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Energy Dissipation of Skimming Flow on Flat and Pooled Stepped Spillways

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ABSTRACT

Background: The dissipation energy of skimming flow, has been investigated by many researchers. But general agreement concerning experimental result has not been achieved. Objective: In this paper, the dissipation energy of skimming flow on flat and pooled stepped spillways are investigated with slopes of 45˚and 30˚ for each number of steps 40 and 20. The experiments were conducted for ten discharges run ranging from 3.457 to 30.669 l/s. The focus of the study is the relationship between relative energy dissipation in skimming flow performance and critical flow depth on various slope and surface roughness. Results: The effect of spillway slopeon skimming flow is evident when the relative energy loss decreases with increasing the slope. In addition, the relative energy loss of flow on pooled steps are dissipate more energy than the flat steps. Conclusion: The dissipation energy are strongly influenced by the number of steps, the slope of the stepped spillways, the critical depth, and roughness of surface.

INTRODUCTION

A hydraulic structure such spillway must be designed to overflow discharge water safely, avoiding any damage to the structure and surroundings. The reduction of kinetic energy of water through transformation to heat, so that the size of the stilling basin can be reduced is the advantages of stepped spillway. Stepped chutes are mostly equipped with flat horizontal steps of uniform step height. In some cases, the steps may be equipped with a sill at the step edge, creating a pool (Felder & Chanson, 2013).

Each step in a pooled stepped corresponds to on isolated stilling basin for hydraulic jump. On stepped spillway the water falls from step to step and the optimal dissipation is expected if a hydraulic jump builds. For steeper stepped spillway and large discharges, so called skimming flow occurs in which the step acts as roughness element and causes a strong mixing of the water with air. The stepped profile allows an increased rate of energy dissipation on the spillway chute (Chanson and Toombes, 2001; Takahashi *et al*., 2004) and the design engineers must assess accurately the turbulent kinetic energy dissipation down the staircase chute, in particular for the large discharges per unit width corresponding to the skimming flow regime.

Experiments on pooled stepped spillways were conducted in the last decade to provide a better understanding of the flow properties and energy dissipation perfomances, namely Kökpinar (2004); Barani *et al*. (2005); Chinnarasri and Wongwises (2006); Thornwarth (2008); Felder and Chanson, (2013); Guenther *et al*. (2013).

Thornwarth (2008) investigated flow instabilities on pooled stepped chutes with flat slopes of 8.9˚ and 14.6˚, which might cause a risk for safety of dam. Guenther *et al*. (2013)studied the air-water flow patterns and energy dissipation on a 26.6˚ slope stepped chutes with in-line and staggered configuration of flat and pooled

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steps. Their result data showed that the residual energy was the lowest for the flat steps. The energy dissipation on the spillway with flat horizontal steps compared to all the other configurations in the skimming flow regime. It is consistent with the re-analysis of physical data down a 30˚ stepped chutes (Kökpinar, 2004), despite some quantitative difference in residual head level.

Chinnarasri and Wongwises(2006) observed a comparatively smaller rate of energy dissipation on the flat stepped chute than on steps with end sill for chute slope 45˚. Barani *et al*. (2005) investigated the energy dissipation of flow over the stepped spillway model with 41.41˚ slope and 21 steps. Comparison of flow energy dissipation over pooled stepped spillway has been dissipated more energy than flat stepped spillway. To find the best energy dissipation efficiency for flow on stepped spillways, the objective of the study is to compare the relative energy dissipation among flat stepped spillway and pooled stepped spillway. The effect of step geometry, number of steps and relative energy loss are presented and discussed. Experiments on physical model were conducted in the last decade to provide a better understanding of the characteristic flow and energy dissipation performances.

In skimming flow regime, in addition to the recirculation between the main flow and the water trapped on the steps, large vortex elements are continuously produced that break off and get carried into the skimming flow. Chanson (1994b) expressed the head loss may be written in term of the friction factor (f), the spillway slope (θ), the critical depth (y_c) and the dam height (H_{dam}):

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\left(\frac{f}{8\sin\theta}\right)^{\frac{1}{3}}\cos\theta + \frac{1}{2}\left(\frac{f}{8\sin\theta}\right)^{-\frac{2}{3}}}{\frac{3}{2} + \frac{H_{dam}}{y_c}} \quad (1)$$

Where $H_{max} = H_{dam} + \frac{3}{2} y_c$ is the maximum head available (m).

Yasuda et.al (2001) have derived a general expression for the energy loss at the downstream end of the spillway with skimming flow, regardless of whether the flow is uniform or not:

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\left(\frac{y_w}{y_c}\right)^{-2} + 2\left(\frac{y_w}{y_c}\right)\cos\theta}{3 + 2\left(\frac{H_{dam}}{y_c}\right)} \quad (2)$$

Where y_w is the clear water depth of flow referred to the pseudo-bottom. The research of Yasuda *et al*. (2001) conducted on stepped spillway model with slopes of5.7˚, 8.5˚, 11.3˚, 19˚, 23˚, 30˚and55˚.

Boes and Hager (2003a,b) with estimation suggested by Chanson (1994b) provides the following equation to determine the head loss in the gradually-varied skimming flow region for slopes 19˚ ≤ θ ≤ 55˚:

$$\frac{\Delta H}{H_{max}} = 1 - \left\{\exp\left[\left(-0.045\left(\frac{k_s}{D_{hwu}}\right)^{0.1}(\sin\theta)^{-0.8}\right)\frac{H_{dam}}{y_c}\right]\right\} \quad \text{for } \frac{H_{dam}}{y_c} < 15-20 \quad (3)$$

$$\frac{\Delta H}{H_{max}} = 1 - \frac{\left[\left(\frac{f}{8\sin\theta}\right)^{1/3}\cos\theta + \frac{\alpha}{2}\left(\frac{f}{8\sin\theta}\right)^{-2/3}\right]}{\frac{H_{dam}}{y_c} + \left[\left(\frac{f}{8\sin\theta}\right)^{1/3}\cos\theta + \frac{\alpha}{2}\left(\frac{f}{8\sin\theta}\right)^{-2/3}\right]} \quad \text{for } \frac{H_{dam}}{y_c} \geq 15-20 \quad (4)$$

Where k_s = h cos θ, D_{hwu} is the clear water depth of flow at the location concerned, and f is Darcy – Weisbach friction factor.

In the present study, the dissipation energy on flat and pooled stepped spillways with a slopes of 45˚ and 30˚, number of step 20 and 40 were investigated with a ratio of critical depth to step height $y_c/h > 0.70$ for h/l = 1.00, respectively. The results emphasize the dissipation energy on flat and pooled steps of skimming flow condition with configuration different of slopes and number of steps.

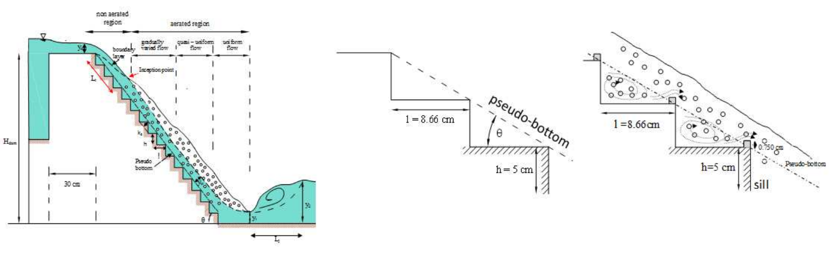

Fig. 1: Definition sketch: (a) stepped channel used for: 30˚, 45˚ ; (b) flat steps with slopes of 30˚, N =20 ; (c) pooled steps with slopes of 30˚, N=20.

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MATERIALS AND METHODS

All experiments were conducted in a prismatic rectangular channel (flume) of 7 m length and 0.5 m width. The tests were carried out in a recirculating flume located at the hydraulic laboratory of Water Resources Engineering Department, Brawijaya University, Indonesia.

The stepped spillways are made of acrylic having thickness of 0.01 m and side walls with height of 0.6 m. The slope of the stepped spillway (θ) is 45˚ and 30˚ with number of steps 20 and 40, respectively. Two step heights h = 25 and 50 mm for flat steps. The characteristic height (w) used have been 7.5 x 7.5 mm^2 for number of step 20 and 3.75 x 3.75 mm^2 for number of step 40. The discharge varied from 3.457 – 30.669 l/s, and was measured by the Rehbock weir tank. Water was pumped from reservoir to upstream tank and flow to the Rehbock measurement and water entered the stepped channel through stilling tank. Schematic of experimental apparatus is shown in Fig. 2.

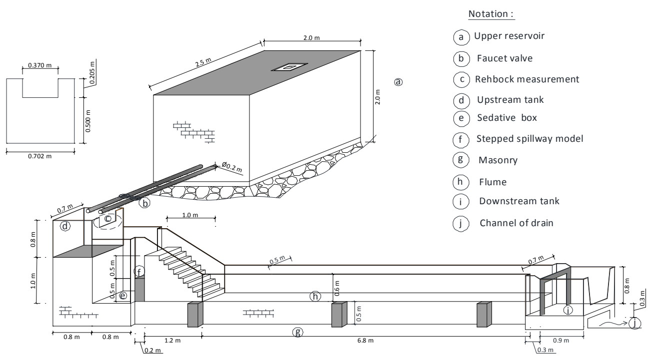

Fig. 2: Experimental setup for stepped spillway facility.

Two types of step were tested in the study, that is, flat and pooled steps. The dimensions of the step can be defined as h/l, where h = step height and l = horizontal length. The characteristic height of end sill or pool weir height (w) were 7.5 mm for number of step (N) 20 and 3.75 mm for number of step (N) 40. Further details a sketch of the stepped configurations is provided in Fig. 3.

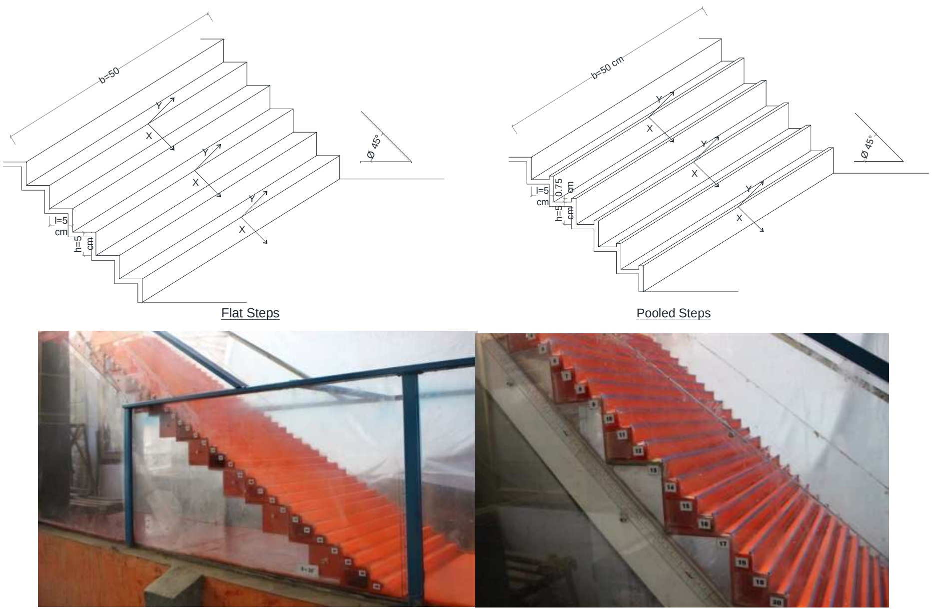

Fig. 3: Sketch of the stepped configuration (a) flat steps and (b) pooled steps.

The water discharge was measured by a rectangular – weir with Rehbock equation. A point gauge was utilized to measure water depth across the channel width. The velocity of flow was measured by two methods, first by a pitot tube and second by calculating discharge flow on the Rehbock measurement. The pitot tube was also used by Chanson *et al*. (2002) as an indirect method to measure the skimming flow depth in stepped spillways. The measured critical depth was compared with analysis from calculation of Rehbock formulas. The result showed relative error between both methods was under 10%.In calculating the relative energy loss, the velocity obtained from the first method. Details of the experiments are summarized in Table 1 to 2.

Table 1: Detail of experimental for the number of steps 40.

		y_c/h	0.700	0.740	0.800	0.850	0.875	1.400	1.600	1.700	2.800	3.000
		H_{dam}/y_c	57.143	54.054	50.000	47.059	45.725	28.571	25.000	23.529	14.286	13.333
		Q (l/s)	3.457	3.757	4.223	4.625	4.829	9.777	11.945	13.083	27.654	30.669
		v (m/s)	0.395	0.406	0.422	0.435	0.442	0.559	0.597	0.616	0.790	0.818
Configuration		Re	1.98E+05	2.03E+05	2.11E+05	2.18E+05	2.21E+05	2.79E+05	2.99E+05	3.08E+05	3.95E+05	4.09E+05
45˚	Flat step	$\frac{\Delta H}{H_{max}}$	0.891	0.881	0.871	0.865	0.859	0.853	0.833	0.852	0.709	0.674
	Pooled step		0.896	0.887	0.902	0.898	0.908	0.905	0.880	0.862	0.800	0.778
30˚	Flat step		0.968	0.966	0.961	0.960	0.962	0.900	0.890	0.891	0.819	0.818
	Pooled step		0.970	0.970	0.967	0.967	0.968	0.942	0.927	0.922	0.845	0.835

Table 2: Detail of experimental for the number of steps 20.

		y_c/h	0.700	0.750	0.800	0.825	0.850	0.875	1.400	1.500
		H_{dam}/y_c	28.571	26.667	25.000	24.242	23.529	22.857	14.286	13.333
		Q (l/s)	9.777	10.843	11.945	12.510	13.083	13.664	27.654	30.669
		v (m/s)	0.559	0.578	0.597	0.607	0.616	0.625	0.790	0.818
Configuration		Re	2.79E+05	2.89E+05	2.99E+05	3.03E+05	3.08E+05	3.12E+05	3.95E+05	4.09E+05
45˚	Flat step	$\frac{\Delta H}{H_{max}}$	0.856	0.847	0.845	0.846	0.803	0.794	0.747	0.733
	Pooled step		0.883	0.885	0.886	0.881	0.877	0.873	0.862	0.835
30˚	Flat step		0.900	0.898	0.898	0.895	0.894	0.891	0.822	0.821
	Pooled step		0.926	0.928	0.921	0.920	0.916	0.914	0.879	0.863

RESULTS AND DISCUSSION

In skimming flow, most energy is dissipated to maintain stable horizontal vortices beneath the pseudo-bottom formed by the external edges of the steps. In this study, the stepped model showed two typical flow pattern: transition flow ($0.5 < y_c/h < 0.8$) and skimming flow ($y_c/h > 0.8$) regimes depending upon the dimensionless flow rate y_c/h. The limit of value was comparable with earlier studies of conducted by Rajaratnam (1990), that defined the onset of skimming flow for values of the ratio $y_c/h = 0.8$. The flow over on flat stepped spillway with a slopes of 30˚ for conditions of transition flow (N=20) and skimming flow (N=40) illustrated in Fig. 4. The flow over on pooled stepped spillway with different type of number of step, 20 and 40 are shown in Fig. 5.

(a) (b)

Fig. 4: The flat stepped spillway (a) 20 steps and (b) 40 steps (θ =30˚, y_c = 3.50 cm, Q = 9.777 l/s)

(a) (b)

Fig. 5: The pooled stepped spillway (a) 20 steps, w = 0.750 cm and (b) 40 steps, w = 0.375 cm (θ =30˚, yc = 3.50 cm, Q = 9.777 l/s)

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Generally, the energy loss of flow on stepped spillway with flat step, $\Delta H/ H_{max}$, depends on the height dam (H_{dam}), the critical depth (y_c), the step heigh (h), the slope of the spillway (θ) or $\tan^{-1}$ (h/l). These variables can be expressed functionally as:

$$\Delta H = f(H_{dam}, y_c, h, \theta) \tag{5}$$

Using the Langhaar matrix, the variables in eq. (5) can be expressed in nondimensional form as:

$$\frac{\Delta H}{H_{max}} = f\left(\frac{H_{dam}}{y_c}, \frac{y_c}{h}, \theta\right) \tag{6}$$

where $\frac{\Delta H}{H_{max}}$ is the relative energy loss. The relationship between the relative energy loss and ratio critical depth to step height (y_c/h) are shown in Fig. 6. Another approach to estimate the energy loss on chutes is the comparison between the relative energy loss and the ratio of the critical flow depth to the step height (y_c/h) presented by Christodolou (1993) and Chanson (1993c).

The increase of discharge in flat steps or pooled steps, causing reduction of the amount of the energy dissipation. As shown on Fig. 6, the dissipated energy of flow over the pooled steps (N = 40) is more than the flat steps for slopes of 30˚ and 45˚, respectively. In Fig. 7, the relative energy loss obtained from the test for slopes of 45˚ with the number of steps 20 and 40, have been compared with another researcher.

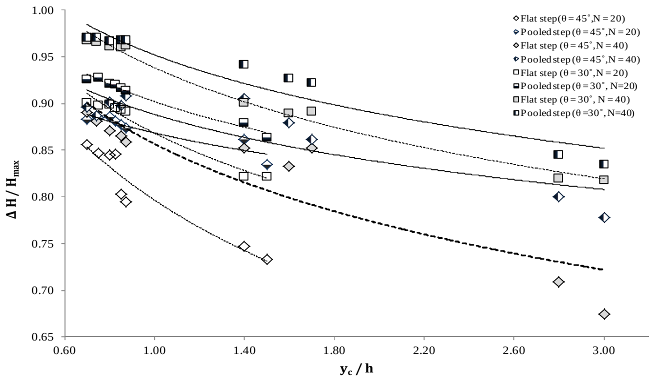

Fig. 6: Comparison of the relative energy losses functions of y_c/h from model test for different of slopes and number of step.

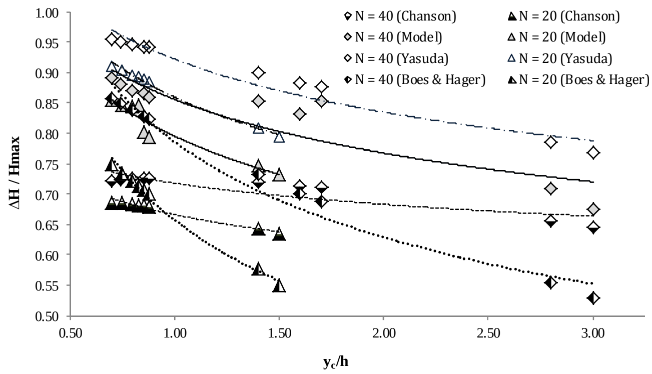

Fig. 7: Energy dissipation on flat stepped spillway (θ=45˚) as a function of (y_c/h): comparison between the equation (1)-(4) from Chanson (1994b), Yasuda (2001), Boes and Hager (2003a,b).

On the Fig. 6 of the energy dissipation in terms of y_c/h for number steps 20 and 40 have also been implied by using sill of the edge of the steps, then the amount of the relative energy loss would be increased. The water drop from the top of the endsill to the surface of step is higher and the impacted jet is deflected more, which creates an additional energy loss. It should be noted that the product obtained from this experiment is comparable with the results presented by André and Schleiss, (2004); Barani *et al*. (2005); Chinnarasri and Wongwises (2006); Hamedi *et al*. (2011); Felder and Chanson (2014).

Fig. 7 shows that the physical model in this study had a trend patterns that are close with the use of the formula of previous researchers. The results of calculations using the equations of previous researchers is tested of calibration to get the value of the relative error of each of the researcher formula. The relative error using calibration coefficients that have been obtained in the equation of each of the researcher is shown in Table 3.

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On the Table 3, the smallest of relative error that is closest to the model of this study on the formula of Ohtsu and Yasuda with value of range 1-2%.

Table 3: The relative error for calculation $\Delta H / H_{max}$ on the formula of previous researcher.

Slope	Configuration	The number of Step	Relative error (%) for $\Delta H/H_{max}$ Chanson	Othsu & Yasuda	Boes & Hager
45˚	Flat step	40	3.508	2.147	8.799
		20	3.082	1.643	4.652
	Pooled step	40	5.581	1.697	13.523
		20	3.722	1.856	7.692
30˚	Flat step	40	1.122	1.674	6.729
		20	1.226	0.487	5.310
	Pooled step	40	2.310	1.751	6.201
		20	2.310	1.751	6.201

Results show that the number of step, chute angles, the critical depth and addition sill on the edge of the step are effective on the rate of the relative energy loss. The empirical correlation for the relative energy loss on stepped spillway is obtained by fitting the experimental data in logarithmic regression form as follows:

a) θ = 45˚, flat steps

$$\frac{\Delta H}{H_{max}} = 0.8568 - 0.123 \ln(\frac{y_c}{h}) \qquad N=40 \qquad (8)$$

$$\frac{\Delta H}{H_{max}} = 0.7965 - 0.16 \ln(\frac{y_c}{h}) \qquad N=20 \qquad (9)$$

b) θ = 45˚, pooled steps

$$\frac{\Delta H}{H_{max}} = 0.8883 - 0.073 \ln(\frac{y_c}{h}) \qquad N=40 \qquad (10)$$

$$\frac{\Delta H}{H_{max}} = 0.8684 - 0.056 \ln(\frac{y_c}{h}) \qquad N=20 \qquad (11)$$

c) θ = 30˚, flat steps

$$\frac{\Delta H}{H_{max}} = 0.9383 - 0.108 \ln(\frac{y_c}{h}) \qquad N=40 \qquad (12)$$

$$\frac{\Delta H}{H_{max}} = 0.8682 - 0.119 \ln(\frac{y_c}{h}) \qquad N=20 \qquad (13)$$

d) θ = 30˚, pooled steps

$$\frac{\Delta H}{H_{max}} = 0.952 - 0.091 \ln(\frac{y_c}{h}) \qquad N=40 \qquad (14)$$

$$\frac{\Delta H}{H_{max}} = 0.9021 - 0.082 \ln(\frac{y_c}{h}) \qquad N=20 \qquad (15)$$

In both chutes of slopes 45˚ and 30˚ in skimming flow conditions, the design of a stepped spillway with slopes of 30˚ is a very efficient to dissipate a large part of the flow energy along the spillway than slopes of 45˚. It is shown with Eq. (12)-(13) have coefficient is larger than Eq. (8)-(9) for spillway model with flat steps and the number of step 20 and 40, respectively. Flow condition above a stepped spillway with a certain slope is affected also by the air entrainment. This result is comparable with the result of experiment by Chanson (1993c).

In Fig. 7 above, the pooled steps has dissipater more energy than flat steps because the characteristic height of the end sill increases the volume of trapped water. Therefore, the circulating vortex was greatest compared to flat steps. The shapes of the end sills have been formed rectangular. In the nappe flow, the characteristic height does not much effect the relative energy loss because most energy loss is due to the occurrence of the hydraulic jump and impact of the jet on the step face. In transition flow, the characteristic height has higher influence on the relative energy loss than in nappe flow. In the skimming flow, the effect of characteristic height is clearly observed.

Despite increases in the energy loss on pooled steps, but the use of this configuration in field has to be observed deeply if it does not provide any advantages significantly in the energy dissipation.

The correlation on Equation (8) -(15) may be used in design practice for initial estimates of the relative energy loss with boundary condition: $0.700 < y_c/h < 3.00$ and θ = 30˚ - 45˚.

Conclusions:

Result obtained from this experiments show that in the case of the number of steps increases, the relative energy loss is increased. This is due to the fact that steps act as macro-roughness that increase friction and then change the kinetic energy into thermal energy. Despite the stepped spillway with a slopes of 30˚ give a better energy loss than a slopes of 45˚ for the same number steps in skimming flow condition. Second, the relative energy loss will decrease with increasing relative critical flow depth. Third, the relative energy loss of flow on pooled steps are dissipate more energy than the flat steps. The characteristic height of the end sill influences the relative energy loss on the pooled steps.

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