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New Exact Travelling Wave Solutions for Some Nonlinear Partial Differential Equations by Using Tanh Method

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ABSTRACT

The solution of nonlinear partial differential equation is current research in Applied Science.Here, we introduce new exact solutions for some nonlinear partial differential equations which obtained by using tanh (or tangent hyperbolic) method. This method depending on the travelling wave solutions and does not use any conditions to solve any system of partial differential equations. The tanh method gives more general exact solutions and does not require any extra effort and it worthwhile to mention that the proposed method can be applied to establish more entirely new exact solutions for other kinds of nonlinear partial differential equations.

INTRODUCTION

It can be said that the partial differential equations occupies an important place in allbranches of engineering and the physical sciences, where most of the relations and the ruling laws between physical problems or engineering appear in the form of differential equations .To understand this problems, we must solve this partial differential equation, or at least to know many of the properties of this solution and that hard get it and process of getting a solution to the problem is not always easy, but that many of the differential equations does not get exact solution. The researchers interested differential equations by studying the existence of the solution or through its properties and nature, or to get through it. Various methods have been used to explore different kinds of solutions of physical problems described by nonlinear PDEs such as: the first integral method (Al-Saif and Abdul-Hussien, 2012), (Al-Saif and Abdul-Hussien, 2012), sine-cosine method (Hosseini *et al*, 2011), the extended tanh function method (Dahmani *et al*, 2009) and the Jacobi elliptic function expansion method (Gui-Qiong andZhi-Bin, 2005), and so on. Most of exact solutions have been obtained by these methods, including the solitary wave solutions, shock wave solutions, periodic wave solutions, and the like.Thetanh method is a powerful solution method for the computation of exact traveling wave solutions (Malfliet, 1992) and based on the fact that in many cases traveling-wave solutions canbe written in terms of a hyperbolic tangents. This method is very suitable for an easier and more effective handling of the solution process of nonlinear equations.

In this paper, to illustrate the effectiveness and convenience of the tanh method, we solve two systems of nonlinear partial differential equations: these systems are coupled Drinfeld-Sokolov-Satsuma-Hirota system, Whitham-BroerKaup equation. This will be useful in numerical studies.

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2- Outline of the tanh method:

The tanh method will be introduced as presented by Malfliet(Malfliet, 1992),(Malfliet, 2004) and by Wazwaz(Wazwaz,2006), (Wazwaz,2006), (Wazwaz,2006). The tanh method is based on a priori assumption that the traveling wave solutions can be expressed in termsof the tanh function to solve the coupled KdV equations. The tanh method is developed by Malfliet(Malfliet, 1992). This method applied to find out exact solutions of a coupled system of nonlinear differential equations with two unknowns.

Consider the system of nonlinear partial differential equations:

$$F_1(u, v, u_t, v_t, u_x, v_x, u_{xx}, v_{xx}, \dots) = 0$$
$$F_2(u, v, u_t, v_t, u_x, v_x, u_{xx}, v_{xx}, \dots) = 0 \qquad (1)$$

Where F_1 and F_2 are polynomials of the variables u, v and their derivatives. Let $u(x,t) = U(\xi)$ and $v(x,t) = V(\xi)$where $\xi = k(x - \lambda t)$. Here k and λ represent the wave number and velocity of the traveling wave.Both are undetermined parameters but we assume that$k > 0$. We use the following changes:

$$\frac{\partial}{\partial t} = -k\lambda \frac{d}{d\xi}, \quad \frac{\partial}{\partial x} = k\frac{d}{d\xi}, \frac{\partial^2}{\partial x^2} = k^2\frac{d^2}{d\xi^2}, \quad \frac{\partial^3}{\partial x^3} = k^3\frac{d^3}{d\xi^3}, \dots \qquad (2)$$

Then (1)become ordinary differential equations:

$$Q_1(U, U', U'', U''', \dots, V, V', V'', V''', \dots) = 0$$
$$Q_2(U, U', U'', U''', \dots, V, V', V'', V''', \dots) = 0 \qquad (3)$$

withQ_1, Q_2 being another polynomials form of their argument, which will be called the reducedordinary differential equations (3).Integrating(3), as long as all terms contain derivatives, the integration constants are considered to be zeros in view of the localized solutions. However, the nonzero constants can be used and handled as well (Wazwaz, 2006). Now finding the traveling wave solution to equation (1) is equivalent to obtaining the solution to the reduced ordinary differential equation(3). For the tanh method, we introduce the new independent variable:

$$Y(x,t) = tanh(\xi) \qquad (4)$$

that leads to a change of variables in the derivatives:

$$\frac{d}{d\xi} = (1 - Y^2)\frac{d}{dY},$$
$$\frac{d^2}{d\xi^2} = -2Y(1 - Y^2)\frac{d}{dY} + (1 - Y^2)^2\frac{d^2}{dY^2},$$
$$\frac{d^3}{d\xi^3} = 2(1 - Y^2)(3Y^2 - 1)\frac{d}{dY} - 6Y(1 - Y^2)^2\frac{d^2}{dY^2} + (1 - Y^2)^3\frac{d^3}{dY^3} \qquad (5)$$

Where the other derivatives can be derived in a similar way.The next crucial step is that the solution we are looking for is expressed in the form:

$$u(x,t) = U(\xi) = \sum_{i=0}^{m} a_i Y^i$$
$$v(x,t) = V(\xi) = \sum_{i=0}^{n} b_i Y^i \qquad (6)$$

where the parameter$m$ and $n$can be found by balancing the highest-order linear term with the nonlinear terms in equation (1), and $k, \lambda, a_0, a_1, \dots, a_m, b_0, b_1, \dots, b_n$are to be determined. Substituting (6) into (3) will yield a set of algebraic equations for $k, \lambda, a_0, a_1, \dots, a_m, b_0, b_1, \dots, b_n$because all coefficients of Y^i have to vanish for $i = 0,1,2,3$ ....Having determinedthese parameters, knowing that m and $n$are positive integers in most cases, and using (6), weobtain the expressions for $u(x,t)$ and $v(x,t)$ in a closed form (Wazwaz,2009). The tanh method seems to be apowerful tool in dealing with coupled nonlinearphysical models.

3- Applications:

To illustrate the tanh technique and the possibilities it offers, we now investigate some well-known examples of nonlinear PDEs in detail.

Example 1:

Let us first consider the coupled Drinfeld-Sokolov-Satsuma-Hirota (DSSH) system(Adem and Khalique, 2014)which reads:

$$u_t - 6uu_x + u_{xxx} - 6v_x = 0$$
$$v_t - 2v_{xxx} + 6uv_x = 0 \qquad (7)$$

This system was introduced independently by Drinfeld and Sokolov(Drinfeld and Sokolov, 1981), and by Satsuma and Hirota(Satsuma and Hirota, 1982). The coupled DSSH system (Drinfeld and Sokolov, 1981)was given as one of numerous examples of nonlinear equations possessing Lax pairs of a special form. Also, the coupled DSSH system (Satsuma and Hirota, 1982) was found as a special case of the four-reduction of the KP hierarchy, and its explicit one-soliton solution was constructed.Wazwaz(Wazwaz, 2009) used three distinct methods, namely the Cole-Hopf transformation, Hirota's bilinear and the exp-function methods and obtained solitons, multiple soliton solutions, multiple singular soliton solutions, and plane periodic solutions. Zheng (Zheng, 2011) used the (G'/G)-expansion method and obtained traveling wave solutions of (7).

Using the traveling wave transformations:

$$u(x,t) = U(\xi) = \sum_{i=0}^{m} a_i Y^i$$

$$v(x,t) = V(\xi) = \sum_{i=0}^{n} b_i Y^i \qquad (8)$$

Where $Y = tanh(\xi)$ and $\xi = k(x - \lambda t)$ (9)

The nonlinear system of partial differential equations(7) is carried to a system of ordinary differential equations:

$$-k\lambda U' - 6kUU' + k^3 U''' - 6k V' = 0$$

$$-k\lambda V' - 2k^3 V''' + 6kU V' = 0 \qquad (10)$$

By postulating tanh series, and using the transformations given by (9), the first equation in (10) reduces to

$$-\lambda(1-Y^2)\frac{dU}{dY} - 6U(1-Y^2)\frac{dU}{dY}$$

$$+ k^2\left[2(1-Y^2)(3Y^2-1)\frac{dU}{dY} - 6Y(1-Y^2)^2\frac{d^2U}{dY^2} + (1-Y^2)^3\frac{d^3U}{dY^3}\right]$$

$$-6(1-Y^2)\frac{dV}{dY} = 0 \qquad (11)$$

while the second equation in (10) reduces to

$$-\lambda(1-Y^2)\frac{dV}{dY} - 2k^2\left[2(1-Y^2)(3Y^2-1)\frac{dV}{dY} - 6Y(1-Y^2)^2\frac{d^2V}{dY^2} + (1-Y^2)^3\frac{d^3V}{dY^3}\right] + 6U(1-Y^2)\frac{dV}{dY} = 0 \qquad (12)$$

Now, to determine the parameter $m$and n, we balance the linear term of highest-order with the highest order nonlinear terms. So, in (11) we balance U''' with V' , to obtain $4 + m - 1 = n + 1 \Longrightarrow n = m + 2$.
While, in (12) we balance V''' with UV' , to obtain
$n + 3 = m + n + 1$, then $m = 2$ and $n = 4$

The tanh method admits the use of the finite expansion for both:

$$u(x,t) = U(Y) = a_0 + a_1 Y + a_2 Y^2 \ , a_2 \neq 0 \qquad (13)$$

$$v(x,t) = V(Y) = b_0 + b_1 Y + b_2 Y^2 + b_3 Y^3 + b_4 Y^4 \ , b_4 \neq 0 \qquad (14)$$

Substituting U, U', U'', U''', V' from equation(13)and (14)into equations(11), then equating the coefficient of $Y^i, i = 0,1,2,3,4,5$leads to the following nonlinear system of algebraic equations:

$$Y^0: -\lambda a_1 - 6a_0 a_1 - 2k^2 a_1 - 6b_1 = 0$$

$$Y^1: -2\lambda a_2 - 12a_0 a_2 - 6a_1^2 - 16a_2 k^2 - 12b_2 = 0$$

$$Y^2: \lambda a_1 + 6a_0 a_1 - 18a_1 a_2 + 8a_1 k^2 - 18b_3 + 6b_1 = 0$$

$$Y^3: 2\lambda a_2 + 12a_0 a_2 + 6a_1^2 - 12a_2^2 + 40a_2 k^2 - 24b_4 + 12b_2 = 0 \qquad (15)$$

$$Y^4: 18a_1 a_2 - 6a_1 k^2 + 18b_3 = 0$$

$$Y^5: 12a_2^2 - 24a_2 k^2 + 24b_4 = 0$$

Substituting $U, V', V'', V'''$from equation(13)and (14)into equations(12), then equating the coefficient of $Y^i, i = 0,1,2,3,4,5,6,7$leads to the following nonlinear system of algebraic equations:

$$Y^0: -\lambda b_1 + 4k^2 b_1 - 12k^2 b_3 + 6a_0 b_1 = 0$$

$$Y^1: -2\lambda b_2 + 32b_2 k^2 - 48k^2 b_4 + 12a_0 b_2 + 6a_1 b_1 = 0$$

$$Y^2: -3\lambda b_3 + \lambda b_1 - 16b_1 k^2 + 120k^2 b_3 - 6a_0 b_1 + 12a_1 b_2 + 6a_2 b_1 + 18a_0 b_3 = 0$$

$$Y^3: -4\lambda b_4 + 2\lambda b_2 - 72k^2 b_2 + 304b_4 k^2 - 12b_2 a_0 + 18a_1 b_3 - 6a_1 b_1 + 12a_2 b_2 + 24a_0 b_4 = 0 \qquad (16)$$

$$Y^4: 3\lambda b_3 - 228b_3 k^2 + 12b_1 k^2 - 18a_0 b_3 + 24a_1 b_4 - 12a_1 b_2 + 18a_2 b_3 - 6a_2 b_1 = 0$$

$$Y^5: 4\lambda b_4 - 496k^2 b_4 + 48k^2 b_2 - 24a_0 b_4 - 18a_1 b_3 + 24b_4 a_2 - 12a_2 b_2 = 0$$

$$Y^6: 120b_3 k^2 - 24b_4 a_1 - 18b_3 a_2 = 0$$

$$Y^7: 240b_4 k^2 - 24b_4 a_2 = 0$$

Solving the nonlinear systems of equations (15) and (16) we get two solution sets

$$a_0 = -\frac{19}{3}k^2, \quad a_1 = 0 \ , \quad a_2 = 10k^2 \ , \quad \lambda = 14k^2$$

$$b_0 = b_0 \ , \ b_1 = 0 \ , \ b_2 = \frac{80}{3}k^4, \ b_3 = 0 \ , \ b_4 = -40k^4 \qquad (17)$$

and

$a_0 = -7k^2$, $a_1 = 0$, $a_2 = 10k^2$, $\lambda = -14k^2$
$b_0 = b_0$, $b_1 = 0$, $b_2 = 80k^4$, $b_3 = 0$, $b_4 = -40k^4$ (18)

From (17) we get the following solution of the system(7):

$u(x,t) = -\frac{19}{3}k^2 + 10k^2tanh^2(kx - 14k^3t),$
$v(x,t) = b_0 + \frac{80}{3}k^4tanh^2(kx - 14k^3t) - 40k^4tanh^4(kx - 14k^3t)$ (19)

and from (18) we get the following solution of the system(7):

$u(x,t) = -7k^2 + 10k^2tanh^2(kx + 14k^3t),$
$v(x,t) = b_0 + 80k^4tanh^2(kx + 14k^3t) - 40k^4tanh^4(kx + 14k^3t)$ (20)

Example 2:

Consider the Whitham-BroerKaup equation (Kadem and Dumitru, 2011):

$u_t + uu_x + v_x + bu_{xx} = 0$
$v_t + (uv)_x + au_{xxx} - bv_{xx} = 0$ (21)

where$u = u(x,t)$is the horizontal velocity, $v = v(x,t)$is the height that deviatesfrom equilibrium position of the liquid, and$a$ and b are constants which arerepresentedin different diffusion powers. This system introduced by Whitham(Whitham, 1967), Broer(Broer, 1975), and Kaup(Kaup, 1975)and describe the propagation of shallow water waves (Rashidi*et al*, 2008), with different dispersion relations.

For $a = b = 1$, equation (*21*) becomes:

$u_t + uu_x + v_x + u_{xx} = 0$
$v_t + (uv)_x + u_{xxx} - v_{xx} = 0$ (22)

Using the traveling wave transformationsin (*8*) and (*9*), the nonlinear system of partialdifferential equations (*22*) is carried to a system of ordinary differential equations:

$-k\lambda U' + kUU' + kV' + k^2U'' = 0$
$-k\lambda V' + kVU' + kUV' + k^3U''' - k^2V'' = 0$ (23)

By postulating tanh series, and using the transformations given by (*9*), the first equation in (*23*) reduces to

$-\lambda(1 - Y^2)\frac{dU}{dY} + U(1 - Y^2)\frac{dU}{dY} + (1 - Y^2)\frac{dV}{dY}$
$+k(1 - Y^2)^2\frac{d^2U}{dY^2} - 2kY(1 - Y^2)\frac{dU}{dY} = 0$ (24)

while the second equation in (23) reduces to

$-\lambda(1 - Y^2)\frac{dV}{dY} + V(1 - Y^2)\frac{dU}{dY} + U(1 - Y^2)\frac{dV}{dY} + 2k^2(3Y^2 - 1)(1 - Y^2)\frac{dU}{dY}$
$-6k^2Y(1 - Y^2)^2\frac{d^2U}{dY^2} + k^2(1 - Y^2)^3\frac{d^3U}{dY^3} - k(1 - Y^2)^2\frac{d^2V}{dY^2}$
$+2kY(1 - Y^2)\frac{dV}{dY} = 0$ (25)

Now, to determine the parameter $m$and n, we find$n = 2, m = 1$.

The tanh method admits the use of the finite expansion for both:

$u(x,t) = U(Y) = a_0 + a_1Y$, $a_1 \neq 0$ (26)
$v(x,t) = V(Y) = b_0 + b_1Y + b_2Y^2$, $b_2 \neq 0$ (27)

Substituting $U, U', U'', V'$from equation(26)and (27)into equations(24), then equating the coefficient of $Y^i, i = 0,1,2,3$leads to the following nonlinear system of algebraic equations:

$Y^0: -\lambda a_1 + a_0a_1 + b_1 = 0$
$Y^1: a_1^2 - 2a_1k + 2b_2 = 0$
$Y^2: \lambda a_1 - a_0a_1 - b_1 = 0$ (28)
$Y^3: a_1^2 - 2a_1k + 2b_2 = 0$

Substituting $U, U', U'', U''', V, V', V''$from equation(26)and (27)into equations(25), then equating the coefficient of $Y^i, i = 0,1,2,3,4$leads to the following nonlinear system of algebraic equations:

$Y^0: -\lambda b_1 + a_1b_0 + a_0b_1 - 2k^2a_1 - 2kb_2 = 0$
$Y^1: -2\lambda b_2 + 2a_1b_1 + 2a_0b_2 + 2kb_1 = 0$
$Y^2: \lambda b_1 + 3a_1b_2 - a_1b_0 - a_0b_1 + 8k^2a_1 + 8kb_2 = 0$
$Y^3: 2\lambda b_2 - 2a_1b_1 - 2a_0b_2 - 2kb_1 = 0$ (29)
$Y^4: -3a_1b_2 - 6k^2a_1 - 6kb_2 = 0$

Solving the nonlinear systems of equations (28) and (29) we get two solution sets

$a_0 = \lambda$, $a_1 = \sqrt{8}k$
$b_0 = 4k^2 - \sqrt{8}k^2$, $b_1 = 0$, $b_2 = \sqrt{8}k^2 - 4k^2$ (30)

and

$a_0 = \lambda \quad , \quad a_1 = -\sqrt{8}k$
$b_0 = 4k^2 + \sqrt{8}k^2 \ , \ b_1 = 0 \ , \ b_2 = -\sqrt{8}k^2 - 4k^2$ (31)

From (30) we get the following solution of the system(22):

$u(x,t) = \lambda + \sqrt{8}k tanh(kx - k\lambda t),$
$v(x,t) = (4k^2 - \sqrt{8}k^2) sech^2(kx - k\lambda t)$ (32)

And from (31) we get the following solution of the system(22):

$u(x,t) = \lambda - \sqrt{8}k tanh(kx - k\lambda t),$
$v(x,t) = (4k^2 + \sqrt{8}k^2) sech^2(kx - k\lambda t)$ (33)

4-Conclusion:

In this study, we applied tanh method for obtaining the anyltical solutionsof coupled Drinfeld-Sokolov-Satsuma-Hirota system andWhitham-BroerKaup equation. Theresults show that this method is a powerful and effective mathematical tool for solving nonlinear evolution equations in science and engineering.The applied methods will be used in further works to establish more entirely new solutions for other kinds of nonlinear equations.

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