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A Rational Cubic Spline for Preserving the Positivity of 2D Data

Anas K. Faris, Zainor Yahya and Nursalasawati Rusli

University Malaysia Perlis, Institute of Engineering Mathematics.02600. Pauh Perlis. Malaysia

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ABSTRACT

In many scientific applications, the data schemes required to be smooth and controllable while preserve the features of these schemes. One of the most important features is positivity. This work intends to address the problem of preserving positivity of positive 2D data, with considering that their display looks smooth and controllable. In order to achieve this goal a C^1 piecewise cubic/cubic spline function had proposed, the proposed function contains three free parameters in each interval of its construction, simple data dependent constraints are derived for single shape parameter to preserve the positivity through given positive data, remaining two parameters are free to change for refinement the curve as designer needs. This work make the designer in capable of getting the required smoothness by resetting the suitable shape parameter values. Numerical examples are provided to demonstrate that anticipate schemes are positivity preserving and smooth.

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INTRODUCTION

In the fields of Computer Aided Geometric Design CAGD, Data Visualization DV and Shape Control, many data schemes required to be smooth and controllable, while preserving the shape of the data representation. The problem of shape preserving had discussed by many authors, such as, (Goodman *et al*., 1998; Hussain and Hussain, 2006; Ibraheem *et al*., 2013); they discussed the preservation of the inherited basic properties of data visualization, positivity, monotony and convexity in addition to constrained over straight line.

These basic properties are existent in the data visualizations coming from any social or scientific fields. For example, positive 2-dimentional data must be seen in the in the levels of the gas discharge in some chemical reactions and progress of irreversible processes, size and density (Sarfraz, 1993).

These were some of the physical quantities that are always positive and there graphical displays are nonsense if they were negative. Many methods with polynomials such as Lagrange approximation and spline function (built in matlab function) provide smooth and visually acceptable data representation but with ability for unexpected undulation and missing the positivity, while piecewise cubic Hermite interpolation polynomial is able to remove the unexpected undulation but the shape display model do not guarantee to preserve positivity. This work used a cubic/cubic rational spline function which preserve positivity and produce smooth curves for given positive data.

(Schmidt and Heβ, 1988) developed sufficient conditions to preserve the positively using cubic spline interpolation of cubic polynomial through the whole interval. (Gregory and Sarfraz, 1990) suggested a rational cubic spline with tension control parameter in each sub interval, for both rational B-spline and interpolatory, they also analyzed the effect of tension parameter on the visualization of the curve with respect to variation of control parameter. Preserved the convexity of the data by interval subdivision technique, that is in any interval convexity was lost the interval had been divided into two sub -intervals via interpolating additional knots in that interval. (Goodman *et al*., 1991) suggested two plans to preserve the shape of data lying on one side of the straight line using a rational cubic function, the first one to preserve the shape of data located above a straight line through scaling weights by some scale factor. The second plan has preserved the shape of data by inserting a new interpolation point. (Butt and Brodlie, 1993) developed sufficient conditions in the term of the first and mixed partial derivative at the rectangular grid to preserve the shape of positive 3D data using piecewise bucolic function.

Corresponding Author: Anas K. Faris, University Malaysia Perlis, Institute of engineering Mathematics, 02600. Pauh, Perlis, Malaysia.
Ph: 0060124529717. E-mail: Anas.math.unimap@gmail.com

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(Sarfraz, 1993) Gave a description for the parametric curve to preserve the shape and started with some introductions of rational cubic interpolation. (Goodman *et al*., 1998) constructed non-planner shape preserving interpolating curve scheme, they obtained a curve through an optimization process involving some fairness criteria, in order to achieve curve by piecewise rational cubic function. (Sarfraz *et al*., 2000) introduced a piecewise cubic spline interpolation with two free shape parameters to preserve positivity and convexity of positive and convex data in them representation respectively.

(Lamberti and Manni, 2001) proposed a parametric cubic curve such that the curve is easily controllable by tension parameters, they described a shape-preserving interpolation based on parametric cubic curve and provides sufficient conditions for positivity, monotony and convexity. (Goodman, 2002) studied the algorithms for the shape-preserving interpolation of 2D data and deals with defined planner of curve parametrically. (Asim and Brodlie, 2003) developed a piecewise rational cubic function to preserve the positivity of positive data and as this function don't preserve the positivity, the authors inserted extra knots to improve this matter, they also developed the constraints on parameters to preserve the shape of 3D data points lying above the plane.

(Sarfraz and Hussain, 2006) developed a piecewise rational cubic function with two free parameters to preserve the positivity of positive data, but with no freedom to use to refine the curve as desired. (Hussain and Sarfraz, 2008) had preserved positivity of curve using a piecewise rational cubic function with four free parameters such that two of them are constrained depending on the other two which are free for the designer choice, further than they had extended that rational cubic function into rational bi-cubic function with eight free parameters for preserving 3D data scheme by arranging the data over a rectangular grid with no need to supply additional knots. (Sarfraz and Hussain, 2010) developed a rational cubic function with two free parameters to preserve the shape of 2D data, they also extended the rational cubic function into rational bi-cubic function to preserve the shape of positive surface data.

Abbas *et al*, 2011) used a rational cubic/cubic piecewise interpolation with three free parameters provides more freedom to preserve the monotony of monotone 2D displayed data. (Sarfraz *et al*., 2012) had used the same plan in (Hussain and Sarfraz, 2008) to preserve inherited curve properties positivity, monotony and convexity. (Shaikh *et al*., 2012) addressed the problem of positivity and convexity of surfaces using rational bi-cubic interpolation involving eight free shape parameters, there was no limitation on the interpolation lying in a specified interval. (Abbas *et al*., 2013) presented a cubic/quadratic piecewise interpolation containing three free parameters in order to solve the problem of constructing positivity of curve through given positive data, he did not insert any extra knots in the proposed interpolation (Ibraheem *et al*., 2013) proposed trigonometric rational interpolation with four free parameters in the goal of preserving the convexity of the curve. (Hussain and Hussain, 2013) proposed a two-dimensional shape preserving algorithm for preserving the positivity, monotony, convexity and constrained over a straight line in the data representation, the interpolation in use rational cubic/linear and involving two free parameters. (Karim *et al*., 2014) studied the use of cubic ball interpolation for convexity preserving of scalar data, they developed the sufficient condition for the convexity-preserving of cubic ball interplant with smoothness degree. (Tahat *et al*., 2014) used rational cubic ball interpolation with four free parameters in order to preserve the shape of positive and constrained 2D data visualization, such that two of these free parameters are free to guarantee the refinement of the curve while the another two are constrained represents positive 2D data. (Zabidi, *et al*, 2014) developed a rational cubic ball interpolation containing three free parameters in order to preserve monotony.

This paper has been arraigned as in I. Rational cubic spline function, II. Determine of Derivatives, III. Positivity preserving rational cubic spline interpolation, IV. Numerical Examples, V. Conclusions

I. Rational cubic/cubic spline function:

Let $\{(x_i, f_i), i = 0,1,2,..., n\}$ be a given set of data points defined over the interval $[a,b]$, where $a = x_0 < x_1 < .... < x_n = b$. A C^1 piecewise rational cubic/cubic function with three free parameters over each subinterval $[x_i, x_{i+1}]$, $i = 0,1,\ldots,n-1$ is defined as:

$$S(x) \equiv S(x_i) = \frac{A_0(1-\theta)^3 + A_1(1-\theta)^2\theta + A_2(1-\theta)\theta^2 + A_3\theta^3}{u_i(1-\theta)^3 + (u_i + w_i + v_i)(1-\theta)\theta + v_i\theta^3} \quad (1)$$

where $\theta = \frac{x - x_i}{h_i}$ and $h_i = x_{i+1} - x_i$ $\forall\ i = 1,2,3,\ldots,n-1$, $(u_i v_i)$ are positive and w_i is constrained shape parameters as will be shown in section III. The interpolation (1) is C^1 if satisfies the following interpolatory conditions

$$S(x_i) = f_i \qquad S(x_{i+1}) = f_{i+1}$$
$$S'(x_i) = d_i \qquad S'(x_{i+1}) = d_{i+1} \quad (2)$$

where $S'(x_i)$ denotes the first order derivative with respect to x and d_i denotes the derivative value at knots. The substitution of interpolatory condition (2) on the function (1) produces the values :

$$A_0 = u_i f_i$$
$$A_1 = (u_i + w_i + v_i) f_i + u_i h_i d_i$$
$$A_2 = (u_i + w_i + v_i) f_{i+1} - v_i h_i d_{i+1}$$
$$A_3 = v_i f_{i+1}$$

These values of A_i, $i = 0,1,2,3$ yields the equation (1) to the form

$$S(x_i) = \frac{p_i(\theta)}{q_i(\theta)}$$

where,

$$p_i(\theta) = \begin{cases} u_i f_i (1-\theta)^3 + ((u_i + w_i + v_i) f_i + u_i h_i d_i)(1-\theta)^2 \theta \\ + ((u_i + w_i + v_i) f_{i+1} - v_i h_i d_{i+1})(1-\theta)\theta^2 + v_i f_{i+1} \theta^2 \end{cases}$$

It is important to show that, if we set $u_i = w_i = v_i = 1$, then the equation (1) clearly became the standard cubic Hermite polynomial.

II. Determine of Derivatives:

Almost, the derivative parameters $\{d_i\}$ are unknown and hence they must be determined either from the data knots $\{(x_i, f_i), i = 0,1,2,...n\}$ or by some other algebraic means. In this article, they had estimated from the given data points such that the C^1 smoothness of interpolation (1) is preserved. The method below is the approximation based on various mathematical theories. The descriptions of such approximations are stated as following:

II.I. Arithmetic Mean method:

This is the three point difference approximation method (M. Dube and P. S. Rana, 2014):

$$d_i = \begin{cases} 0 & if \quad \Delta_{i-1} = 0 \quad \Delta_i = 0 \\ \dfrac{h_i \Delta_{i-1} + h_{i-1} \Delta_i}{(h_i + h_{i-1})} & otherwise \end{cases}$$

$$d_1 = \begin{cases} 0 & if \quad \Delta_i = 0 \quad \text{sgn}(d_1) \neq \text{sgn}(\Delta_1) \\ \dfrac{\Delta_1 + (\Delta_1 - \Delta_2) h_1}{(h_1 + h_2)} & otherwise \end{cases}$$

$$d_n = \begin{cases} 0 & if \quad or \quad \text{sgn}(d_n) \neq \text{sgn}(\Delta_{n-1}) \quad \Delta_{n-1} = 0 \\ \dfrac{\Delta_{n-1} + (\Delta_{n-1} - \Delta_{n-2}) n_{n-1}}{(h_{n-1} + h_{n-2})} & otherwise \end{cases}$$

$$\Delta_i = \frac{f_{i+1} - f_i}{h_i}, \quad i = 1,2,\ldots,n-1$$

III. Positivity Preserving using Rational Cubic Spline Interpolation:

For given data point $\{(x_i, f_i), \ i = 0,1,2,\ldots,n\}$ with $x_0 < x_1 < ..... < x_n$ and $f_o > 0, f_1 > 0, ..., f_n > 0$. The main idea is to construct an interpolation S, which is positive through each subinterval $I_i = [x_i, x_{i+1}]$. That is

$$S_i(x) > 0, \quad x \in [x_i, x_n], \quad i = 0,1,2,\ldots,n \quad (3)$$

The piecewise rational function (1) does not ensures preserving the shape of positive data unless the inequality (3) is satisfied. Let us suppose that the data set to be positive as $(x_i, f_i) > 0$ such that the interpolation defined in (1) redact the positive data as positive curve. The equation (3) is satisfied if and only if the numerator and denominator are positive, that are:

$p_i(\theta) > 0$ and $q_i(\theta) > 0$

Considering that the nominator is positive, the challenge is to remark suitable values for the shape parameters u_i, w_i and v_i in order to ensures preserving the positivity of denominator, that are:

$u_i > 0$, $v_i > 0$ and $w_i > -(u_i + v_i)$.

Through the result modified by (Schmidt and Heβ, 1988), $p_i(\theta) > 0$ in $I_i = [x_i x_{i+1}]$, if $(p_i'(0), p_i'(1)) \in R_1 \cup R_2$

Where

$$R_1 = \left\{ (a,b), a > \frac{-3 f_i}{h_i}, b < \frac{3 f_{i+1}}{h_i} \right\}$$

$$R_2 = \begin{cases} (a,b) : 36 f_i f_{i+1} (a^2 + b^2 + ab - 3\Delta_i (a+b) + 3\Delta_i^2) + \\ 4h_i (a^3 f_{i+1} - b^3 f_i) + 3(af_{i+1} - bf_i)(2h_i ab - 3af_{i+1} + 3bf_i) \\ -h_i^2 a^2 b^2 \geq 0 \end{cases}.$$

where $a = p_i'(0)$, $b = p_i'(1)$

It is easy to show that

$$p_i'(0) = \frac{1}{h_i}(-3u_i f_i + (u_i + w_i + v_i) f_i + u_i h_i d_i)$$

$$p_i'(1) = (-(u_i + w_i + v_i) f_{i+1} + v_i h_i d_{i+1} + 3v_i f_{i+1}) \frac{1}{h_i}$$

$(p_i'(0), p_i'(1)) \in R_1$ if

$$\frac{1}{h_i}(-3u_i f_i + (u_i + w_i + v_i) f_i + u_i h_i d_i) > \frac{-3 f_i}{h_i}$$
$$or$$
$$(-(u_i + w_i + v_i) f_{i+1} + v_i h_i d_{i+1} + 3v_i f_{i+1}) \frac{1}{h_i} < \frac{3 f_{i+1}}{h_i}$$

Thus:

$$w_i > \frac{-u_i h_i d_i}{f_i} + (2u_i - v_i), \quad or \quad w_i > \frac{v_i h_i d_{i+1}}{f_{i+1}} - (u_i - 2v_i)$$

Theorem 1: The C^1 piecewise rational cubic/cubic polynomial (1) is positive if the following positivity conditions are achieved:

$$w_i > \max\left\{ -(u_i + v_i), \frac{-u_i h_i d_i}{f_i} + (2u_i - v_i), \frac{v_i h_i d_{i+1}}{f_{i+1}} - (u_i - 2v_i) \right\}$$

The inequality above can be reformed as:

$$w_i = C_i + \max\left\{ -(u_i + v_i), \frac{-u_i h_i d_i}{f_i} + (2u_i - v_i), \frac{v_i h_i d_{i+1}}{f_{i+1}} - (u_i - 2v_i) \right\}$$

IV. Numerical Examples:

Example 1:

Fig. 1(a) presents the visualization of the set of positive data contained in table I, using cubic Hermite spline scheme. It is obvious that Fig. 1(a) does not preserve the positivity of the data. For avoiding the negative part of the data visualization, the results in section III had used to generate Fig. 1(b) and Fig. 1(c), with values of the free shape parameters set as:

$u_i = v_i = 0.5$ and $u_i = [0.5,1,0.5,2.0,5,1]$, $v_i = [0.5,5,0.5,0.01,5,1]$, respectively. It clearly that Fig. 1(c) is more pleasant and smooth as compared to Fig. 1(b).

Table I: A set of positive data (Sarfraz and Hussain, 2012).

i	1	2	3	4	5	6	7
x_i	2	3	7	8	9	13	14
f_i	10	2	3	7	2	3	10

Table II: Oxygen levels in flue gas ((Jafaari *et al*., 2014)).

i	1	2	3	4	5	6	7
x_i	0	2	4	10	28	30	32
f_i	20.8	8.8	4.2	0.5	3.9	6.2	9.6

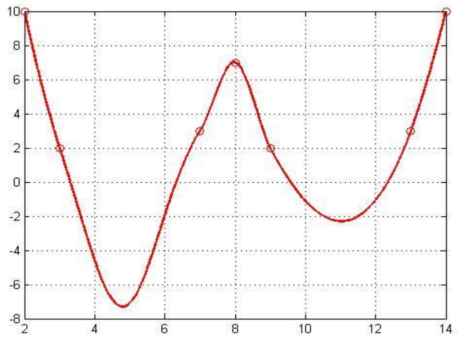

Fig. 1(a): Generated using cubic Hermite spline.

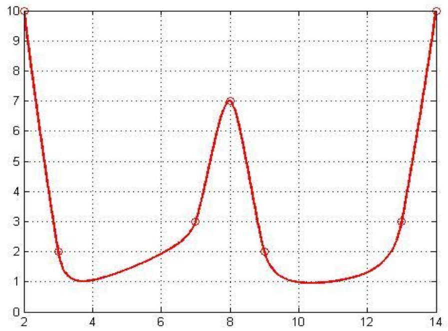

Fig. 1(b): Is drawn by using the scheme developed in section III, with the values of free parameters set as: $u_i = v_i = 0.5$.

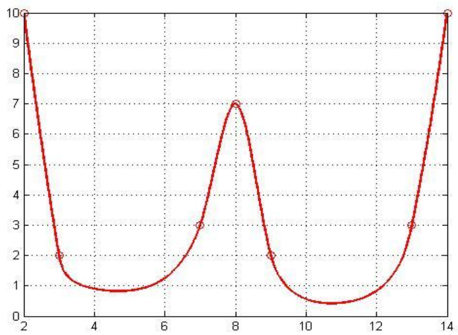

Fig. 1(c): Is drawn through using the scheme developed in section III with the values of free shape parameters set as: $u_i = [0.5,1,0.5,2.0,5,1]$ $v_i = [0.5,5,0.5,0.01,5,1]$

Example 2:

Fig. 2(a) presents the visualization of the set of positive data contained in table II, using cubic

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Hermite spline scheme. It is obvious that Fig. 2(a) does not preserve the positivity of the data. In order to preserve the curve positivity, the results in section III had used to generate Fig. 2(b) and Fig. 2(c) with values of the free shape parameters set as: $u_i = v_i = 0.5$ and $u_i = [5,2,1,10,5,5]$, $v_i = [5,4,0.1,100,5,5]$ respectively. One can easily notice that Fig. 2(c) is more pleasant and smooth than Fig. 2(b).

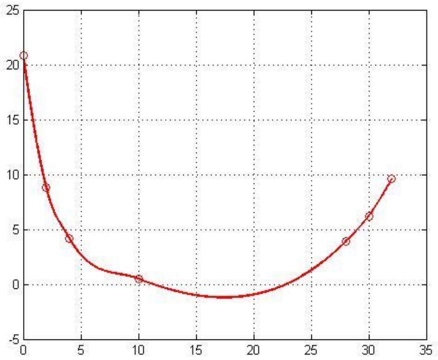

Fig. 2(a): Product by using cubic Hermite spline scheme.

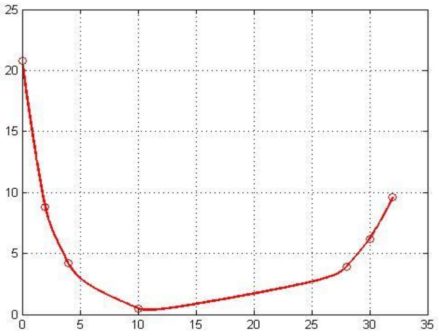

Fig. 2(b): Drawn using the results developed in section III to preserve the positivity of the data shape with the values of shape parameters $u_i = v_i = 0.5$.

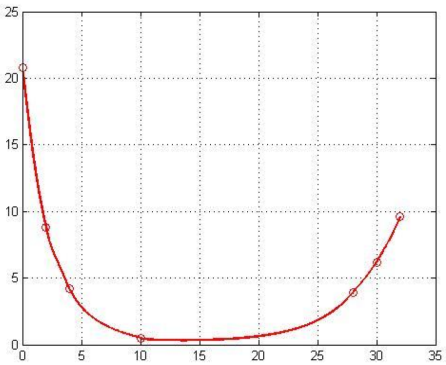

Fig. 2(c): Drawn using the results developed in section IV to preserve the positivity of the data shape with the values of shape parameters: $u_i = [5,2,1,10,5,5]$, $v_i = [5,4,0.1,100,5,5]$.

V. Conclusion:

A piecewise rational cubic/cubic interpolation involved three shape parameters, has been developed to obtain a smooth and positive curve. One of this shape parameter is dependent to ensure the shape positivity of the data while other two are left under designer's will to refine the curve as need. It is obvious that the increase and decrease of constrained value of the shape parameters may affect the smoothness and positivity of the data visualization curves.

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