M,$ then either $m\in N$ or $r^nM\subseteq N,$ for some positive integer $n.$

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J. Saffar Ardabili, S. Motmaen and A. Yousefian Darani in (2011) defined a different class of subsemimodules and called it classical prime. A proper subsemimodule N of M is said to be classical prime when for $a,b \in R$ and $m \in M, abm \in N$ implies that $am \in N$ or $bm \in N.$ In this paper we introduce the concept of classical prime subsemimodules of a semimodule $M,$ and study some basic properties of this class of subsemimodules. Moreover, we investigate relationships between classical primary and primary subsemimodules of $M.$ Finally, we obtain necessary and sufficient conditions of a classical primary to be a primary in $M.$

Basic properties of subsemimodules:

In this section we refer to (Atani, 2010; Dubey and Sarohe, 2013) for some elementary aspects and quote few theorem and lemmas which are essential to step up this study. For more details we refer to the papers in the references.

Definition 2.1: (Dubey and Sarohe, 2013) Let M be an R-semimodule and N be a proper subsemimodule of $M.$ An associated ideal of N is defined as $(N:M)=\{a \in R: \ aM \subseteq N\}.$

Lemma 2.2: (Dubey and Sarohe, 2013) Let M be an R-semimodule and N be a proper subsemimodule of $M.$ If N is a subtractive subsemimodule of $M,$ and let $m \in M.$ Then the following hold:

1. $(N:M)$ is a subtractive ideal of $R.$
2. $(0:M)$ and $(N:m)$ are subtractive ideals of $R.$

Lemma 2.3: (Atani, 2010) Let N and K be subsemimodules of a semimodule M over a semiring R with $N \subseteq K.$ Then K/N is a subsemimodule of $M/N.$

Definition 2.4: Let N be any subsemimodule of an R-semimodule $M.$ For $a \in R$ and an ideal I of $R,$ the sets $[N:a]$ and $[N:I]$ are defined by

1. $[N:a]=\{m \in M: am \in N\}$ and
2. $[N:I]=\{m \in M: Im \subseteq N\}.$

Remark: Let N be any subsemimodule of an R-semimodule M and let I be an ideal of $R, a \in R.$ Then

1. $[N:a] \neq \emptyset$ and $[N:I] \neq \emptyset$
2. $N \subseteq [N:a]$ and $N \subseteq [N:I]$

Definition 2.4: (Saffar Ardabili, Motmaen and Yousefian Darani, 2011) Let N be any subsemimodule of an R-semimodule $M.$ A subtractive subsemimodule (k-subsemimodule) N is a subsemimodule of M such that if $m, m+n \in N,$ then $n \in N.$

Theorem 2.5: Let N be a k-subsemimodule of semimodule M over a semiring $R.$ Then $[N:a]$ is a k-subsemimodule of $M,$ when $a \in R.$

Proof: Let $x, x+y \in [N:a].$ Then $ax, a(x+y) \in N$ so $ax, ax+ay \in N.$ Since N is a k-subsemimodule of $M,$ we have $ay \in N.$ Which implies that $y \in [N:a].$

Corollary 2.6: Let N be a k-subsemimodule of semimodule M over a semiring $R.$ Then $[N:I]$ is a k-subsemimodule of $M,$ when I is an ideal of $R.$

Proof: This follows from Theorem 2.5.

Basic properties of classical subsemimodules:

We start with the following theorem that gives a relation between classical primary and primary subsemimodules of $M.$ Our starting points is the following lemma:

Lemma 3.1: Let P be a proper ideal of a semiring $R.$ The following statements are equivalent.

1. P is a primary ideal.
2. P is a classical primary ideal.
3. $(P:I)$ is a primary ideal, for each ideal I of R such that $I \not\subset P.$
4. P is a classical quasi-primary ideal.

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5. $\sqrt{(P:I)}$ is a prime ideal, for each ideal I of R such that $I \not\subset P.$

Proof: 1.$\Longrightarrow$ 2. Suppose that P is a primary ideal of $R.$ We will show that P is a classical primary ideal of $R.$ Let $abJ \subseteq P,$ where $a,b \in R$ and J is an ideal of R such that $bJ \not\subset P.$ Then, there exists $x \in bJ$ such that $x \in P.$ Since P is primary ideal and $ax \in P,$ we conclude that $a^n \in P,$ for some positive integer $n.$ It follows that $a^n J \subseteq P,$ for some positive integer $n.$ Thus, P is a classical primary ideal of $R.$

2.$\Longrightarrow$ 3. Suppose that P is a classical primary ideal of $R.$ We will show that $(P:I)$ is a primary ideal, for each ideal I of R such that $I \not\subset P.$ Let $ab \in (P:I),$ where $a,b \in R.$ Then, $abI \subseteq P,$ so that $aI \subseteq P$ or $b^n I \subseteq P,$ for some positive integer $n.$ It follows that $a \in (P:I)$ or $b^n \in (P:I),$ for some positive integer $n.$ Thus, $(P:I)$ is a primary ideal of $R.$

3.$\Longrightarrow$ 1. Suppose that $(P:I)$ is a primary ideal, for each ideal I of R such that $I \not\subset P.$ We will show that P is a primary ideal of $R.$ Take $I = R$ and so by 3, $P = (P:R)$ is a primary ideal of $R.$

3.$\Longrightarrow$ 5. is evident.

4.$\Longrightarrow$ 5. Suppose that P is a classical quasi-primary ideal of $R.$ We will show that $\sqrt{(P:I)}$ is a prime ideal, for each ideal I of R such that $I \not\subset P.$ Let $ab \in \sqrt{(P:I)},$ where $a,b \in R.$ Then, $(ab)^n \in (P:I),$ for some positive integer $n,$ so that $(ab)^n I \subseteq P.$ Since P is a classical quasi-primary ideal of $R,$ there exists $k \in \mathbb{N}$ such that either $a^{nk} I \subseteq P$ or $b^{nk} I \subseteq P,$ i.e., either $a \in \sqrt{(P:I)}$ or $b \in \sqrt{(P:I)}.$ Thus, $\sqrt{(P:I)}$ is a prime ideal of $R.$

5.$\Longrightarrow$ 4. Suppose that $\sqrt{(P:I)}$ is a prime ideal, for each ideal I of R such that $I \not\subset P.$ We will show that P is a classical quasi-primary ideal of $R.$ Let $abI \subseteq P,$ where $a,b \in R.$ Then, $ab \in (P:I) \subseteq \sqrt{(P:I)}.$ Since $\sqrt{(P:I)}$ is a prime ideal of $R,$ we have $\sqrt{(P:I)}$ is either R or a prime ideal of $R,$ depending on whether $P \subseteq I$ or not, we conclude that $a \in \sqrt{(P:I)}$ or $b \in \sqrt{(P:I)},$ i.e., $a^n I \subseteq P$ or $b^n I \subseteq P,$ for some positive integer $n.$ Thus, P is a classical quasi-primary ideal of $R.$

Definition 3.2: A proper subsemimodule N of an R-semimodule M is a classical primary subsemimodule of M if for $m \in M$ and $a,b \in R$ such that $abm \in N,$ then $am \in N,$ or $b^n m \in N,$ for some positive integer $n.$ An R-semimodule M is a classical primary semimodule if every proper subsemimodule N of M is a classical primary subsemimodule of $M.$

Remark: It is easy to see that every primary is classical primary subsemimodule of $M.$

Lemma 3.3: Let N be a k-subsemimodule of semimodule M over a semiring $R.$ The following statements are equivalent.

1. N is a classical primary subsemimodule.
2. For every subsemimodule K of M and $a,b \in R.$ If $abK \subseteq N,$ then $aK \subseteq N$ or $b^n K \subseteq N,$ for some positive integer $n.$

Proof: 1. $\Longrightarrow$ 2. Assume that N is a classical primary subsemimodule of $M.$ Let $a,b \in R$ and let K be a subsemimodule of M such that $abK \subseteq N.$ If $aK \not\subset N$ and $b^n K \not\subset N,$ for all positive integer $n,$ then there exist $x,y \in K$ such that $ax \notin N$ and $b^n y \notin N$ for all positive integer $n.$ In this case from $abx, aby \in abK \subseteq N$ we get $b^k x \in N$ and $ay \in N$ for some positive integer $k.$ In this case it follows from $ab(x+y) \in abK \subseteq N$ that either $ax + ay = a(x+y) \in N$ or $b^k x + b^k y = b^k (x+y) \in N$ for

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some positive integer $k.$ If $ax+ay\in N,$ then $ax\in N$ since $ay\in N$ and N is a k-subsemimodule, a contradiction. If $b^k x+b^k y\in N$ we get a contradiction in a similar way.

2. $\Longrightarrow$ 1. is evident

Theorem 3.4: Let N be a k-subsemimodule of semimodule M over a semiring $R.$ Then, N is classical primary if and only if for every subsemimodule K of M such that $K\not\subset N,(N:K)$ is a primary ideal of $R.$

Proof: $\Longrightarrow$ Suppose that N is a classical primary subsemimodule of $M.$ We will show that $(N:K)$ a primary ideal, for each subsemimodule K of M such that $K\not\subset N.$ Let $ab\in(N:K),$ where $a,b\in R.$ Then $abK\subseteq N.$ By Lemma 3.3, we have $aK\subseteq N$ or $b^nK\subseteq N,$ for some positive integer $n.$ Thus $a\in(N:K)$ or $b^n\in(N:K),$ for some positive integer $n.$ Hence $(N:K)$ a primary ideal of $R.$

$\Longleftarrow$ Suppose that $(N:K)$ is a primary ideal of $R,$ for each subsemimodule K of M such that $K\not\subset N.$ We will show that N is classical primary of $M.$ Let $abK\subseteq N,$ where $a,b\in R.$ Then $ab\in(N:K).$ Since $(N:K)$ is a primary ideal of $R,$ we have $a\in(N:K)$ or $b^n\in(N:K),$ for some positive integer $n.$ It follows that $aK\subseteq N$ or $b^nK\subseteq N,$ for some positive integer $n.$ By Lemma 3.3., we have N is classical primary of $M.$

Theorem 3.5: Let N be a subsemimodule of semimodule M over a semiring $R.$ Then, N is classical primary if and only if for every $m\in M$ such that $m\notin N,(N:m)$ is a primary ideal of $R.$

Proof: $\Longrightarrow$ Suppose that N is a classical primary subsemimodule of $M.$ We will show that $(N:m)$ a primary ideal, for each $m\in M$ such that $m\notin N.$ Let $ab\in(N:m),$ where $a,b\in R.$ Then $abm\in N.$ By Definition 3.2, we have $am\in N$ or $b^nm\in N,$ for some positive integer $n.$ Thus $a\in(N:m)$ or $b^n\in(N:m),$ for some positive integer $n.$ Hence $(N:m)$ a primary ideal of $R.$

$\Longleftarrow$ Suppose that $(N:m)$ is a primary ideal of $R,$ for each $m\in M$ such that $m\notin N.$ We will show that N is classical primary of $M.$ Let $abm\in N,$ where $a,b\in R.$ Then $ab\in(N:m).$ Since $(N:m)$ is a primary ideal of $R,$ we have $a\in(N:m)$ or $b^n\in(N:m),$ for some positive integer $n.$ It follows that $am\in N$ or $b^nm\in N,$ for some positive integer $n.$ By Definition 3.2, we have N is classical primary of $M.$

Proposition 3.6: Let N be a proper subsemimodule of semimodule M over a semiring $R.$ If N is classical primary subsemimodule of $M,$ then $[N:c]$ is a classical primary subsemimodule of $M,$ where $c\in R.$

Proof: Let $abm\in[N:c],$ where $a,b\in R$ and $m\in M.$ By Definition 2.4, we have $(ca)bm=c(abm)\in N.$ Since $(ca)bm\in N$ and N is classical primary subsemimodule of $M,$ we have $cam\in N$ or $b^nm\in N,$ for some positive integer $n.$ Then $am\in[N:c]$ or $b^nm\in N\subseteq[N:c],$ for some positive integer $n.$ Hence $[N:c]$ is a classical primary subsemimodule of $M.$

Proposition 3.7: Let R be a semiring, with identity, and let N be a proper subsemimodule of semimodule M over $R.$ If $[N:c]$ is a classical primary subsemimodule of $M,$ then N is classical primary subsemimodule of $M,$ where $c\in R.$

Proof: Let $abm\in N,$ where $a,b\in R$ and $m\in M.$ Then $1bm=bm\in[N:a].$ Since $1bm\in[N:a]$ and $[N:a]$ is classical primary subsemimodule of $M,$ we have $m=1m\in N\subseteq[N:a]$ or $b^nm\in N,$ for some positive integer $n.$ Thus $am\in N$ or $b^nm\in N,$ for some positive integer $n,$ and hence N is classical primary of $M.$

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Lemma 3.8: Let N be a classical primary subsemimodule of semimodule M over a commutative semiring R and let $r \in R.$ If $r \in (N:x)\setminus\sqrt{(N:y)},$ where $x \in M$ and $y \in M \setminus N,$ then $(N:y)=(N:ry).$

Proof: Let $r \in (N:x)\setminus\sqrt{(N:y)},$ where $x \in M$ and $y \in M \setminus N,$ First, we will show that $(N:y) \subseteq (N:ry).$ Let $a \in (N:y).$ By Definition 2.1, we have $ay \in N.$ Since N is a subsemimodule of $M,$ we have $a(ry) = r(ay) \in rN \subseteq N,$ for all $r \in R.$ Again, by Definition 2.1, we have $a \in (N:ry)$ which implies that $(N:y) \subseteq (N:ry).$ Next, we will show that $(N:ry) \subseteq (N:y).$ Let $a \in (N:ry).$ By Definition 2.1, we have $(ar)y = a(ry) \in N.$ Again, by Definition 2.1, we have $ar \in (N:y).$ By Theorem 3.5, It follows that $a \in (N:y)$ or $r^n \in (N:y),$ for some positive integer $n.$ By the assumption, we have $a \in (N:y).$ Therefore $(N:ry) \subseteq (N:y)$ and hence $(N:y)=(N:ry).$

Theorem 3.9: Let N be a classical primary k-subsemimodule of semimodule M over a commutative semiring $R,$ $x \in M$ and $y \in M \setminus N.$ If $(N:x)\setminus\sqrt{(N:y)} \neq \varnothing,$ then $N = (N+Rx)\cap(N+Ry).$

Proof: It is easy to see that $N \subseteq (N+Rx)\cap(N+Ry).$ Next, we will show that $(N+Rx)\cap(N+Ry) \subseteq N.$ Let $t \in (N+Rx)\cap(N+Ry).$ Then, $n_1 + r_1x = t = n_2 + r_2y,$ for some $n_1, n_2 \in N$ and $r_1, r_2 \in R.$ Since $(N:x)\setminus\sqrt{(N:y)} \neq \varnothing,$ there exists $r \in (N:x)\setminus\sqrt{(N:y)}$ such that $rx \in N$ and $rr_1x = r_1rx \in N$ so that $rn_1, rr_1x, rn_2 \in N.$ Since $rt = rn_1 + rr_1x = rn_2 + rr_2y,$ we have $rn_2 + rr_2y \in N.$ It follows that $rr_2y \in N.$ By Definition 2.1, we have $r_2 \in (N:ry).$ By Lemma 3.8, we have $(N:y)=(N:ry).$ This implies that $(N+Rx)\cap(N+Ry) \subseteq N$ and hence $N = (N+Rx)\cap(N+Ry).$

Theorem 3.10: Let N be a proper k-subsemimodule of semimodule M over a commutative semiring $R.$ The followings are equivalent;

1. N is a classical primary subsemimodule of $M.$
2. For any $x \in M$ and $y \in M \setminus N,$ if $(N:x)\setminus\sqrt{(N:y)} \neq \varnothing,$ then $N = (N+Rx)\cap(N+R)$

Proof: 1.$\Longrightarrow$2. It follows from Theorem 3.9.

2.$\Longrightarrow$1. To show that N is a classical primary subsemimodule of $M.$ That is; for all $y \in M \setminus N,$ we will show that $(N:y)$ is a primary ideal of $R.$ Let $a, b \in R$ such that $ab \in (N:y).$ To show that $a \in (N:y)$ or $b^n \in (N:y)$ for some positive integer $n.$ Assume that $b \notin \sqrt{(N:y)}.$ Since $ab \in (N:y),$ we have $b \in (N:ay)$ so that $(N:ay)\setminus\sqrt{(N:y)} \neq \varnothing.$ This implies that $N = (N+Ray)\cap(N+Ry).$ Since $ay \in (N+Ray)\cap(N+Ry),$ it follows that $ay \in N.$ $(N:y)$ is a primary ideal of $R.$ By Theorem 3.5, we have N is a classical primary subsemimodule of $M.$

Theorem 3.11: Let N be a classical primary k-subsemimodule of semimodule M over a commutative semiring $R,$ $x \in M$ and $y \in M \setminus N.$ If $rx \in N,$ then $N = (N+Rx)\cap(N+Rr^ny)$ for some positive integer $n.$

Proof: Let $rx \in N$ and $r \in R.$ It is clear that $N \subseteq (N+Rx)\cap(N+Rr^ny),$ for some positive integer $n.$ On the other hand, we show that $(N+Rx)\cap(N+Rr^ny) \subseteq N,$ for some positive integer $n.$ We divide our proof into two cases.

Case 1: There exists positive integer n such that $r^ny \in N.$ Since N is a subsemimodule of $M,$ we have $Rr^ny \subseteq N.$ It follows that $(N+Rx)\cap(N+Rr^ny) \subseteq N,$ for some positive integer $n.$

Case 2: No positive integer n such that $r^ny \in N.$ By Definition 2.1, we have $r^n \notin \sqrt{(N:y)}.$ Since $rx \in N,$ we have $r \in (N:x).$ Then $(N:x)\setminus\sqrt{(N:y)} \neq \varnothing.$ By Theorem 3.10, it follows that $N = (N+Rx)\cap(N+Ry).$ Now, since $(N+Rr^ny) \subseteq (N+Ry),$ for all positive integer $n,$ we have

$(N+Rx)\cap(N+Rr^n y)\subseteq(N+Rx)\cap(N+Ry)=N,$ for all positive integer $n.$ Hence $N=(N+Rx)\cap(N+Rr^n y)$ for some positive integer $n.$

Definition 3.12: A proper subsemimodule N of an R-semimodule M is irreducible if for any subsemimodules N_1 and N_2 of M such that $N=N_1\cap N_2,$ then $N=N_1$ or $N=N_2.$

Theorem 3.12: Let N be a classical primary k-subsemimodule of semimodule M over a commutative semiring $R,$ with identity. If N is an irreducible subsemimodule of $M,$ then N is a primary subsemimodule of $M.$

Proof: Clearly, N is a proper subsemimodule of $M.$ Thus there exists $y\in M\setminus N.$ Let $r\in R$ and $x\in M$ such that $rx\in N.$ We will show that $x\in N$ or $r^n M\subseteq N,$ for some positive integer $n.$ Assume that $r^n M\nsubseteq N,$ for all positive integer $n.$ By Theorem 3.11, we have $N=(N+Rx)\cap(N+Rr^k y),$ for some positive integer $k.$ Since N is an irreducible subsemimodule of $M,$ it follows that $N=N+Rx$ or $N=N+Rr^k y.$ Now since $r^n M\nsubseteq N,$ for all positive integer $n,$ we have $r^k y\notin N.$ This implies that $N=N+Rx.$ It follows that $x=1x\in Rx\subseteq N$ and hence N is a primary subsemimodule of $M.$

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