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Bayes Estimators For The Maxwell Distribution Under Quadratic Loss Function Using Different Priors

[1]Huda, A. Rasheed and [2]Zainab, N. Khalifa

[1]*AL-Mustansiriyah University Collage of Science, Dept. of Math.*
[2]*AL-Mustansiriyah University Collage of Science, Dept. of Math.*

Address For Correspondence:
Huda, A. Rasheed, AL-Mustansiriyah University Collage of Science, Dept. of Math.

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ABSTRACT

For estimating an unknown parameter for the Maxwell distribution, we obtained some Bayes estimators under Quadratic loss function using Non-informative prior, represented by Jefferys prior and Informative priors as Gumbel Type II and Conjugate (Inverted Gamma and Inverted Levy) priors. All these estimators compared with Maximum likelihood estimator. According to Monte-Carlo simulation study, the performance of these estimates is compared depending on the mean squared errors (MSE's) and reached to, the Bayes estimator under Inverted Gamma prior is the best estimator, comparing to other priors.

INTRODUCTION

Maxwell distribution was first introduced by J. C. Maxwell (1860) and then described by Boltzmann (1870) with a few assumptions.

Maxwell distribution plays an important role in Physics and chemistry. It gives the distribution of speeds of molecules in thermal equilibrium as given by statistical mechanics. For example, this distribution explains many fundamental gas properties in kinetic theory of gases, distribution of energies and moments,…etc.

The Bayesian deduction requires appropriate choice of priors for the parameters. In the last several decades, Bayesian analysis focused on priors that are un-informative. But if we have enough information about the parameter, then it is better to make use of the informative priors. The parameters of the prior distribution are called hyper-parameters.

Maxwell distribution (2008), and Maxwell-Boltzmann (2008) are giving a summary of this applications. In (2005) Bekker and Roux, studied empirical Bayes estimation for Maxwell distribution, and we have assumed that complete sample information is available, Sanku Dey (2011) studies on Bayes estimators of the parameter of a Maxwell distribution and obtain associated based on conjugate prior under scale invariant symmetric and a symmetric loss functions. Huda, A. Rashed (2013) derived Minimax estimation of the parameter of the Maxwell distribution under Quadratic loss function.

Model Description (Maxwell–Boltzmann, 2008; Maxwell distribution, 2008):

The Maxwell (or Maxwell – Boltzmann) distribution gives the distribution of speeds of molecules in thermal equilibrium as given by statistical mechanics.

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Defining $\theta = \frac{2KT}{m}$, where K is the Maxwell constant, T is temperature, m is the mass of a molecule. The probability density function and the cumulative distribution function of Maxwell distribution over the rang x∈ $[0,\infty)$ are given by :

$$f((x|\theta) = \frac{4}{\sqrt{\pi}}\frac{1}{\theta^{3/2}}x^2 e^{-\frac{x^2}{\theta}} \quad ; \quad 0 < X, \theta \tag{1}$$

$$F(x)= \frac{1}{\Gamma(\frac{1}{2})}\Gamma(\frac{x^2}{\theta},\frac{3}{2}) \tag{2}$$

Where $\Gamma(x\ ,\alpha)= \int_0^x e^{-u}\, u^{\alpha-1}\, du$ is the incomplete gamma function (Krishna Hare and Malik Manish, 2008).

It can also be expressed as follows

$$F(x;\theta) = 2erf\left(\frac{x}{\sqrt{\theta}}\right) - \frac{2}{\sqrt{\pi}}\frac{x}{\theta}e^{-\frac{x^2}{\theta}}$$

Where $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-w^2}$ dw ,is the error function.

Properties of Maxwell distribution[7]:

We can summarize the most important properties by the following points:

1- The n^{th} row moment is:

$$\acute{\mu}_n = \frac{2}{\sqrt{\pi}}\Gamma\left(\frac{n+3}{2}\right)\theta^{\frac{n}{2}}\,; n > -3$$

So that:

If n = 1 ,then $\acute{\mu}_1 = 2\sqrt{\frac{\theta}{\pi}}$, and if n=2 ,then $\acute{\mu}_2 = 3\sqrt{\frac{\theta}{\pi}}$;..., etc.

2- Mean=$2\sqrt{\frac{\theta}{\pi}}$

3- Variance =$\frac{\theta}{2\pi}(3\pi - 8)$

4- Mode = $\sqrt{\theta}$

5- Median=$\frac{1}{3}(Mode + 2\ Mean)$

$=\frac{1}{3}\sqrt{\frac{\theta}{\pi}}\ (\sqrt{\pi}+4)$

$=1.0856\sqrt{\pi}$

6- R(t)=P(x>t)=$\int_t^\infty \frac{4}{\sqrt{\pi}}\ \frac{1}{\theta^{\frac{3}{2}}}\ x^2\, e^{-\frac{x^2}{\theta}}\, dx$

$=\frac{4}{\sqrt{\pi}}\ \frac{1}{\theta^{\frac{3}{2}}}\ J(t,2,\theta);\ t > 0$

Where $J(t,2,\theta) = \int_t^\infty e^{-\frac{w^2}{\theta}} w^k\, dw$, is the Jacobian function

7- $h(t) = \frac{f(t)}{R(t)} = \frac{t^2\, e^{-\frac{t^2}{\theta}}}{J(t,2,\theta)}$

Estimation of Parameter:

In this section we can used two methods to estimation parameter θ

1.Maximum likelihood estimation (Rasheed, H.A., 2013):

We introduce the concept of maximum likelihood estimation with Maxwell distribution. Let n items have an independent and identically distributed, then the likelihood of the sample from Maxwell distribution with parameter θ is given by:

$$L\left(x_{i;}\theta\right) = \pi_{i=1}^n f(x_i;\theta) = \left(\frac{4}{\sqrt{\pi}}\right)^n \frac{1}{\theta^{\frac{3n}{2}}}(\pi_{i=1}^n x_i^2)\, e^{-\frac{\sum_{i=1}^n x_i^2}{\theta}}$$

From which we calculate the log-likelihood function:

$$ln\ L(x_i;\theta) = nln\left(\frac{4}{\sqrt{\pi}}\right) + \ln 1 - \frac{3n}{2}ln\theta + 2\sum_{i=1}^n lnx_i - \frac{\sum_{i=1}^n x_i^2}{\theta} \tag{3}$$

Now, differentiating partially equation(3)with respect to θ:

$$\frac{\partial lnL(x_i;\theta)}{\partial\theta} = \frac{-3n}{2\theta} + \frac{\sum_{i=1}^n x_i^2}{\theta^2} \tag{4}$$

Then, the MLE of θ is the solution of equation (1) after equating the first derivative to zero. Hence:

$$\hat{\theta}_{ML}=\frac{2\sum_{i=1}^{n}x_i^2}{3n}=\frac{2T}{3n}, \text{ where } T=\sum_{i=1}^{n}x_i^2 \quad (5)$$

2. Bayes Estimators:

Let $x_1, x_2, \dots, x_n$ be a random sample of size n with probability density function given in equation (1) and likelihood function given in equation (5), then, the Bayes estimators of the parameter $\boldsymbol{\theta}$ under different prior distributions which is mentioned below, can be obtained as follows:

2.1. Jefferys Prior Information (Al-Baldawi, T., H.K, 2013):

Assume that, the unknown scale parameter θ has no-information prior density defined as using Jefferys prior information $g(\theta)$ which is derived to be

$$g_1 \propto \sqrt{I(\theta)} \quad (6)$$

Where $I(\theta) = -nE\left(\frac{\partial^2 \ln f(x;\theta)}{\partial\theta^2}\right)$ is the Fisher's information matrix.

Hence,

$$g_1(\theta) = k\sqrt{-nE\left(\frac{\partial^2 \ln f(x;a,\theta)}{\partial\theta^2}\right)} \quad (7)$$

With k is a constant.

By taking the logarithm for distribution and taking the partial derivative with respect to θ, we get:

$$\frac{\partial \ln f(x|\theta)}{\partial\theta} = -\frac{3}{2\theta} + \frac{x^2}{\theta^2}$$

$$\frac{\partial^2 \ln f(x|\theta)}{\partial\theta^2} = \frac{3}{2\theta^2} - \frac{2x^2}{\theta^3}$$

$$E\left(\frac{\partial^2 \ln f(x|\theta)}{\partial\theta^2}\right) = E\left(\frac{3}{2\theta^2}\right) - E\left(\frac{2x^2}{\theta^3}\right)$$

$$= \frac{3}{2\theta^2} - \frac{2}{\theta^3}E(x^2) = \frac{-3}{2\theta^2} \quad (8)$$

After Substitution (8) into(6), we get

$$g_1(\theta) = k\sqrt{-n\left(-\frac{3}{2\theta^2}\right)} = \frac{k}{\theta}\sqrt{\frac{3n}{2}} \quad (9)$$

Now, combining the prior (9) with the likelihood function(5), we have the posterior distribution of θ with Jefferys prior information which is given by :

$$h_1(\theta|\underline{x}) = \frac{g_1(\theta)L(\theta; x_1x_2 \dots\dots.. x_n)}{\int_0^\infty g_1(\theta)L(\theta; x_1x_2 \dots\dots.. x_n)d\theta}$$

$$h_1(\theta|\underline{x}) = \frac{\frac{1}{\theta^{(\frac{3n}{2}+1)}} exp\left(-\frac{\sum_{i=1}^{n}x^2}{\theta}\right)}{\int_0^\infty \frac{1}{\theta^{(\frac{3n}{2}+1)}} exp\left(-\frac{\sum_{i=1}^{n}x^2}{\theta}\right)d\theta} \quad (10)$$

On Simplification ,we have

$$h_1(\theta|\underline{x}) = \frac{(\sum_{i=1}^{n}x^2)^{\frac{3n}{2}} e^{-\sum_{i=1}^{n}x^2/\theta}}{\theta^{(\frac{3n}{2}+1)}\Gamma(\frac{3n}{2})} = \frac{T^{\frac{3n}{2}} e^{-\frac{T}{\theta}}}{\theta^{(\frac{3n}{2}+1)}\Gamma(\frac{3n}{2})} \quad (11)$$

This posterior density is recognized as the density of the Inverse Gamma distribution, i.e.:

$\theta|\underline{x} \sim IG\left(\frac{3n}{2}, T\right)$

Where , $E(\theta|\underline{x})=\frac{T}{(\frac{3n}{2}-1)}$, $Var(\theta|\underline{x})\ \frac{T^2}{\left(\frac{3n}{2}-1\right)^2\left(\frac{3n}{2}-2\right)}$, $n > 1$

2.2. Gumbel Type II Prior Information:

Using Gumbel type II prior with hyper parameter (b), which is defined as (Ali, S., *et al.*, 2012; Rasheed, H.A. and E.F. Ai-Shareefi, 2015),

$$g_2(\theta) = b\left(\frac{1}{\theta}\right)^2 Exp\left[\frac{-b}{\theta}\right] \quad b, \theta > 0 \quad (12)$$

Now, Combining the prior (12) with the likelihood function (5) yields the posterior distribution of θ with Gumbel Type II prior distribution information denoted by $h_2(\theta|\underline{x})$, where:

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$$h_2(\theta|\underline{x}) = \frac{\frac{1}{\theta^{(\frac{3n}{2}+2)}} e^{-(\sum_{i=1}^{n} x^2+b)/\theta}}{\int_0^\infty \frac{1}{\theta^{(\frac{3n}{2}+2)}} e^{-(\sum_{i=1}^{n} x^2+b)/\theta} d\theta} \tag{13}$$

After simplification, we get:

$$h_2(\theta|\underline{x}) = \frac{(\sum_{i=1}^{n} x^2+b)^{\frac{3n}{2}+1} e^{-(\sum_{i=1}^{n} x^2+b)/\theta}}{\theta^{(\frac{3n}{2}+2)}\,\Gamma(\frac{3n}{2}+1)} = \frac{(T+b)^{\frac{3n}{2}+1} e^{-\frac{(T+b)}{\theta}}}{\theta^{(\frac{3n}{2}+2)}\,\Gamma(\frac{3n}{2}+1)} \tag{14}$$

This posterior is recognized as the density of the Inverse Gamma distribution, i.e.:

$\theta|\underline{x} \sim IG\ (\frac{3n}{2}+1, \boldsymbol{T+b})$

Where, $E(\theta|\underline{x}) = \frac{T+b}{\frac{3n}{2}}$, $Var(\theta|\underline{x}) = \frac{(T+b)^2}{(\frac{3n}{2})^2(\frac{3n}{2}-1)}$, $n > 1$

2.3. Inverted Gamma Prior information (Al-Baldawi, T., H.K, 2015):

This conjugate prior distribution is the distribution of the reciprocal of a variable distributed according to Gamma distribution that is assumed to be:

$$g_3(\theta) = \frac{\alpha^\beta}{\Gamma\beta}\frac{1}{\theta^{\beta+1}} e^{-\alpha/\theta}\ ,\ \alpha,\beta,\ \theta > 0 \tag{15}$$

With scale parameter α and shape parameter β.

The posterior distribution under the assumption of gamma prior is:

$$h_3(\theta|\underline{x}) = \frac{\frac{1}{\theta^{(\frac{3n}{2}+\beta+1)}}\, exp - \frac{(\sum_{i=1}^{n} x^2+\alpha)}{\theta}}{\int_0^\infty \frac{1}{\theta^{(\frac{3n}{2}+\beta+1)}}\, exp - \frac{(\sum_{i=1}^{n} x^2+\alpha)}{\theta} d\theta} \tag{16}$$

On Simplification , we have:

$$h_3(\theta|\underline{x}) = \frac{(\sum_{i=1}^{n} x^2+\alpha)^{\frac{3n}{2}+\beta} e^{-(\sum_{i=1}^{n} x^2+\alpha)/\theta}}{\theta^{(\frac{3n}{2}+\beta+1)}\,\Gamma(\frac{3n}{2}+\beta)} = \frac{(T+\alpha)^{\frac{3n}{2}} e^{-\frac{(T+\alpha)}{\theta}}}{\theta^{(\frac{3n}{2}+\beta+1)}\,\Gamma(\frac{3n}{2}+\beta)} \tag{17}$$

This posterior density is recognized as the density of the Inverse Gamma distribution: $\theta|\underline{x} \sim IG\ (\frac{3n}{2}+\beta, T+\alpha)$

Where, $E(\theta|\underline{x}) = \frac{T+\alpha}{\frac{3n}{2}+\beta-1}$, $Var(\theta|\underline{x}) = \frac{(T+\alpha)^2}{(\frac{3n}{2}+\beta-1)^2(\frac{3n}{2}+\beta-2)}$, $n > 1$

2.4. Inverted Levy Prior Information:

The inverted Levy prior is assumed to be (Sindhu, T.N., M. Aslam, 2013)

$$g_4(\theta) = \sqrt{\frac{\lambda}{2\pi}}\frac{1}{\sqrt{\theta}} e^{-\frac{\lambda}{2\theta}} \qquad \theta,\lambda > 0 \tag{18}$$

Where λ is the hyper parameter.

The posterior distribution of θ is given by:

$$h_4(\theta|\underline{x}) = \frac{\frac{1}{\theta^{(\frac{3n}{2}+\frac{1}{2})}}\, exp\left(-\frac{\sum_{i=1}^{n} x^2+\frac{\lambda}{2}}{\theta}\right)}{\int_0^\infty \frac{1}{\theta^{(\frac{3n}{2}+\frac{1}{2})}}\, exp\left(-\frac{\sum_{i=1}^{n} x^2+\frac{\lambda}{2}}{\theta}\right) d\theta} \tag{19}$$

On Simplification, we have

$$h_4(\theta|\underline{x}) = \frac{\left(\sum_{i=1}^{n} x^2+\frac{\lambda}{2}\right)^{\frac{3n}{2}-\frac{1}{2}} e^{-(\sum_{i=1}^{n} x^2+\frac{\lambda}{2})/\theta}}{\theta^{(\frac{3n}{2}+\frac{1}{2})}\,\Gamma(\frac{3n}{2}-\frac{1}{2})} = \frac{(T+\frac{\lambda}{2})^{\frac{3n}{2}} e^{-\frac{(T+\frac{\lambda}{2})}{\theta}}}{\theta^{(\frac{3n}{2}+\frac{1}{2})}\,\Gamma(\frac{3n}{2}-\frac{1}{2})} \tag{20}$$

Notice that, $h_4(\theta|\underline{x})$ is recognized as the density of the Inverse Gamma distribution, namely, $\theta|\underline{x} \sim IG\ (\frac{3n}{2}-\frac{1}{2}, T+\frac{\lambda}{2})$

Where, $E(\theta|\underline{x}) = \frac{T+\frac{\lambda}{2}}{(\frac{3n}{2}-\frac{3}{2})}$, $Var(\theta|\underline{x}) = \frac{\left(T+\frac{\lambda}{2}\right)^2}{(\frac{3n}{2}-\frac{3}{2})^2(\frac{3n}{2}-\frac{5}{2})}$

2.5. Bayes Estimator Under Quadratic Loss Function:

De Groote (1970) discussed different types of loss function and obtained the Bayes estimates under these loss function which is a non- negative symmetric and continuous loss function [4],[8] and is defined as :

$$L(\hat{\theta}, \theta) = \left(\frac{\hat{\theta}-\theta}{\theta}\right)^2 = \left(1-\frac{\hat{\theta}}{\theta}\right)^2 \tag{21}$$

The Risk function under the Quadratic Loss function denoted by $R_Q(\hat{\theta}, \theta)$, can be obtained as follows:

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$$R_Q(\hat{\theta}, \theta) = E\left(1 - \frac{\hat{\theta}}{\theta}\right)^2 \quad (22)$$

$$= \int_0^\infty \left(1 - \frac{\hat{\theta}}{\theta}\right)^2 h_1(\theta|\underline{x})d\theta$$

$$\frac{\partial R_Q(\hat{\theta}, \theta)}{\partial \hat{\theta}} = 2\int_0^\infty \left(1 - \frac{\hat{\theta}}{\theta}\right)\left(-\frac{1}{\theta}\right) h_1(\theta|\underline{x})d\theta$$

Let $\frac{\partial R_Q(\hat{\theta}, \theta)}{\partial \hat{\theta}} = 0$, yields:

$$\Rightarrow \hat{\theta}\int_0^\infty \frac{1}{\theta^2} h_1(\theta|\underline{x})d\theta - \int_0^\infty \frac{1}{\theta} h_1(\theta|\underline{x})d\theta = 0$$

Hence, Bayes estimator under the Quadratic loss function will be:

$$\hat{\theta}_Q = \frac{E\left(\frac{1}{\theta}|\underline{x}\right)}{E\left(\frac{1}{\theta^2}|\underline{x}\right)} \quad (23)$$

(*i*) *with Jefferys prior information:*

According to the posterior density function (11), the Bayes estimator of θ of Maxwell distribution under Quadratic loss function is obtained by applying (23) as follows:

$$E\left(\frac{1}{\theta^m}|\underline{x}\right) = \int_0^\infty \frac{1}{\theta^m} h_1(\theta|\underline{x})d\theta$$

$$= \int_0^\infty \frac{1}{\theta^m} \frac{(\sum_{i=1}^n x^2)^{\frac{3n}{2}}}{\Gamma\left(\frac{3n}{2}\right)\theta^{\left(\frac{3n}{2}+1\right)}} e^{\frac{-\sum_{i=1}^n x^2}{\theta}} d\theta$$

Hence,

$$E\left(\frac{1}{\theta^m}|\underline{x}\right) = \frac{\Gamma\left(\frac{3n}{2}+m\right)}{\Gamma\left(\frac{3n}{2}\right)(\sum_{i=1}^n x^2)^m}, \quad m = 0,1,2,\ldots \quad (24)$$

After substituting, we get the Bayes estimator of parameter θ under Quadratic loss function with Jefferys prior information as follows:

$$\hat{\theta}_{QJ} = \frac{\sum_{i=1}^n x^2}{\left(\frac{3n}{2}+1\right)} \quad (25)$$

(ii) *With Gumbel Type -II prior information:*

Based on the posterior density function (14), the Bayes estimator of **θ** of Maxwell distribution under Quadratic loss function is obtained as:

$$E\left(\frac{1}{\theta^m}|\underline{x}\right) = \int_0^\infty \frac{1}{\theta} h_2(\theta|\underline{x})d\theta$$

$$= \int_0^\infty \frac{1}{\theta^m} \frac{(\sum_{i=1}^n x^2+b)^{\frac{3n}{2}+1}}{\Gamma\left(\frac{3n}{2}+1\right)\theta^{\left(\frac{3n}{2}+2\right)}} e^{\frac{-(\sum_{i=1}^n x^2+b)}{\theta}} d\theta$$

Hence,

$$E\left(\frac{1}{\theta^m}|\underline{x}\right) = \frac{\Gamma\left(\frac{3n}{2}+m+1\right)}{\Gamma\left(\frac{3n}{2}+1\right)(\sum_{i=1}^n x^2+b)^m} \quad (26)$$

After substituting, we get the Bayes estimator of parameter θ under Quadratic loss function with Gumbel Type- II information as:

$$\hat{\theta}_{QGL} = \frac{(\sum_{i=1}^n x^2+b)}{\frac{3n}{2}+2} \quad (27)$$

(iii) With Inverted Gamma Prior Information:

From (17) posterior density function, the Bayes estimator of **θ** of Maxwell distribution under Quadratic loss function is obtained as:

$$E\left(\frac{1}{\theta^m}|\underline{x}\right) = \int_0^\infty \frac{1}{\theta^m} h_3(\theta|\underline{x})d\theta$$

$$= \int_0^\infty \frac{1}{\theta^m} \frac{(\sum_{i=1}^n x^2+\alpha)^{\frac{3n}{2}+\beta}}{\Gamma\left(\frac{3n}{2}+\beta\right)\theta^{\left(\frac{3n}{2}+\beta+1\right)}} e^{\frac{-(\sum_{i=1}^n x^2+\alpha)}{\theta}} d\theta$$

Hence, $E\left(\frac{1}{\theta^m}|\underline{x}\right) = \frac{\Gamma\left(\frac{3n}{2}+m+\beta\right)}{\Gamma\left(\frac{3n}{2}+\beta\right)(\sum_{i=1}^n x^2+\alpha)^m}, \quad m = 0,1,2,\ldots$ (28)

After substituting, we get the Bayes estimator of parameter $\boldsymbol{\theta}$ under Quadratic loss function with Inverted gamma prior information as follows:

$$\hat{\theta}_{QGA} = \frac{(\sum_{i=1}^{n} x^2+\alpha)}{(\frac{3n}{2}+\beta+1)} \quad (29)$$

(V) With Inverted Levy Prior Information:

From (20) posterior density function, the Bayes estimator of **θ** of Maxwell distribution under Quadratic loss function is obtained as:

$$\mathrm{E}\left(\frac{1}{\theta^{m}}|\underline{x}\right) = \int_{0}^{\infty} \frac{1}{\theta^{m}} \mathrm{h}_4(\theta|\underline{\mathrm{x}})d\theta$$

$$= \int_{0}^{\infty} \frac{1}{\theta^{m}} \frac{\left(\sum_{i=1}^{n} x^2 + \frac{\lambda}{2}\right)^{\frac{3n}{2}-\frac{1}{2}}}{\theta^{\frac{3n}{2}-\frac{1}{2}}\Gamma(\frac{3n}{2}-\frac{1}{2})} e^{\frac{-(\sum_{i=1}^{n} x^2+\frac{\lambda}{2})}{\theta}} d\theta$$

Hence,

$$\mathrm{E}\left(\frac{1}{\theta^{m}}|\underline{x}\right) = \frac{\Gamma(\frac{3n}{2}+m-\frac{1}{2})}{\Gamma(\frac{3n}{2}-\frac{1}{2})\left(\sum_{i=1}^{n} x^2+\frac{\lambda}{2}\right)^{m}} \quad , \quad m = 0, 1, 2, \ldots \quad (30)$$

After substituting ,we get the Bayes estimator of parameter $\boldsymbol{\theta}$ under Quadratic loss function with Inverted Levy prior information as follows:

$$\hat{\theta}_{QIL} = \frac{(\sum_{i=1}^{n} x^2+\frac{\lambda}{2})}{(\frac{3n}{2}+\frac{1}{2})} \quad (31)$$

4. Simulation Study:

Mean Squared Errors (MSE's), are considered to compare the different estimators of the parameter θ that obtained by the method of Maximum likelihood and Bayes Estimators for Quadratic Loss function methods. In this simulation study, the number of replication used was I = 5000 samples of sizes n = 5,10, 20, 50, 100 from the Maxwell distribution with different values of θ where, θ = 0.5, 1.5, 3.

In this section, Monte-Carlo simulation study is performed to compare the methods of estimation by using mean squared errors (MSE's) as an index for precision to compare the efficiency of each of estimators, where:

$$\mathrm{MSE}(\theta) = \frac{\sum_{i=1}^{I}(\hat{\theta}_i-\theta)^2}{I}$$

The results are summarized and tabulated in tables (1-3) which contain the expected values and (MSE's) for estimating the scale parameter θ. and we have observed that:

1- Table (1), shows that, the performance of Bayes estimator under Inverse Gamma prior when ($\theta = 0.5$) and ($\alpha = 0.9$ and $\beta = 2$) is the best estimator.

2- Table (2), shows that ,the performance of Bayes estimator under Inverted Gamma prior when ($\theta = 1.5$) and ($\alpha = 3$ and $\beta = 2$) is the best estimator.

3- Table(3), shows that ,the performance of Bayes estimator under Inverted Gamma prior when ($\theta = 3$) and ($\alpha = 3$ and β=0.9) is the best estimator.

4- For all parameter values, an obvious that, reduction in (MSE) is observed with the increase in all sample sizes. Finally ,It is observed that, (MSE) of all estimators of the scale parameter is increasing with the increase of the scale parameter value with all sample sizes.

Generally, the comparison results showed that, the performance of Bayes estimator under Quadratic loss functions based on Inverted Gamma prior is more appropriate than using other priors, on condition that, the ratio between the scale parameter α to the location parameter β is nearly, equivalent to the estimated parameter θ.

Table 1: Estimates and MSEs under different priors for different sample sizes with θ=0.5

n	Estimator / Criteria	MLE	Jefferys	Gumbel type-II		Inverted Gamma				Inverted Levy	
						$\beta = 0.9$		$\beta = 2$			
				b=0.9	b=3	$\alpha = 0.9$	$\alpha = 3$	$\alpha = 0.9$	$\alpha = 3$	λ=1	λ=2
5	EXP	0.49982	0.44102	0.48933	0.71039	0.49454	0.71795	0.44273	0.64273	0.53108	0.59358
	MSE	0.53410	0.03017	0.02148	0.06563	0.02185	0.06932	0.02077	0.03786	0.03110	0.38888
10	EXP	0.49893	0.46774	0.49317	0.61670	0.49609	0.62035	0.46577	0.58244	0.51509	0.54735
	MSE	0.51613	0.01616	0.01344	0.02701	0.01357	0.02804	0.01312	0.01875	0.01634	0.0 1836
20	EXP	0.49965	0.48353	0.49655	0.56217	0.49811	0.56395	0.48150	0.54514	0.50786	0.52425
	MSE	0.50802	0.00811	0.00737	0.01122	0.00740	0.01149	0.00726	0.00895	0.00816	0.00868
50	EXP	0.49971	0.49313	0.49842	0.52569	0.49907	0.52637	0.49203	0.51895	0.50302	0.50964
	MSE	0.50300	0.00326	0.00313	0.00379	0.00314	0.00381	0.00311	0.00341	0.00326	0.00334
100	EXP	0.49956	0.49625	0.49891	0.51272	0.49923	0.51306	0.49565	0.5094	0.50122	0.50454
	MSE	0.50120	0.00164	0.00160	0.00176	0.00161	0.00178	0.00160	0.00167	0.00164	0.00166

Table 2: Estimates and MSEs under different priors for different sample sizes with θ=1.5

n	Estimator / Criteria	MLE	Jefferys	Gumbel type-II		Inverted Gamma				Inverted Levy	
						$\beta=0.9$		$\beta=2$			
				b=0.9	b=3	$\alpha=0.9$	$\alpha=3$	$\alpha=0.9$	$\alpha=3$	λ=1	λ=2
5	EXP	1.49947	1.32306	1.27853	1.49958	1.29213	1.51553	1.15676	1.35676	1.46825	1.54075
	MSE	1.80799	0.27151	0.24135	0.19230	0.23962	0.19665	0.27522	0.17793	0.27218	0.27212
10	EXP	1.49677	1.40323	1.37363	1.49716	1.38175	1.50602	1.29731	1.41398	1.48075	1.51202
	MSE	1.65164	0.14547	0.13654	0.12058	0.13598	0.12203	0.14863	0.11494	0.14540	0.14250
20	EXP	1.49896	1.45061	1.43340	1.49902	1.43790	1.50373	1.38997	1.45360	1.49078	1.50717
	MSE	1.57427	0.07297	0.07062	0.06619	0.07046	0.06662	0.07435	0.06439	0.06909	0.07291
50	EXP	1.49913	1.47940	1.47188	1.49915	1.47379	1.50110	1.45301	1.47993	1.49582	1.50244
	MSE	1.52878	0.02930	0.28926	0.02814	0.02889	0.02821	0.02963	0.02782	0.02868	0.02927
100	EXP	1.49867	1.48875	1.48488	1.49869	1.48585	1.49968	1.47517	1.48890	1.49702	1.50034
	MSE	1.51348	0.01474	0.01465	0.00144	0.01464	0.01444	0.01485	0.01436	0.01459	0.01464

Table 3: Estimates and MSEs under different priors for different sample sizes with θ=3

n	Estimator / Criteria	MLE	Jefferys	Gumbel type-II		Inverted Gamma				Inverted Levy	
						$\beta=0.9$		$\beta=2$			
				b=0.9	b=3	$\alpha=0.9$	$\alpha=3$	$\alpha=0.9$	$\alpha=3$	λ=1	λ=2
5	EXP	2.99894	2.64613	2.46232	2.68337	2.48852	2.71192	2.22782	2.42781	2.87401	2.93651
	MSE	4.23306	1.08605	1.05829	0.86944	1.04726	0.86863	1.22593	0.95705	1.10055	1.08871
10	EXP	2.99355	2.80645	2.69431	2.81784	2.71025	2.83451	2.54463	2.66129	2.92924	2.96150
	MSE	3.61303	0.58190	0.57572	0.51545	2.71025	0.51538	0.63754	0.54490	0.58514	0.58161
20	EXP	2.99792	2.90121	2.83868	0.27392	2.84757	2.91340	2.75266	2.81630	0.29265	2.98156
	MSE	3.29916	0.29187	0.29078	2.95933	0.28965	0.27392	0.31013	0.28270	0.29265	0.29178
50	EXP	2.99825	2.95880	2.93206	2.95933	2.93588	2.96318	2.89447	2.92140	2.98502	2.99164
	MSE	3.11688	0.11722	0.11716	0.11419	0.11695	0.11419	0.12081	0.11585	0.11728	0.11713
100	EXP	2.99735	2.97750	2.96383	2.97765	2.96578	2.97961	2.94446	2.95818	2.99072	2.99404
	MSE	3.05659	0.05896	0.05995	0.05819	0.05893	0.05818	0.06002	0.05868	0.05893	0.05888

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