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Robustness of PI Controller for Various Tuning Methods for a Nonlinear Process

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ABSTRACT

In many of the industries some of the systems are nonlinear. In this the nonlinear process is conical tank. Conical tank is nonlinear in shape. As height change the area also changes due to change in radius and the mathematical relation is nonlinear. The main objective is to maintain the liquid level in the conical tank at the desired value. In this paper, various tuning strategies like Zeigler Nichols, Cohen-coon, CHR and Kappa-Tau applied to the analog PI Controller to achieve the desired value for the process. The performance indices and time domain analysis are compared for all methods. Robustness of the controller was compared for all methods by varying the time constant and gain of the nonlinear process.

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INTRODUCTION

In process industries liquids are stored in tanks and pumped one tank to another tank. The tanks may be linear or nonlinear. In many industries the basic problems are maintain the liquid level and maintain the flow rate of the fluid in the tank. If the process is linear, design is simple to maintain the parameters mentioned above at a desired value. If the process is nonlinear the design is complicated. In this paper, conical tank is used. Conical tank is nonlinear in shape. These tanks find wide applications in hydrometallurgical industries, food process industries, concrete mixing industries and wastewater treatment industries.

In this paper the height of the fluid in the tank is maintained at desired value. For the process, height of the tank is controlled variable and inflow rate is manipulated variable. Analog controller is used to manipulate the manipulated variable and in turn maintain the controlled variable at desired value. Conventional PID controllers are widely used in industries since they are simple, robust and familiar to the field operator. In this work a PI controller is used.

Performance of the overall process mainly depends on the controller parameters. Closed loop system becomes unstable if the controller parameters are not tuned properly. Controller parameters are properly tuned to get the closed loop system stable and performance indices are minimum. So, tuning is essential for getting good dynamics. In this paper controller parameters are tuned using four different methods like Zeigler Nichols closed loop method, Cohen-Coon method, Chein-Hrones-Reswick method and Kappa-Tau method (Sreenivasulu, 2003). The performance indices Integral Square Error (ISE), Integral Absolute Error (IAE) and Integral Time Absolute Error (ITAE) of these methods are analyzed and compared.

The paper is organized as following. Mathematical modeling the process was discussed in the section II. Section III presents controller used in the paper. The tuning methods are discussed in section IV. Computer simulations and results are given in section V. Conclusions are summarized in section VI.

Mathematical modelling:

The conical tank system is shown in Figure 1. For developing a mathematical model of the process, flow rate of the liquid into the tank as manipulated variable and height of the liquid in the tank as controlled variable [1, 3,10].

The operating parameters are

F_{in} – Inflow rate of the tank
F_{out} – Outflow rate of the tank
H – Total height of the conical tank.

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R – Top radius of the conical tank.
h – Height of the liquid in the tank.
r – Radius of the liquid in the tank.
K – Valve coefficient

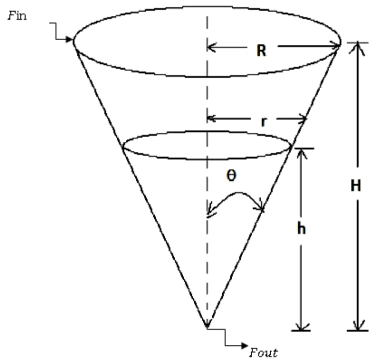

Fig. 1: Schematic diagram of conical level system.

The cross sectional area of the conical tank is

$$A = \pi r^2 \quad (1)$$

$$\tan\theta = \frac{r}{h} = \frac{R}{H} \quad (2)$$

$$r = R * \frac{h}{H} \quad (3)$$

$$A = \pi \frac{h^2 * R^2}{H^2} \quad (4)$$

According to mass balance equation, Rate of accumulation = inflow rate - outflow rate.

$$A\frac{dh}{dt} = F_{in} - F_{out} \quad (5)$$

$$F_{out} = K\sqrt{h} \quad (6)$$

using (4) & (6) in (5),

$$\frac{dh}{dt} = \frac{F_{in} - K\sqrt{h}}{A} \quad (7)$$

$$\frac{dh}{dt} = \frac{F_{in} - K\sqrt{h}}{\pi R^2 h^2 / H^2} \quad (8)$$

by integrating (8), the mathematical model can be written as follows,

$$h = \int \left(F_{in} - K\sqrt{h}\right) * \frac{1}{\pi} * \frac{H^2}{h^2} * \frac{1}{R^2} \quad (9)$$

Equation (7) is a nonlinear equation. Using Taylor's series to linearize the equation around h = hs

$$f(h,F) = \frac{F_{in} - K\sqrt{h}}{A} \quad (10)$$

By Taylor's expansion,

$$f(h,F) = \frac{F_{in} - K\sqrt{h}}{A} + \frac{1}{A}(F - F_{ins}) - \frac{K(h - h_s)}{2A\sqrt{h_s}} \quad (11)$$

The first term on RHS is zero, because the linearization is about a steady state point

$$f(h,F) = \frac{1}{A}F' - \frac{K{h_s}'}{2A\sqrt{h_s}} \quad (12)$$

$$\frac{d{h_s}'}{dt} = \frac{1}{A}F' - \frac{K{h_s}'}{2A\sqrt{h_s}} \quad (13)$$

This is similar to the first order equation

$$\frac{\tau dy}{dt} + y = ku \quad (14)$$

Hence the transfer function of the above system is

$$\frac{h(s)}{F_{in}(s)} = \frac{k}{\tau s + 1} \quad (15)$$

Where,

$$\tau = \frac{2A\sqrt{h}}{K} ; \quad k = \frac{2\sqrt{h}}{K}$$

The transfer function is to be a first order plus dead time delay model (FOPDT) $G(S) = \frac{K\, e^{-LS}}{\tau S + 1}$ and the transfer function of a real time experiment is used here for analysis (Nithya, 2010) is $G(S) = \frac{12\, e^{-2.05S}}{53.6S + 1}$.

Controller:

Conventional controllers are simple in operation. So many industries are used these types of controllers. There are continuous and discontinuous modes of controllers. P, PI and PID continuous mode of controllers are used in many industries. In this paper PI controller is selected.

The general equation of PI controller is

$$u(t) = K_c e(t) + \frac{K_c}{\tau_i}\int_0^t e(t)\,dt + u(0)$$

Where,

K_c = proportional gain

τ_i = integral time

The variable e(t) represents the tracking error which is the difference between the desired input value and the actual output. This error signal will be sent to the PI controller. The signal u(t) from the controller is applied to the plant. The block diagram of the closed loop system is shown in Figure 2.

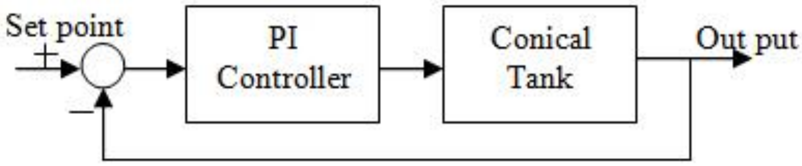

Fig. 2: Block diagram of the conical tank system.

Proportional gain K_c: Large proportional gain can increase response speed and reduce the steady state error, but will lead to oscillation of the system.

Integral gain $K_i(1/\tau_i)$: Integral control is favorable for diminishing the steady state error but will lengthen the transient response.

Tunning methods:

Ziegler Nichols method: In 1942, Ziegler and Nichols presented two classical methods to tune a PID controller. In open loop method, the controller parameters are calculated based on the parameters of K, L and τ of the process reaction curve (Ziegler, 1942). In closed loop method, the controller parameters are calculated based on the ultimate gain Ku and ultimate period Tu.

Cohen-Coon method: Cohen and Coon design method is the second popular method after Ziegler Nichols method. This method is similar to the Ziegler Nichols reaction curve method in that it makes of the FOPDT model to develop the tuning parameters (Cohen, 1953). The controller settings are based on the three parameters K, L, and τ of the open loop step response. The main design requirement is the rejection of load disturbances. Despite a better model, the results of the Cohen Coon method are not much better than the Ziegler Nichols method.

Chien-Hrones-Reswick method: This method of tuning was developed from the Ziegler Nichols open loop method for better performance of response speed and overshoot. The quickest aperiodic response is labeled with 0% overshoot and the quickest oscillatory process is labeled with 20% overshoot (Dr. Satya Sheel, 2012).

Kappa-Tau method: The Kappa-Tau method is developed by K. Astrom and T. Hagglund. The method is developed based on dominant pole design with criterion on the rejection of load disturbance and constraints on the maximum sensitivity (Ms) (Astrom, 1995). Typical values of Ms are in the range of 1.2 to 2. Larger values of Ms give systems that are faster but less robust. This method avoids the poor damping obtained with the Ziegler Nichols method and it gives good tuning for processes with long dead time.

The PI controller tuning rules (Aidan O'Dwyer, 2009; Wayne Bequette, 2003) of the above mentioned methods for the FOPDT model $G(S) = \frac{K\,e^{-LS}}{\tau S+1}$ is given in the Table 1.

Table 1: Different tuning methods for Analog PI Controller.

Tuning Methods	K_c	τ_i
Ziegler Nichols (Open Loop)	$\frac{0.9\tau}{KL}$	$3.33L$
Cohen-coon	$\frac{1}{K}\left(\frac{\tau}{L}\right)\left[1+\frac{L}{3\tau}\right]$	$L\left[\frac{(30+\frac{3L}{\tau})}{(9+\frac{20L}{\tau})}\right]$
CHR (0% overshoot)	$0.35\tau/KL$	1.2τ
	$0.6\tau/KL$	$4L$
CHR (20% overshoot)	$0.6\tau/KL$	τ
	$0.7\tau/KL$	$2.3L$

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Kappa-Tau (Ms=1.4) $x = L/(L+\tau)$	$[(0.29\tau/LK)\exp(-2.7x + 3.7x^2)]$	$[8.9L\exp(-6.6x + 3x^2)]$
Kappa-Tau (Ms=2) $x = L/(L+\tau)$	$[(0.7\tau/LK)\exp(-4.1x + 5.7x^2)]$	$[8.9L\exp(-6.6x + 3x^2)]$

Results:

The robustness of the controller is studied by varying steady state gain and time constant. Using Table 1, the values of Proportional controller gain and integral time of the PI controller are calculated for the system $G(S) = \frac{K\,e^{-LS}}{\tau S+1}$ with different values of gain constant, K and time constant, τ and is given in Table 2.

All the four tuning methods are simulated in MATLAB environment. The time domain analysis and the performance indices are calculated for servo operation and regulatory operation for the step input. The servo and regulatory response of the process for all four tuning methods is shown in Figure 3 and Figure 4 respectively. The time domain analysis of servo and performance indices are calculated and shown in Table 3, Table 4 and Table 5 respectively.

From Figure 3 and Figure 4 it is observed that in Cohen and Coon method produces maximum overshoot and long time to settle compared with Zeigler Nichols method. CHR (0% overshoot) method makes the system to settle without overshoot and more over takes less time to reach the steady state value. CHR (20% overshoot) method has least overshoot, settling time and ITAE compared with other methods. Kappa-tau method (Ms=2) values better than the Kappa-tau method (Ms=1.4). Peak overshoot is more in this method compared to near to CHR method.

Table 2: Values of Analog PI Controller parameters (K_c, τ_i).

	K= 9, L= 2.05, τ = 53.6		K=12, L=2.05, τ = 53.6		K=15, L=2.05, τ = 53.6		K=12, L=2.05, τ = 45		K=12, L=2.05, τ = 60	
	K_c	T_i	K_c	T_i	K_c	T_i	K_c	T_i	K_c	T_i
ZN	2.614	6.826	1.961	6.826	1.568	6.826	1.646	6.826	2.195	6.826
CC	2.942	6.322	2.206	6.322	1.765	6.322	1.857	6.233	2.466	6.372
CHR (0%)	1.016	64.32	0.762	64.32	0.61	64.32	0.64	54	0.853	72
CHR (20%)	1.743	53.6	1.307	53.6	1.045	53.6	1.097	45	1.463	60
KT (1.4)	0/766	14.365	0.574	14.365	0.459	14.365	0.474	13.763	0.649	14.718
KT (2.0)	1.762	14.365	1.321	14.365	1.057	14.365	1.082	13.763	1.5	14.718

Table 3: Time domain analysis for servo operation for different values of steady state gain, K.

	K =12, L=2.05, τ =53.6					K =9, L=2.05, τ =53.6					K =15, L=2.05, τ =53.6				
	Rise Time	Settling Time	Peak over shoot	Peak value	Peak time	Rise Time	Settling Time	Peak over shoot	Peak value	Peak time	Rise Time	Settling Time	Peak over shoot	Peak value	Peak time
ZN	1.65	34.67	82.49	1.825	9	1.654	34.67	82.546	1.826	9	1.654	34.67	82.555	1.826	9
CC	1.47	53.17	96.07	1.955	9	1.469	53.32	96.267	1.957	9	1.469	53.31	96.258	1.957	9
CHR (0%)	7.59	17.86	0	0.993	>70	7.601	17.95	0	0.993	>70	7.588	17.91	0	0.993	>70
CHR (20%)	3.07	13.86	11.84	1.118	10	3.07	13.88	11.918	1.119	10	3.069	13.88	11.923	1.119	10
KT (1.4)	7.21	50.22	19.96	1.195	23	7.201	50.18	19.972	1.196	23	7.202	50.21	19.92	1.195	23
KT (2.0)	2.67	33.55	29.27	1.293	11	2.668	33.54	29.308	1.294	10	2.667	33.54	29.305	1.294	10

Table 4: Time domain analysis for servo operation for different values of time constant, τ

	K =12, L=2.05, τ =45					K =9, L=2.05, τ =53.6					K =15, L=2.05, τ =60				
	Rise Time	Settling Time	Peak over shoot	Peak value	Peak time	Rise Time	Settling Time	Peak over shoot	Peak value	Peak time	Rise Time	Settling Time	Peak over shoot	Peak value	Peak time
ZN	1.65	34.47	80.97	1.809	9	1.654	34.67	82.49	1.825	9	1.65	34.79	83.4	1.83	9
CC	1.46	53.11	95.3	1.948	9	1.47	53.17	96.07	1.955	9	1.46	53.38	96.69	1.96	9
CHR (0%)	1.65	18.52	0	0.992	>70	7.594	17.868	0	0.993	>70	7.55	17.65	0	0.99	>70
CHR (20%)	3.07	13.85	11.84	1.118	10	3.071	13.862	11.84	1.118	10	3.07	13.88	11.91	1.12	10
KT (1.4)	7.44	50.04	18.68	1.183	24	7.209	50.22	19.96	1.195	23	7.11	50.54	20.44	1.2	23
KT (2.0)	2.76	32.94	27.91	1.28	11	2.671	33.55	29.27	1.293	11	2.62	33.95	30.31	1.3	10

Table 5: Performance Indices analysis for servo operation.

	K =9, L=2.05, τ =53.6			K=12, L=2.05, τ = 53.6			K=15, L=2.05, τ =53.6			K =12, L=2.05, τ =45			K=12, L=2.05, τ =60		
	ISE	IAE	ITAE	ISE	IAE	ITAE	ISE	IAE	ITAE	ISE	IAE	ITAE	ISE	IAE	ITAE
ZN	5.172	8.556	77.89	5.225	8.563	77.55	5.172	8.557	77.91	5.121	8.418	75.29	5.291	8.67	79.32
CC	6.533	11.22	138.5	6.686	11.31	137.5	6.531	11.22	138.4	6.672	11.35	139.1	6.745	11.44	140.8
CHR (0%)	4.226	6.558	52.99	4.221	6.511	51.33	4.223	6.548	52.75	4.227	6.612	54.46	4.217	6.494	51.16
CHR (20%)	3.24	4.345	17.29	3.241	4.335	17.83	3.24	4.345	17.29	3.241	4.335	17.87	3.242	4.319	16.98
KT (1.4)	5.208	10.28	143.7	5.208	10.28	143.8	5.205	10.27	143.6	5.19	10.1	138.3	5.21	10.36	146.7
KT (2.0)	3.575	6.453	56.75	3.574	6.445	56.68	3.574	6.453	56.75	3.565	6.373	54.87	3.575	6.479	57.79

Table 6: Time domain analysis for regulatory operation.

Tuning methods	Max. peak value	Peak time (sec)	Settling time (sec)
ZN	0.663	6.09	31.6
CC	0.64	5.71	45.4
CHR (0%)	0.763	6.93	25
CHR (20%)	0.702	6.08	39
KT (1.4)	1.2	11.8	44.6
KT (2.0)	0.775	7.09	51.1

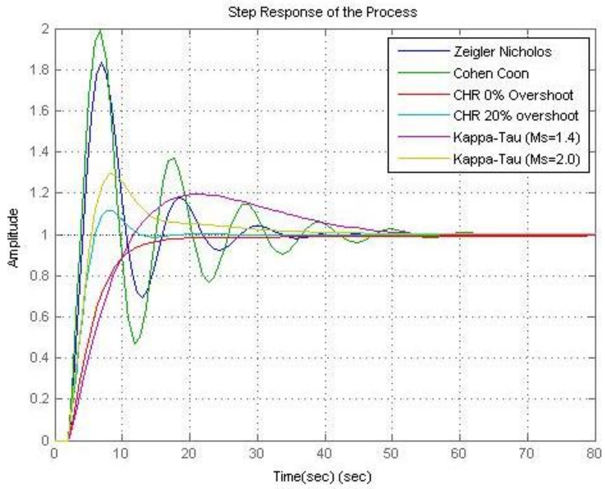

Fig. 3: Step response of the process for servo operation (K=12, L=2.05, τ =53.6).

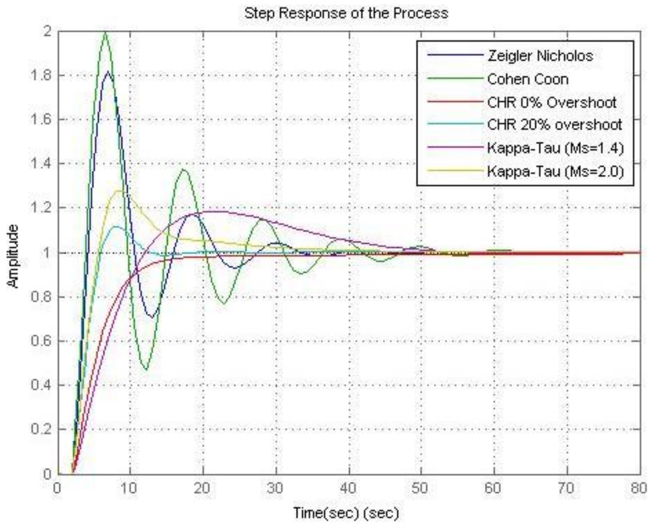

Fig. 4: Step response of the process for servo operation (K=12, L=2.05, τ =45)

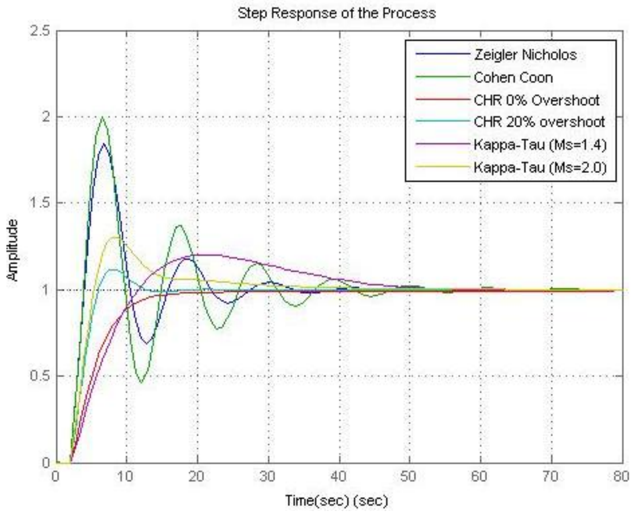

Fig. 5: Step response of the process for servo operation (K=12, L=2.05, τ =60)

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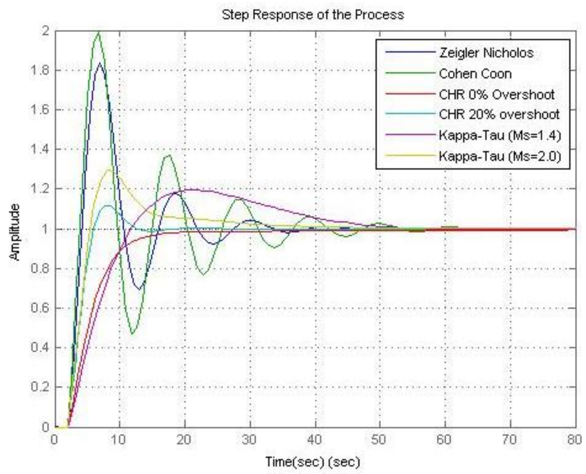

Fig. 6: Step response of the process for servo operation (K=9, L=2.05, τ =53.6)

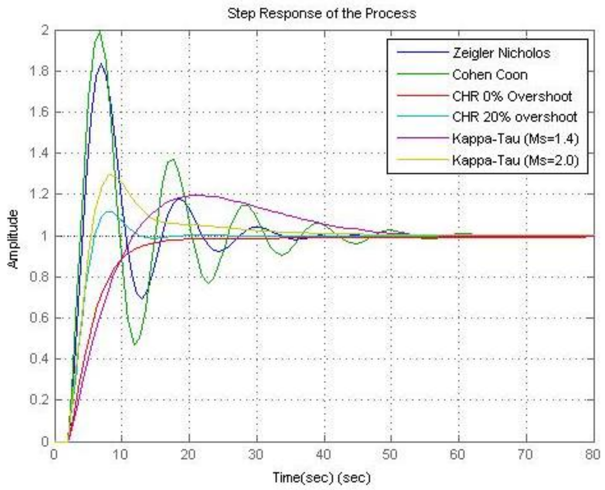

Fig. 7: Step response of the process for servo operation (K=15, L=2.05, τ =60)

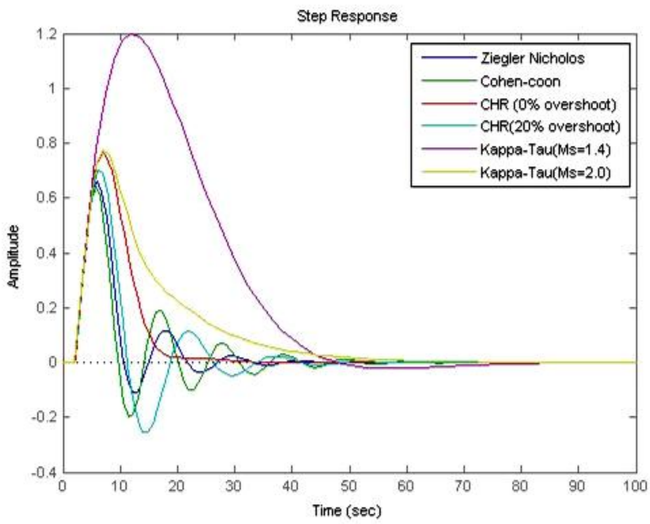

Fig. 8: Step response of the process for regulatory operation.

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Conclusions:

PI Controller with different tuning methods was simulated for nonlinear system. Robustness of the controller also studied for varying the gain and time constant. It is observed and concluded that CHR (20% overshoot) method is appropriate for servo and regulatory operation compared with other methods mentioned in this paper. This method has least overshoot, settling time, ISE, IAE and ITAE.

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