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Jeffrey Fluid Flow and Heat Transfer Over a Stretching Sheet with Non-Uniform Heat Source/Sink

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ABSTRACT

This paper presents a steady two-dimensional boundary layer flow and heat transfer of a Jeffrey fluid over a stretched sheet with non-uniform heat source/sink. The governing equations that govern the fluid flow and heat transfer are solved using the Keller-box method after being reduced to a set of non-linear ordinary differential equations by a similarity transformation. The effects of Deborah number β, Eckert number *Ec*, Prandtl number *Pr* and non-uniform heat source/sink parameters *A**, *B** on the flow and heat transfer characteristics are investigated.

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INTRODUCTION

In industry processes, such as paper production, hot rolling, polymer extrusion and drawing of plastic films, the final product depends on the rate of stretching and heat transfer. Sakiadis (1961) was the first to study flow induced by a moving surface and Crane (1970) examined flow generated by a linearly stretching sheet. Industrially, the flow of non-Newtonian fluid over stretching sheet has become more and more important. This is due to the fact that Newtonian fluid flow alone is inadequate to describe fluids with long chains of molecules that contain fine particles. There are a few constitutive equations of non-Newtonian fluids, available in related literature, that are complicated and much more non-linear than those of viscous fluid. One of them is the Jeffrey fluid, which is a relatively simpler linear model using time derivatives instead of convected derivatives used by most fluid models. Many researchers have conducted experiments using only flow on a stretching sheet or a combination of various effects on a stretching sheet. Hayat *et al*. (2012) investigated three-dimensional flow of a Jeffrey fluid over a linearly stretching sheet and Qasim (2013) investigated the combined effects of heat and mass transfer in Jeffrey fluid over a stretching sheet in the presence of heat source/sink. The unsteady flow and heat transfer of Jeffrey fluid over a stretching sheet was conducted by Hayat *et al*. (2014) and recently, Ahmad and Ishak (2015) studied MHD flow and heat transfer of a Jeffrey fluid towards a stretching vertical surface.

In view of several physical problems, such as fluids undergoing exorthemic and endothermic chemical reactions, the study of heat generation or absorption has gained considerable attention. The existence of significant temperature differences between the surface and ambient fluid necessitates the consideration of temperature-dependent heat sources or sinks, which may exert strong influence on heat transfer characteristics (Vajravelu and Nayfeh, 1992). Although the exact modelling of internal heat generation or absorption is difficult, some simple mathematical models can express its average behavior for most physical situations (Abo-Eldahab and El Aziz 2004). The idea by Abo-Eldahab and El Aziz (2004) introducing non-uniform heat souce/sink

$$q''' = \left(\frac{k\,b}{\nu}\right)\left[A^*(T_w - T_\infty)\frac{u}{u_w} + B^*(T - T_\infty)\right],$$

Incorporated the effects of the space dependent of the internal heat generation or absorption, which is given by the first term, in addition to the temperature-dependent heat source/sink, given by latter term. Abel and Nandeppananvar (2009) presented the study of MHD viscoelastic boundary layer flow over a stretching sheet with non-uniform heat source/sink and Aurangzaib and Shafie (2012) discussed the effects of heat and mass transfer in a MHD non-Darcian flow of a micropolar fluid over an unsteady stretching sheet with thermophoresis and non-uniform heat source/sink. Zheng *et al.* (2013)

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presented an analysis for the unsteady mixed boundary layer flow and radiation heat transfer of generalised Maxwell fluids towards an unsteady stretching permeable surface in the presence of boundary slip and non-uniform heat source/sink. Recently, Dessie and Kishan (2015) investigated the unsteady MHD flow of heat and mass transfer of nanofluid over stretching sheet with a non-uniform heat source/sink considering viscous dissipation and chemical reaction.

Based on our close observations, no study has been made for Jeffrey fluid flow and heat transfer over a stretching sheet with non-uniform heat source/sink. However, the effect of heat source/sink has been conducted by Qasim (2013).

Analysis:

Consider the steady two-dimensional laminar boundary layer flow of an incompressible Jeffrey fluid flow over an impermeable flat sheet coinciding with the plane $y = 0$ with the flow being confined to $y > 0$. The flow is generated by stretching the sheet away from the leading edge with linear velocity $u_w(x) = a\,x$ where a is a positive constant. The x-axis runs along the stretching sheet in the direction of motion, while the y-axis is taken as normal to the sheet. Under these assumptions and the usual boundary layer assumptions, the boundary layer equations governing the flow motion are (Qasim 2013)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\nu}{1+\lambda_1}\left[\frac{\partial^2 u}{\partial y^2}+\lambda_2\left(u\frac{\partial^3 u}{\partial x\partial y^2}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\right)\right], \qquad (2)$$

subject to the boundary conditions

$$u = u_w, \quad v = 0 \quad \text{at} \quad y = 0,$$
$$u \to 0, \quad \frac{\partial u}{\partial y} \to 0 \quad \text{as} \quad y \to \infty, \qquad (3)$$

where u and v are velocity components in the x and y directions, respectively. ν is the kinematic viscosity, λ_1 is the ratio of relaxation and retardation times and λ_2 is the relaxation time.

To get the effect of the temperature difference between the surface and the ambient fluid, heat transfer analysis is performed. The energy is considered to be stored in the fluid by means of frictional heating due to viscous dissipation, and the temperature is considered to depend on heat source/sink throughout the flow. Further, the plate is assumed to have a temperature distribution in the quadratic form $T_w = T_\infty + A\left(\frac{x}{l}\right)^2$ at $y = 0$. Therefore, the thermal boundary layer under consideration is given by (Abel *et al*. 2007)

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+q''', \qquad (4)$$

subject to the following boundary conditions

$$T = T_w \quad \text{at} \quad y = 0,$$
$$T \to T_\infty \quad \text{as} \quad y \to \infty. \qquad (5)$$

Here, k is the thermal conductivity, μ is the dynamic viscosity, c_p is the specific heat at constant pressure, T is the fluid temperature and q''' is the space and temperature dependent internal heat generation/absorption (non-uniform heat source/sink), which is given as (Abo Eldahab and El Aziz 2004, Abel *et al*. 2007, Zheng *et al*. 2011)

$$q'''=\left(\frac{k\,u_w}{x\nu}\right)\left[A^*(T_w-T_\infty)\,f'(\eta)+B^*(T-T_\infty)\right], \qquad (6)$$

where A^* and B^* are parameters of the space and temperature dependent on internal heat generation/absorption. The case $A^* > 0$ and $B^* > 0$ corresponds to the internal heat source while $A^* < 0$ and $B^* < 0$ corresponds to internal heat sink.

The second term on the right side of Eq. (4) is the viscous dissipation term, which is always positive and represents a source of heat due to friction between the fluid particles.

Eqs. (1), (2) and (4), subject to the boundary conditions (3) and (5), can be solved by introducing the following similarity transformation

$$\eta=\sqrt{\frac{a}{\nu}}\,y, \quad \psi=-\sqrt{a\nu}\,xf(\eta), \quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \qquad (7)$$

where η is the similarity variable, f is the dimensionless stream function, θ is the dimensionless temperature, ψ is the stream function defined in usual way as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$. Thus, we have

$$u = axf'(\eta), \quad v=-\sqrt{a\nu}\,f(\eta), \qquad (8)$$

where prime denotes differentiation with respect to η. Using (7) and (8), Eq. (1) is trivially satisfied, while Eqs. (2) and (4) are reduced to

$$f'''-(1+\lambda_1)\left(f'^2-ff''\right)+\beta\left(f''^2-ff^{iv}\right)=0, \qquad (9)$$

$$\theta''+\Pr f\theta'+\left(B^*-2\Pr f'\right)\theta+\Pr Ec\, f''^2+A^* f'=0, \qquad (10)$$

and the transformed boundary conditions (3) and (5) can be written as

$$f(0)=0, \quad f'(0)=1, \quad \theta(0)=1 \quad \text{at} \quad \eta=0,$$
$$f'(\eta)\to 0, \quad f''(\eta)\to 0, \quad \theta(\eta)\to 0 \quad \text{as} \quad \eta\to\infty, \qquad (11)$$

where $\beta = a\lambda_2$ is the Deborah number, $\Pr=\mu c_p/k$ is the Prandtl number and $Ec = a^2 l^2 / A c_p$ is the Eckert number.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho u_w^2 / 2}, \quad Nu_x = \frac{x q_w}{k(T - T_\infty)}, \tag{12}$$

where τ_w is wall shear stress and q_w is the heat flux from the surface, which are given by

$$\tau_w = \mu \frac{\partial u}{\partial y}, \quad q_w = -k \frac{\partial T}{\partial y}, \tag{13}$$

with μ and k being the dynamic viscosity and the thermal conductivity. Substituting (7) and (8) into (12), the scaled skin friction coefficient and the local Nusselt number are reduced to

$$\frac{1}{2} C_f \, \mathrm{Re}_x^{1/2} = f''(0), \qquad Nu_x \, \mathrm{Re}_x^{-1/2} = -\theta'(0), \tag{14}$$

where $\mathrm{Re}_x = u_w x / \nu$ is the local Reynolds number.

RESULTS AND DISCUSSION

Eqs. (9) – (10), subject to the boundary conditions (11), have been solved numerically using a finite-difference method, namely the Keller-box method, for some arbitrary values of the Jeffrey fluid parameter β, the Prandtl number *Pr*, the Eckert number *Ec*, the value of the space dependent heat source/sink parameter A^* and the value of the temperature dependent heat source/sink parameter B^* with the ratio of the relaxation and retardation times λ_1 held fixed (=0). In order to validate the numerical code used, the results obtained for $\theta'(0)$ when $\beta = Ec = A^* = 0$ are compared with those reported by Tsai *et al*. (2008) when $S = A^* = 0$ in their paper, and it is found that the calculations are in good agreement, as shown in Table 1. Nevertheless, the values of $f''(0)$ remain constant at -1.000035 due to fact that the values of B^* and *Pr* only affect the thermal flow.

Table 1: Numerical output obtained for local Nusselt number $\theta'(0)$ when $\beta = Ec = A^* = 0$

B^*	Pr	Tsai *et al*. (2008)	Present results
-1	1	-1.710937	-1.710935
-2	2	-2.485997	-2.486000
-3	3	-3.028177	-3.028180
-4	4	-3.585192	-3.585187
-5	5	-4.028540	-4.028552

The velocity profile $f'(\eta)$ is plotted for various values of β, as depicted in Fig. 1. The graph is valid for any values of the A^*, B^*, *Pr* and *Ec* numbers. This is clear from Eqs. (9)-(10), which show that those parameters are independent of the velocity field and more pronounced to the thermal field. As such, various results are expected for the heat transfer rather than the skin friction at the boundary for various values of the A^*, B^*, *Pr* and *Ec* numbers. The effect of the Deborah number β on the fluid flow can be seen in Fig. 1, which shows that the velocity and the boundary layer thickness are increasing functions of the Deborah number β.

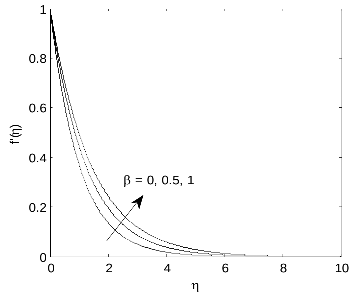

Fig. 1: Velocity profile $f'(\eta)$ for various values of β.

Fig. 2 presents the graph of the temperature profiles for a few values of A^*, B^* and β when $Pr = 6.8$ and $Ec = 1$. It is observed that an increment of the Deborah number β decreases the temperature within the boundary when the flow is generated by the heat sink ($A^*, B^* < 0$). However, for $A^*, B^* > 0$, obvious changes of temperature are observed at the beginning of the formation of the boundary layer as β changes from 0 to 1. Initially, the temperature for $\beta = 0$ is slightly higher compared with $\beta = 1$ and, after reaching a certain thickness, the temperature behaved reversely. For fixed β, it is noticed that the boundary layer absorbs the energy, which causes the temperature profiles to fall with the decreasing values of A^*, B^* (heat sink) whereas for $A^*, B^* > 0$ (heat source), the boundary layer releases the energy, resulting in the rise in temperature.

The temperature profiles for flow generated by heat absorption ($A^*, B^* = -1$) and heat generation ($A^*, B^* = 1$) are depicted in Figs. 3(a) and (b), respectively, for Pr = 1, 10, 100. Both of the graphs reveal that the temperature slowly decreases until it

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reaches a certain thickness and gradually goes to 0 at the outside of the boundary layer. However, the phenomenon is not valid for flow generated by heat source (A^*, B^* = 1) when Pr = 1. It is found that that a peak is formed at the beginning of the development of the boundary layer and subsides after reaching a certain level of thickness.

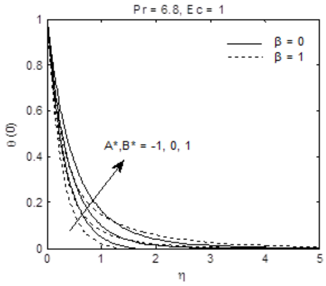

Fig. 2: Temperature profile $\theta(\eta)$ for various values of A^*, B^* and β.

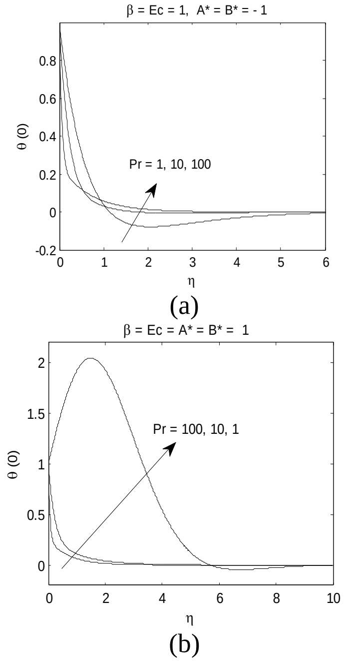

Fig. 3: Temperature profile $\theta(\eta)$ for various values of Pr number when $\beta = Ec = 1$ for (a) A^*, B^* = -1 and (b) $A^*=B^*$ = 1.

The effect of the Ec number towards the distribution of the temperature in the boundary layer are displayed in Fig. 4. Both of the flows, which are generated by heat source/sink, agree that the effect of the Ec number is an increase in temperature distribution in the flow region. This is due to the fact that heat energy is stored in the liquid due to frictional heating, which causes an increment in temperature at any point on the thermal boundary layer (Pal and Mondal 2012). For high Ec numbers (Ec = 10), an overshoot in the temperature is formed at the beginning of the flow motion for both heat source/sink.

The effect of the Deborah number β when Pr = 6.8 and Ec = 1 are depicted in Fig. 5 for various values of A^* when B^* = -1 and 1, and Fig.6 for several values of B^* when A^* = -1 and 1. From Fig. 5, it is seen that the Nusselt number decreases as A^* increases. Also, it is clear that for fixed A^*, the heat transfer at the boundary is found to increase with the increment of β. However, the heat transfer is found to be smaller for B^* = 1 as compared with B^* = - 1 at fixed A^* and β. The same trend is observed for fixed B^* and β, as described in Fig. 6.

Fig. 7 is plotted to examine the effect of the Pr number when $A^* = B^* = \pm 1$ and Ec = 1 for Newtonian and non-Newtonian fluid. It reveals that the heat transfer rate on the surface increases as Pr increases for both flows generated with heat source ($A^*= B^* > 0$) and heat sink ($A^* = B^* < 0$). The introduction of the non-Newtonian fluid to the flow, namely the Jeffrey fluid, which is given by the Deborah number β, is found to increase the heat transfer rate at the surface for both flows generated by heat source/sink. However, the heat rate is more pronounced for flows with a high *Pr* number (*Pr* >> 7) as compared with *Pr* = 2.

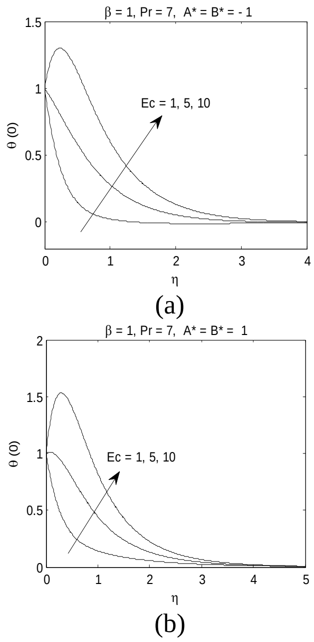

Fig. 4: Temperature profile $\theta(\eta)$ for various values of *Ec* number when $\beta = 1$, *Pr* = 7 for *A**, *B** = -1 and (b) *A**=*B** = 1

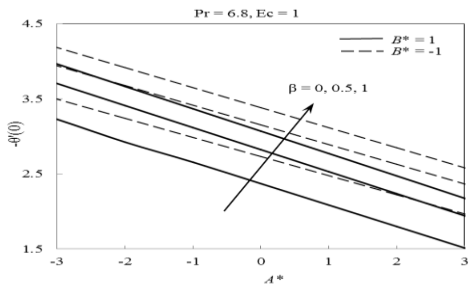

Fig. 5: The variations of local nusselt number $-\theta'(0)$ for various values of β and *A** when *Pr* = 6.8 and *Ec* = 1 for B^* = 1 and -1, respectively.

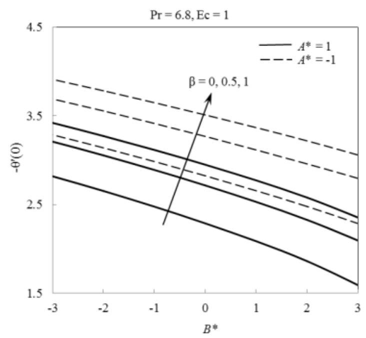

Fig. 6: The variations of local nusselt number $-\theta'(0)$ for various values of β and *B** when *Pr* = 6.8 and *Ec* = 1 for A^* = 1 and -1, respectively.

Fig. 8 displays the effect of the *Ec* number and *Pr* number for few values of the Deborah number β when *A** = *B** = 0.2 (heat source) towards the heat transfer rate, $-\theta'(0)$. It is observed that for *Ec* >> 5, the heat transfer rate decreases as *Pr* increases. However, an opposite trend is observed when *Ec* = 1, as depicted in Fig. 3. Further, the Nusselt number is an increasing function of Deborah number β as *Pr* increases for a fixed value of the *Ec* number, which is in line with the result reported in Fig. 7.

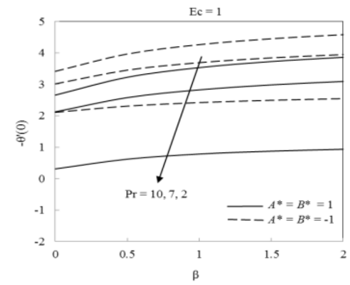

Fig. 7: The values of $-\theta'(0)$ with Deborah number, β when *Ec* =1 for various values of *Pr*.

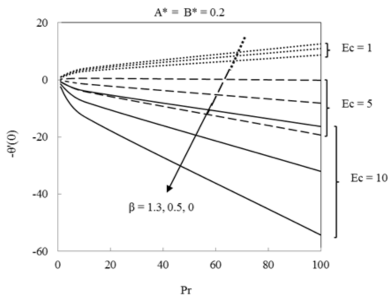

Fig. 8: The values of $-\theta'(0)$ with *Pr* number when $A^* = B^* = 0.2$ for various values of *Ec* number and Deborah number β.

Conclusion:

Jeffrey fluid flow and heat transfer over a stretching sheet with non-uniform heat source/sink with viscous dissipation is studied. Keller-box method is used to solve the governing boundary layer equations. A few graphs are depicted for the skin friction coefficient and the local Nusselt number, along with velocity and temperature profiles, taking the effects of the Deborah number β, Eckert number *Ec,* Prandtl number *Pr* and the heat source/sink parameter A^* and B^*. The results indicate that the heat transfer rate at the surface decreases for flow generated by heat source compared with flow generated by heat sink. However, the introduction of Jeffrey fluid to the fluid flow is found to increase the local Nusselt number.

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