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Multi-Objective Optimization Indices: A Comparative Analysis

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ABSTRACT

In this paper, the multi objective optimization problem is studied further with respect to quality indices. In particular, the maximum Pareto front error, accuracy of Pareto frontier, spacing, overall spread, objective spread, maximum spread, and number of distinct choices, cluster, hyper area, dominant area, non-dominated evaluation, and crowding distances are studied. Two algorithms are employed to solve the multi objective problem, namely: the NSGA II and SPEA algorithms. These performance indices are analyzed in relation to a set of benchmark problems and conclusions are drawn. Five out of twelve indices can be eliminated.

Nomenclature:

EA: Evolutionary
MOP: Multi-objective optimizations
MOOP: Multi-objective optimization problems
VEGA: Vector Evaluated Genetic Algorithm
GA: Genetic Algorithm
WBGA: Weight-Based Genetic Algorithm
Sh: Sharing Function
Nc_i: Niche Count
MOGA: Multiple Objectives Genetic Algorithm
NSGA: Non-Dominated Sorting Genetic Algorithm
NPGA: Niched-Pareto Genetic Algorithm
SPEA: Strength Pareto Evolutionary Algorithm
NSGAII: Elitist Non-Dominated Sorting Genetic Algorithm
MOEA: Multi-objective Evolutionary Algorithm
ER: Error Ratio
GD: Generational Distance
HV: Scaled Hyper-Volume
Do. Scaled Area: Dominant Area
AC: Accuracy of observed Pareto frontier
NDEM: Non-Dominated Evaluation Metric
MPFE: Maximum Pareto-Optimal Front Error
NDC: Number of Distinct Choices
CLu (P): Number of Cluster on the obtained Pareto Frontier

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S: Spacing
Δ: Spread
OS: overall Pareto spread
$\overline{\mathbf{D}}$: Maximum Spread
W: Weighted Metric

INTRODUCTIONS

Various performance indices for measuring the quality of Pareto-Optimal set have been proposed. The quality indices can be used to compare between two or more optimal sets produced by one algorithm to choose the best set, or to compare between two optimal sets produced between two or more different algorithms to determine which algorithm has better accuracy (Kalyanmoy Deb, 2001).

Evolutionary Algorithms

Elitist Strength Pareto Evolutionary Algorithm (SPEA):

Proposed by Zitzler and Thiele (1998a), The SPEA assigns a fitness (called the strength) Si to each member i of an external population first. The strength Si is proportional to the niche count $n_i$as follow:

$$Si = \frac{n_i}{N+1} \quad (1)$$

Where n_i, is the niche count for solution i and n is the population size.Division by (N + 1) ensures that the maximum value of the strength of any external population member is never greater than one. In addition, a non-dominated solution dominating a fewer solutions has a smaller (better) fitness.

The fitness of a current population member j is assigned as one more than the sum of the strength values of all external population members which weakly dominates j:

$$Fj=1+\sum_{i\in\overline{P_t} \wedge i\prec j} S_i \quad (2)$$

Strength Pareto Evolutionary Algorithm (SPEA) (Kalyanmoy Deb, 2001):

Step 1 Find the best non-dominated set $F_i(P_t)$ of $P_t$and copy these solutions to $\overline{P_t}$or perform

$$\overline{P_t} = \overline{P_t} U F_i(P_t) \quad (3)$$

Step 2 Find the best non-dominated solutions $F_i(P_t)$ of the modified population $P_t$and delete all dominated solutions, or perform

$$\overline{P_t} = F_i(P_t). \quad (4)$$

Step 3 If $|\overline{P_t}|>\overline{N}$, use a clustering technique to reduce the size to $\overline{N}$Otherwise, keep $\overline{P_t}$unchanged. The resulting population is the external population $\overline{P}_{t+1}$of the next generation.

Step 4 Assign fitness to each elite solution i∊$\overline{P}_{t+1}$by using equation, then assign fitness to each population member j ∊$P_t$by using equation (2).

Step 5 Apply a binary tournament selection with these fitness values (in a minimization sense), a crossover and P_{t+1}a mutation operator to create the new population of size N from the combined population $\overline{(P}_{t+1} U P_t)$ of size ($\overline{N}+N$).

Steps 3 and 5 result in the new external and current populations which are then processed in the next generation. This algorithm continues until a stopping criterion is satisfied (Kalyanmoy Deb, 2001).

Elitist Non-Dominated Sorting Genetic Algorithm (NSGAII)(Kalyanmoy Deb, 2001):

Deb et al. suggested an elitist non-dominated sorting GA (NSGA-Il) (Deb et al, 2000a, 2000b). Unlike the above method of using only an elite-preservation strategy, NSGA-II also uses an explicit diversity-preserving mechanism. This algorithm does not have much similarity with the original NSGA, but the authors keep the name NSGA-Il to highlight its genesis and place of origin.

The NSGA-II procedure is (Kalyanmoy Deb, 2001):

Step 1 Combine parent and offspring populations and create $R_t = P_t \, U \, Q_t$. Perform a non- dominated sorting to R_t and identify different fronts: F_i i =1, 2, 3…., etc.

Step 2 Set new population $P_{t+1} = \Theta$ Set a counter i = 1. Until$|P_{t+j}| + |F_i| < N$, perform $P_{t+j} \, U \, F_i$ and i = i+ 1.

Step 3 Perform the Crowding-sort $F_i < c$ procedure and include the most widely spread (N - $|P_{t+1}|$) solutions by using the crowding distance values in the sorted Fi to P_{t+1}

Step 4 Create offspring population Q_{t+1} from $P_{t+1}$by using the crowded tournament selection, crossover and mutation operators.

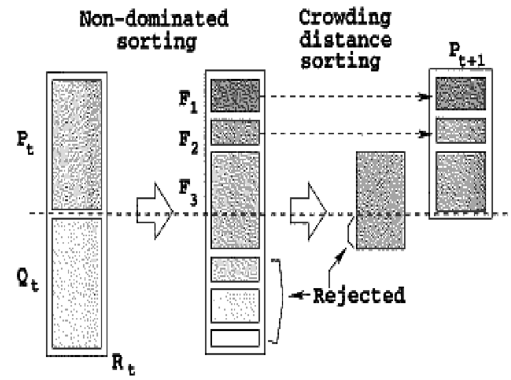

Fig. 1: Schematic of NSGA-II procedure (Kalyanmoy Deb, 2001)

Crowded Tournament Selection Operator (Kalyanmoy Deb, 2001):

The crowded comparison operator (<c) compares two solutions and returns the winner of the tournament. It assumes that every solution i has two attributes:

1. A non-domination rank $r_i$in the population.
2. A local crowding distance (d_i)

The crowding distance (d_i) of a solution i is a measure of the search space around i which is not occupied by any other solution in the population. Based on these two attributes, Crowded Tournament Selection Operator: A solution iwins a tournament with another solution j if any of the following conditions are true:

1. If solution i has a better rank, that is, $r_i<r_j$
2. If they have the same rank but solution i has a better crowding distance than solution j, that is, $r_i=r_j$and di >dj.

Crowding Distance Assignment Procedure (Kalyanmoy Deb, 2001):

Step Cl Call the number of solutions in *F* as *l* = |F|. For each i in the set, first assign d_i= o.

Step C2 for each objective function m = 1, 2, M, sort the set in worse order of $f_m$or, find the sorted indices vector: I^m = sort ($f_m >$).

Step C3 for m = 1, 2… M, assign a large distance to the boundary solutions, or$d_{I_1^m}$ = $d_{I_l^m}$ = 0, and for all other solutions j = 2 to (*l*-1), assign:

$$d_{I_j}^m = d_{I_j}^m + \frac{f_m^{I_{j+1}^m} - f_m^{I_{j-1}^m}}{f_m^{max} - f_m^{min}} \qquad (5)$$

The index I_J denotes the solution index of the j-th member in the sorted list. Thus, for any objective, I_1 and $I_l$denote the lowest and highest objective function values, respectively. The second term on the right side of equation (5) is the difference in objective function values between two neighboring solutions on either side ofSolution I_J. Thus, this metric denotes half of the perimeter of the enclosing cuboid with the nearest neighboring solutions placed on the vertices of the cuboid. The parameters $f_m^{max}$and f_m^{min}can be set as the population-maximum and population-minimum values of the m-th objective function (Kalyanmoy Deb, 2001).

Quality Indices in Multi-Objective Optimization:

There are two distinct goals in multi objective optimization: (i) to discover solutions as close to the Pareto-optimal solutions as possible, and (ii) to find solutions as diverse as possible in the non-dominated front. These two goals are orthogonal to each other. The first goal requires a search towards the Pareto-optimal region; while the second goal requires a search along the Pareto-optimal front (Kalyanmoy Deb, 2001). No single metric can measure the performance of an algorithm in an absolute sense. An MOEA will be termed a good MOEA, if both goals are satisfied adequately.

Metrics evaluating closeness to Pareto Front:

Maximum Pareto-Optimal Front Error (Kalyanmoy Deb, 2001; Carlos, A.):

This metric calculates the worst distance d_i between all each solution of set Q to all solutions of set P, and then the minimum distance among these distances will be chosen.

Accuracy of Pareto frontier (Shapour Azarm1 Professor, 2001):

This metric is a measure of goodness of Pareto optimal set.

According to the definitions and calculations of inferior region (S_{in}(P)), non-inferior region ($S_{non\text{-}in}$(P)) and dominant region (S_{do}(P)) which are mentioned later (Hyper Volume Calculation) (Shapour Azarm1 Professor, 2001).

Suppose that AP (P) is the approximation of the observed Pareto solution set P.

$$AP(P) = 1 - \text{Space}(S_{in}(P)) - \text{Space}(S_{do}(P)) \quad (6)$$

Where

$$\text{Space}(S_{in}(P)) = \left\{ \max_{r=I}^{\overline{np}} \left\{ (-1)^{r+1} \times \left[\sum_{K_I=1}^{\overline{np}-r+1} \dots \sum_{k_I=k_{I-1}+I}^{\overline{np}-(r-I+1)+1} \dots \times \sum_{k_r=k_{r-1}}^{\overline{np}} \prod_{k_r=k_{r-1}}^{m} \left[1 - \max_{J=1}^{r} \left(\bar{f}_I \left(X_{k_J} \right) \right) \right] \right] \right\} \right\} \quad (7)$$

And by the same procedure

$$\text{Space}(S_{do}(P)) = \left\{ \max_{r=I}^{\overline{np}} \left\{ (-1)^{r+1} \times \left[\sum_{K_I=1}^{\overline{np}-r+1} \dots \sum_{k_I=k_{I-1}+I}^{\overline{np}-(r-I+1)+1} \dots \times \sum_{k_r=k_{r-1}}^{\overline{np}} \prod_{k_r=k_{r-1}}^{m} \left[1 - \min_{J=1}^{r} \left(\bar{f}_I \left(X_{k_J} \right) \right) \right] \right] \right\} \right\} \quad (8)$$

$$AC(P) = \frac{1}{AP(P)} \quad (9)$$

Where K and j refer to solution number of total solutions $\overline{np}$, AC (P) is the Accuracy of Pareto frontier, S_{in} (P) is the inferior area, S_{do} (P) is the dominant area,and AP(P) is defined as the approximation of the observed Pareto solution set (P).

Metrics evaluating diversity among Non-dominated solutions (Kalyanmoy Deb, 2001; Miqing Li):

Spacing (S):

This metric calculates the relative distances between the consecutive solutions in non-dominated solution set (Kalyanmoy Deb, 2001; Carlos, A.).

The distance is

$$S = \sqrt{\frac{1}{|Q|} \sum_{i=1}^{|Q|} (di - \bar{d})^2} \quad (10)$$

Where d_i =min of (f_1^i-f_1^{i+1}) + (f_2^i-f_2^{i+1}) for all i=1 to I = n where n is the number of solutions.

Spread (Kalyanmoy Deb, 2001; Carlos, A.):

This metric was suggested by Deb et al. (2000a) to measure the spread of solution over the solution space.

$$\Delta = \frac{\sum_{M=1}^{M} d_M^e + \sum_{i=1}^{Q} |d_i - \bar{d}|}{\sum_{M=1}^{M} d_M^e + |Q|\bar{d}} \quad (11)$$

Where d_i can be taken as the consecutive Euclidean distance between i-th and the (i+1) solutions, and $\bar{d}$the mean value of these distance measures d_m^e Is the distance between the extreme solution each set of solution (here Q).M represents the number of objective functions, and Q is an observed Pareto solution set.

Crowding distance (Kalyanmoy Deb, 2001):

The crowding distance di is a measure of the search space around solution iwhich is not occupied by any other solution in the population.

Overall Pareto spread (Shapour Azarm1 Professor, 2001):

This metric quantifies the observed Pareto solution set spreads over the objective space when the design objective functions are considered altogether.

Figure (2) defines the Overall spread as the volume ratio of two hyper-rectangles. One of these rectangles is HR_{gb}(P) that is defined by the good and bad points with respect to each design objective. Similarly, the extreme points for an observed Pareto solution set define the other hyper-rectangle that is denoted by HR_{ex} (P) (Shapour Azarm1 Professor, 2001).

Therefore overall spread (OS (P)):

$$OS(P) = \frac{HR_{ex}(P)}{HR_{gb}(P)} \quad (12)$$

Where P refers to an observed Pareto solution set, by using the objective values to interpret $HR_{ex}(P)$ and $HR_{gb}(P)$, Eq. (12) can be expressed as:

$$OS(P) = \frac{\prod_{i=1}^{m} \left| \max_{k=1}^{\overline{np}} (P_k)_i - \min_{k=1}^{\overline{np}} (P_k)_i \right|}{\prod_{i=1}^{m} \left| (P_b)_i - (P_g)_i \right|} \quad (13)$$

in the scaled spaces OS can be expressed as:

$$OS(P) = \prod_{i=1}^{m} \left| \max_{k=1}^{\overline{np}} \bar{f}_i(x_k) - \min_{k=1}^{\overline{np}} \bar{f}_i(x_k) \right| \quad (14)$$

Where $\max_{k=1}^{\overline{np}} \bar{f}_i(x_k)$ and $\min_{k=1}^{\overline{np}} \bar{f}_i(x_k)$ are the maximum and minimum objective values for objective respectively, I =1, 2, 3…, m, and $\overline{np}$ are the total solution number.

In case of two objective spaces (Figure 2 shows this case)

$OS(P) = \frac{h_1 h_2}{H_1 H_2}$ where $h_1 = |\bar{f}_{1max} - \bar{f}_{1min}|$, $h2 = |\bar{f}_{2max} - \bar{f}_{2min}|$,

$H_1 = |(P_b)_1 - (P_g)_1|$, and $H_2 = |(P_b)2 - (P_g)2|$ (15)

K^th^ObjectivePareto Spread [3]:

This metric quantitatively depicts the solution range with respect to each individual design objective. The K^{th}objective Pareto spread metric, K=1, 2, 3…., m can be expressed as follow (Shapour Azarm1 Professor, 2001):

$$OS_K(P) = \frac{\left| \max_{k=1}^{\overline{np}} (P_k)_i - \min_{k=1}^{\overline{np}} (P_k)_i \right|}{\left| (P_b)_i - (P_g)_i \right|} \quad (16)$$

Where $\max_{i=1}^{\overline{np}} (P_i)_k$and $\min_{k=1}^{\overline{np}} (P_k)_i$ are the maximumand minimum objectivevalues for an objective i respectively, and $\overline{np}$ is the number of total solutions.

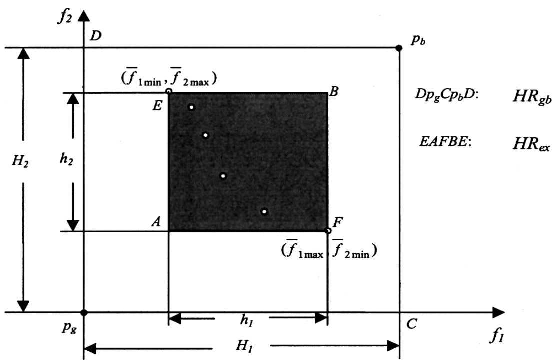

Fig. 2: K^{th} Objective Pareto spread (Shapour Azarm1 Professor, 2001)

Maximum Spread (Kalyanmoy Deb, 2001; Carlos, A.):

Zitzler (1999) defined a metric measuring the length of the diagonal of a hyper box.Formed by the extreme function values observed in the non-dominated set:

$$\bar{D} = \sqrt{\frac{1}{m} \sum_{m=1}^{m} \left(\frac{\left| \max_{i=1}^{Q} f_m^i - \min_{i=1}^{Q} f_m^i \right|}{\left| (P_b)k - (P_g)_k \right|} \right)^2} \quad (17)$$

This metric evaluates the length of diagonal of the hyper box which contains the observed Pareto set.

Number of distinct choices NDCμ (P)(Shapour Azarm1 Professor, 2001):

The more solutions contained in an observed Pareto solution set, the more the number of design options to choose from. However, if the observed Pareto solutions are too close to one another in the objective space, then the variations between the observed Pareto solutions may be indistinguishable, which make them infeasible. Therefore, the more number of solutions in set of solution, the more the number of design choices.

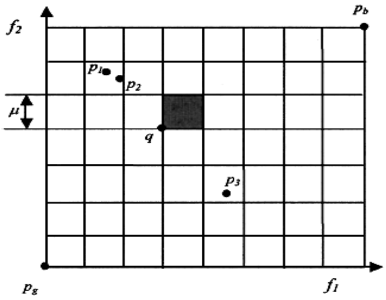

Fig. 3: Indifference region T_μ (q), as shown by a shaded area (Shapour Azarm1 Professor, 2001)

In figure (3), the objective space is divided into squares, where the true length of each one is 1/ μ where μ (0< μ<1).

All the solutions contained in one division is considered similar to one another or the designer is indifferent to such solutions.

$$NT\,\mu\,(q, P) = \begin{cases} 1, \ \exists\ pk\ \in P & pk\ \in T\,\mu\,(q) \\ 0, \ \nabla\ pk\ \in P & pk\ \notin T\,\mu\,(q) \end{cases} \tag{18}$$

The quality metric $NDC_\mu(P)$, that is the number of distinct choices for a pre-specified value of m, can then be defined as:

$$NDC_\mu\,(P) = \sum_{i_m=0}^{\nu-1} \dots \dots \sum_{i_2=0}^{\nu-1} \sum_{i_1=0}^{\nu-1} NT\,\mu\,(q, P) \tag{19}$$

Where q= (q_1, q_2, q_3… q_m) with $q_i = l_i / \nu$ where $\nu = \frac{1}{\mu}$

Cluster (CL_μ) (Shapour Azarm1 Professor, 2001):

This metric evaluates if there are more solutions that lie in one cluster, cluster analysis can be applied to the results of a multi-objective optimization algorithm to organize or partition solutions based on their objective function values. The goal of clustering is to create an efficient representation that characterizes the population being sampled; such a representation allows a decision maker to further understand the decision by making available the attainable limits for each objective (Dharaskar, Dr. R.V., et al., 2010).

A cluster is comprised of a number of similar objects collected or grouped together (Anil, K., et al.,).

Suppose that an observed Pareto set of N = 100 solutions and NDC_μ = 20 solutions, the Cluster is calculated as:

$$CL_\mu\,(P) = \left(\frac{N\,(P)}{NDC\mu\,(P)}\right) = \frac{100}{20} = 5 \tag{20}$$

In the ideal case where every Pareto solution obtained is distinct, the value of quantity CL_μ (P) is equal to 1. In all other cases, CL_μ

(P) is greater than 1. Also, the higher the value of the cluster quantity CLμ(P) is, the more clustered the solution set is, and hence the less preferred the solution set (Shapour Azarm1 Professor, 2001).

4.3 Metrics evaluating closeness and diversity:

Hyper Area and Hyper volume (Kalyanmoy Deb, 2001; Shapour Azarm1 Professor, 2001):

Hyper area or hyper volume metric is a good measure to evaluate both closeness and diversity with respect to Pareto optimal frontier.

For each solution i∈ Q, a hypercube $V_i$is constructed with a reference point W and the solution ***i***as the diagonal corners of the hypercube (HV) where

$$HV = \text{volume}\,\left(\bigcup_{i=1}^{|Q|} V_I\right) \tag{21}$$

Figure (4) shows the regions of the solution space in which pk is any solution in the space, and P_b is the bad point which represents the worst solution for a multi objective problem.In the other hand,Pg represents the good solution in the objective space.

1- Inferior Region:Space (S_{in}) which contains all the solutions dominated by Solution P_J.

Australian Journal of Basic and Applied Sciences, 10(15) October 2016, Pages: 10-25

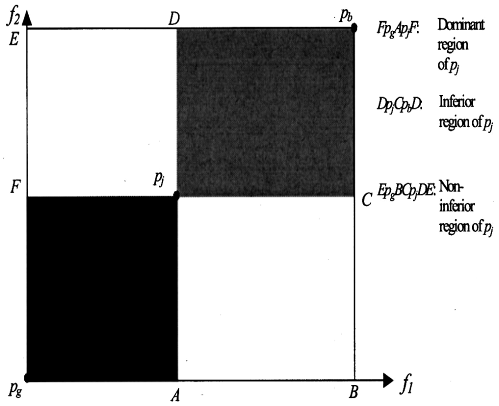

Fig. 4: inferior and inferior areas (Shapour Azarm1 Professor, 2001)

Non-inferior region: The non-inferior region (S_{nin}) of a point $p_J$is defined as the complementary region of the p_J's inferior region

2- The dominant region of a point pj is defined as a hyper rectangle Sdo(pj) such that for all PK□ Sdo(pj), there must be: pk>pj and pk<pg .

With the concept of inferior region, hyper area difference can be quantified as the space difference between the inferior regions of the true Pareto solution set Pt the inferior region of the observed Pareto solution set P. Let HD (P) represent the hyper area difference Quantity, then:

$$HD\ (P) = Space\ (S_{in}(P_t) - (S_{in}) = Space(S_{in}(P_t) - Space\ (S_{in}) \tag{22}$$

For scaled objective space, the inferior region for the true Pareto (P_t) is defined as Space ($S_{in}(P_t)$) and is equal to $S_{in}(P_t)$ =1

And the inferior region for observed Pareto set

$$Space\ (S_{in}) = \left\{ \max_{r=I}^{\overline{np}} \left\{ (-1)^{r+1} \text{ x} \left[\sum_{K_I=1}^{\overline{np}-r+1} \dots \sum_{k_I=k_{I-1}+I}^{\overline{np}-(r-I+1)+1} \dots \text{ x} \sum_{k_r=k_{r-1}}^{\overline{np}} \prod_{k_r=k_{r-1}}^{m} \left[1 - \max_{J=1}^{r} \left(\bar{f}_I \left(X_{k_J} \right) \right) \right] \right] \right\} \right\} \tag{23}$$

Therefor from the above equations

S_{in} (P_t) =1

HD (P) = Space (Sin (Pt) - (Sin) = Space (Sin (Pt) – Space (Sin) =1- Space (Sin)

$$HD\ (P) = 1 - \left\{ \max_{r=I}^{\overline{np}} \left\{ (-1)^{r+1} \text{ x} \left[\sum_{K_I=1}^{\overline{np}-r+1} \dots \sum_{k_I=k_{I-1}+I}^{\overline{np}-(r-I+1)+1} \dots \text{ x} \sum_{k_r=k_{r-1}}^{\overline{np}} \prod_{k_r=k_{r-1}}^{m} \left[1 - \max_{J=1}^{r} \left(\bar{f}_I \left(X_{k_J} \right) \right) \right] \right] \right\} \right\} \tag{24}$$

Also the ***dominant region*(S_{do}(P))**can be expressed as:

$$Space\ (S_{do}(P)) = \left\{ \max_{r=I}^{\overline{np}} \left\{ (-1)^{r+1} \text{ x} \left[\sum_{K_I=1}^{\overline{np}-r+1} \dots \sum_{k_I=k_{I-1}+I}^{\overline{np}-(r-I+1)+1} \dots \text{ x} \sum_{k_r=k_{r-1}}^{\overline{np}} \prod_{k_r=k_{r-1}}^{m} \left[1 - \min_{J=1}^{r} \left(\bar{f}_I \left(X_{k_J} \right) \right) \right] \right] \right\} \right\} \tag{25}$$

Where $\overline{np}$ is the total number of solutions, r is the number of solution of an observed Pareto optimal set, and k is the number of objective functions.

Non-Dominated Evaluation Metric (Kalyanmoy Deb, 2001; Carlos, A):

This metric compares both conflicting goals converging and diversity produced and calculated by an algorithm with those for another one.If the metric value for one algorithm dominates that of other algorithm, then the former is undoubtedly better than the latter. Otherwise, no affirmative conclusion can be made about the two algorithms.

Figure (5) shows the performance of three algorithms on a hypothetical problem. Clearly, Algorithm A dominates algorithm B, but algorithms A and C cannot be judged which is better.In the ideal case, where every Pareto solution obtained is distinct,and then the value of the quantity CL_{μ} (P) is equal to 1. In all other cases, CL_{μ} (P) is greater than 1. Also, the higher the value of thecluster quantity CLμ (P) is, the more clustered the solution set is,and hence the less preferred the solution set.

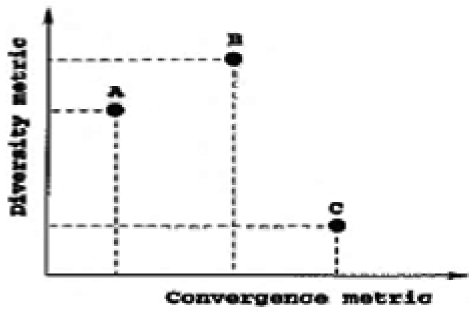

Fig. 5: Algorithms A and C produces a non-dominated outcome (Kalyanmoy Deb, 2001)

This metric can be used to compare between two observed set of optimal solutions, and two observed optimal sets obtained by one or different algorithms.

Comparative Analyses Of Multi-Bjective Indices:

Any real-world problems involve simultaneous optimization of several competing objectives: often, there is no single optimal solution, rather a set of Pareto optimal solutions (also called Pareto front in the objective space). In general, the Pareto front is often infeasible, since the complexity of the underlying application prevents exact methods from being applicable. Heuristic search methods try to find a good frontier.

In this paper, 12 quality metrics are used to evaluate the goodness of the observed Pareto frontier obtained by NSGAII, and SPEA2.The objective is to measure the accuracy and reliability in finding a good Pareto Optimal set. The analysis studies the relation between no. of variables, no. of objective functions, no. of constraints, and nature of constraints versus the quality indices.

Table 1 gives a description of the benchmark problems used for comparative analysis.

Table 1: Description of the benchmark problems used for comparative analysis

S/N	Test function	No. of Decision Variables	No. of objective functions	Nature of objective functions	No. of constraints	Nature of constraints	Constraint type	Type of optimization Algorithm	Overall Spread & Objective Spread	Max Spread	Crowding distances	HV	Do. Area	AC	NDEM	MPFE	NDC	Spacing	CLu (P)
1	MOP1	1 , [x]	2	Non-linear	Un-Constrained	Un-Constrained	N/A	NSGA2	1	4.003	0.27337	0.82388	0.15738	53.3728	50	0	32	1.2796	1.5625
1	MOP1	1 , [x]	2	Non-linear	Un-Constrained	Un-Constrained	N/A	SPEA	1	3.8394	0.31676	0.80536	0.16659	35.6574	50	0.032434	32	1.215	1.5625
2	MOP2	4 , [xi, i=1,2,3, and 4]	2	Non-linear	Un-Constrained	Un-Constrained	N/A	NSGA2	1	0.947	0.069665	0.31362	0.66512	47.0513	50	0	34	0.29422	1.4706
2	MOP2	4 , [xi, i=1,2,3, and 4]	2	Non-linear	Un-Constrained	Un-Constrained	N/A	SPEA	1	0.97219	0.10749	0.30743	0.66716	39.3529	50	0.008805	34	0.30065	1.4706
3	MOP3	2, [x,y]	2	Non-linear	2	Non-linear	Equality	NSGA2	1	141.7017	Inf	0.65799	0.33657	183.7668	50	0	32	58.6393	1.5625
3	MOP3	2, [x,y]	2	Non-linear	2	Non-linear	Equality	SPEA	1	37.8774	1.8456	0.42241	0.52466	18.8948	50	0.20001	34	15.7121	1.4706
4	MOP4	6, [x(i+1), i =1,2,3,4,5]	2	Non-linear	Un-Constrained	Un-Constrained	N/A	NSGA2	1	17.1567	1.5067	0.47035	0.51011	51.1573	50	0	33	16.8612	1.5152
4	MOP4	6, [x(i+1), i=1,2,3,4,	2	Non-linea	Un-Constr	Un-Constr	N/A	SPEA	1	17.3897	3.9853	0.4693	0.50217	35.0924	50	0.14826	34	16.3895	1.470

Australian Journal of Basic and Applied Sciences, 10(15) October 2016, Pages: 10-25

		5]		r	ained	ained						3							6
5	MOP5	2, [x,y]	3	Non-linear	Un-Constrained	Un-Constrained	N/A	NSGA2	1	21.4092	0.58163	0.89934	0.06543	28.3875	50	0.030016	34	9.0819	1.4706
5	MOP5	2, [x,y]	3	Non-linear	Un-Constrained	Un-Constrained	N/A	SPEA	1	11.1585	0.085445	0.95183	0.03063	57.0216	50	0.013759	34	7.0149	1.4706
6	MOP6	2, [x,y]	2	Linear+Non-linear	Un-Constrained	Un-Constrained	N/A	NSGA2	1	1.2797	0.11375	0.25169	0.73438	71.7881	49	0.00602	33	0.38172	1.4848
6	MOP6	2, [x,y]	2	Linear+Non-linear	Un-Constrained	Un-Constrained	N/A	SPEA	1	1.1088	0.02654	0.32813	0.64467	36.767	50	0.003579	34	0.33085	1.4706
7	MOP7	2, [x,y]	3	Non-linear	Un-Constrained	Un-Constrained	N/A	NSGA2	1	56.7287	4.1765	0.85031	0.13062	52.4309	50	0.090492	32	26.6269	1.5625
7	MOP7	2, [x,y]	3	Non-linear	Un-Constrained	Un-Constrained	N/A	SPEA	1	44.8741	4.3834	0.81133	0.165	42.2478	50	0.37478	32	22.2079	1.5625
8	TP_KUR	4, [xi, i=1,2,3,4]	2	Non-linear	Un-Constrained	Un-Constrained	N/A	NSGA2	1	12.2915	1.6198	0.47172	0.51031	55.6512	50	0.040765	32	9.5368	1.5625
8	TP_KUR	4, [xi, i=1,2,3,4]	2	Non-linear	Un-Constrained	Un-Constrained	N/A	SPEA	1	10.8808	0.59396	0.49489	0.47378	31.9199	50	0.078963	32	9.742	1.5625
9	ZDT1	31, [x, xi, i=1,2……30]	2	Non-linear	2	linear	Equality	NSGA2	1	1.9626	0.69712	0.72773	0.23564	4.21897	46	0.0073	28	1.1053	1.6429
9	ZDT1	31, [x, xi, i=1,2……30]	2	Non-linear	2	linear	Equality	SPEA	1	0.92168	0.18084	0.61779	0.35031	31.3561	50	0.005187	34	0.27487	1.4706
10	ZDT2	31, [x, xi, i=1,2……30]	2	Non-linear	1	Non-linear	Equality	NSGA2	1	1.5331	Inf	0.32828	0.64289	34.6929	50	0	31	0.59053	1.6129
10	ZDT2	31, [x, xi, i=1,2……30]	2	Non-linear	1	Non-linear	Equality	SPEA	1	0.93834	0.041885	0.33388	0.63511	32.249	50	0.006418	34	0.31293	1.4706
11	ZDT3	31, [x, xi, i=1,2……30]	2	Non-linear	1	Non-linear	Equality	NSGA2	1	1.5236	0.22346	0.44702	0.42749	0.8214	44	0.002739	33	1.1708	1.3333
11	ZDT3	31, [x, xi, i=1,2……30]	2	Non-linear	1	Non-linear	Equality	SPEA	1	0.82813	0.082493	0.47109	0.50121	36.1014	50	0.001567	33	0.26735	1.5152
12	ZDT4	11, [x, xi, i=1,2……10]	2	Non-linear	1	Non-linear	Equality	NSGA2	1	27.0759	17.2762	0.85346	0.10547	3.8314	13	0.14376	10	14.1466	1.3
12	ZDT4	11, [x, xi, i=1,2……10]	2	Non-linear	1	Non-linear	Equality	SPEA	1	32.8279	42.2432	0.92393	0.00707	14.4927	50	0.000619	36	8.0168	1.3889
13	ZDT6	11, [x, xi, i=1,2……10]	2	Non-linear	1	Non-linear	Equality	NSGA2	1	0.7176	Inf	0.346	0.33674	0.29877	4	0.41629	3	1.7332	1.3333
13	ZDT6	11, [x, xi, i=1,2……10]	2	Non-linear	1	Non-linear	Equality	SPEA	1	1.5215	1.1871	0.71381	0.25666	33.8602	50	0.004315	33	0.31704	1.5152
14	DTLZ2-2	11, [x, xi, i=1,2……10]	2	Non-linear	1	Non-linear	Equality	NSGA2	1	2.5701	0.17632	0.28087	0.70034	53.2016	50	0.021061	32	0.76773	1.5625
14	DTLZ2-2	11, [x, xi, i=1,2……10]	2	Non-linear	1	Non-linear	Equality	SPEA	1	1.4677	0.57629	0.54881	0.41941	31.4749	50	0.00935	31	0.36731	1.6129
15	DTLZ2-3 Obj	13, [x, y, xi, i=1,2……10]	3	Non-linear	1	Non-linear	Equality	NSGA2	1	3.6502	Inf	0.27472	0.55093	5.7356	50	0.009161	34	1.2326	1.4706
15	DTLZ2-3 Obj	13, [x, y, xi, i=1,2……10]	3	Non-linear	1	Non-linear	Equality	SPEA	1	2.6416	1.4307	0.77675	0.09381	7.7256	50	0.096566	34	0.7376	1.4706
16	DTLZ7	12, [x, xi, i=1,2……11]	2	Non-linear	2	Non-linear	Equality	NSGA2	1	3.0858	0.30173	0.67785	0.26675	18.0504	26	0.023391	16	3.7123	1.625
16	DTLZ7	12, [x, xi, i=1,2……11]	2	Non-linear	2	Non-linear	Equality	SPEA	1	0.34815	0.015855	0.53429	0.43556	33.1705	50	0.001644	33	1.8252	1.5152
17	Two Bar Truss	3, [x, y, z]	2	Non-linear	1	Non-linear	Inequality	NSGA2	1	0.81675	0.010292	0.83745	0.14759	66.8579	50	0.000628	32	0.76734	1.5625
17	Two Bar Truss	3, [x, y, z]	2	Non-linear	1	Non-linear	Inequality	SPEA	1	1.013	0.002881	0.86111	0.12257	61.2476	50	0.00062	33	0.88765	1.5152
18	MOPC1	2, [x, y]	2	Non-linear	2	Non-linear	Inequality	NSGA2	1	91.8705	2.886	0.8022	0.18128	60.5357	50	0.51259	33	29.9169	1.5152
18	MOPC1	2, [x, y]	2	Non-linear	2	Non-linear	Inequality	SPEA	1	60.8156	2.5804	0.73828	0.23808	42.298	50	0.40045	34	20.6321	1.4706
19	MOPC2	6, [x(1),x(2),x(3),x(4),x(5),x(6)]	2	Non-linear	6	Linear+Non-linear	Inequality	NSGA2	1	2458.828	571.7654	0.40671	0.56198	31.9404	50	1.6337	33	1283.719	1.5152

19	MOPC2	6, [x(1),x(2),x(3),x(4),x(5),x(6)]	2	Non-linear	6	Linear +Non-linear	Inequality	SPEA	1	830.5493	88.1682	0.48445	0.48946	38.3188	50	5.6061	34	499.3091	1.4706
20	MOPC3	2, [x, y]	3	Non-linear	2	linear	Inequality	NSGA2	1	67.0268	17.1626	0.58598	0.39095	43.3482	50	0.29189	34	65.3349	1.4706
20	MOPC3	2, [x, y]	3	Non-linear	2	linear	Inequality	SPEA	1	2.2359	0.39537	0.72026	0.19073	11.2343	50	0.062232	36	12.2085	1.3889
21	Chakong and Haimes	2, [x, y]	2	Non-linear	2	Non-linear	Inequality	NSGA2	1	214.8502	Inf	0.52215	0.45344	40.9759	50	0	34	129.1188	1.4706
21	Chakong and Haimes	2, [x, y]	2	Non-linear	2	Non-linear	Inequality	SPEA	1	182.5098	22.6616	0.47895	0.48328	26.4753	50	1.195	34	130.3994	1.4706
22	Binh-Korn	2, [x, y]	2	Non-linear	2	Non-linear	Inequality	NSGA2	1	143.7881	4.2326	0.81785	0.16495	58.1258	50	0.35189	34	49.0462	1.4706
22	Binh-Korn	2, [x, y]	2	Non-linear	2	Non-linear	Inequality	SPEA	1	97.0188	7.8794	0.78858	0.18883	44.2562	50	0.53176	31	30.9265	1.6129
23	CTP1	2, [x, y]	2	Non-linear	2	Non-linear	Inequality	NSGA2	1	3.7034	0.46595	0.71482	0.26533	50.3748	50	0.014709	32	1.8881	1.5625
23	CTP1	2, [x, y]	2	Non-linear	2	Non-linear	Inequality	SPEA	1	0.56141	0.045115	0.60438	0.36021	28.2411	50	0.002916	33	0.27906	1.5152
24	Schaffer2	1, [x]	2	Non-linear	Un-Constrained	Un-Constrained	N/A	NSGA2	1	11.668	1.536	0.65128	0.31803	32.5846	50	0	34	4.8035	1.4706
24	Schaffer2	1, [x]	2	Non-linear	Un-Constrained	Un-Constrained	N/A	SPEA	1	10.9078	0.82128	0.64056	0.32752	31.3331	50	0.042825	32	3.9061	1.5625
25	Osyczka and Kundu	11, [x(1),x(2),x(3),x(4),x(5),x(6), xi, i=1,2,3,4,5]	2	Non-linear	6	Linear + Non-linear	Inequality	NSGA2	1	575.57458	58.4924	0.43707	0.54254	49.0422	50	0	31	408.4098	1.6129
25	Osyczka and Kundu	11, [x(1),x(2),x(3),x(4),x(5),x(6), xi, i=1,2,3,4,5]	2	Non-linear	6	Linear + Non-linear	Inequality	SPEA	1	800.4296	61.6466	0.54433	0.42797	36.103	50	6.6927	33	505.6097	1.5152
26	CONSTR_Ex	2, [x, y]	2	linear	2	Linear	Inequality	NSGA2	1	5.4129	Inf	0.6642	0.31306	43.9778	50	0	33	2.5274	1.5152
26	CONSTR_Ex	2, [x, y]	2	linear	2	Linear	Inequality	SPEA	1	3.6934	0.46447	0.72492	0.2438	31.962	50	0.010107	32	2.4028	1.5625
27	OKA2	3, [x(1),x(2),x(3)]	2	Non-linear	Un-Constrained	Un-Constrained	N/A	NSGA2	1	2.059	0.20551	0.53588	0.41734	21.379	40	0	25	2.9637	1.6
27	OKA2	3, [x(1),x(2),x(3)]	2	Non-linear	Un-Constrained	Un-Constrained	N/A	SPEA	1	0.79781	0.10474	0.23281	0.72065	21.4868	50	0.000211	34	2.2966	1.4706
28	Test function	2, [x(1),x(2)]	2	Non-linear	3	Non-linear	Inequality	NSGA2	1	10.7495	0.43283	0.72536	0.25766	58.9177	43	0.038768	29	4.697	1.4828
28	Test function	2, [x(1),x(2)]	2	Non-linear	3	Non-linear	Inequality	SPEA	1	18.9776	1.0544	0.62234	0.34678	32.3765	50	0.093908	34	9.1574	1.4706
29	DTLZ 1-3 Obj	8, [x(1),x(2), xi, i=1,2,3,4,5,6]	3	Non-linear	1	Non-linear	Equality	NSGA2	1	235.3075	36.355	0.44111	0.45959	10.0704	50	0	33	99.6371	1.5152
29	DTLZ 1-3 Obj	8, [x(1),x(2), xi, i=1,2,3,4,5,6]	3	Non-linear	1	Non-linear	Equality	SPEA	1	6.5917	1.0515	0.66444	0.04398	3.4296	50	0.027846	35	0.93508	1.4286
30	Comet	3, [x(1),x(2),x(3)]	3	Non-linear	1	Linear	Equality	NSGA2	1	115.6428	Inf	0.47442	0.4908	28.7559	50	0.62943	35	80.988	1.4286
30	Comet	3, [x(1),x(2),x(3)]	3	Non-linear	1	Linear	Equality	SPEA	1	120.5318	55.9149	0.44037	0.46518	10.5869	50	4.3387	32	68.7748	1.5625

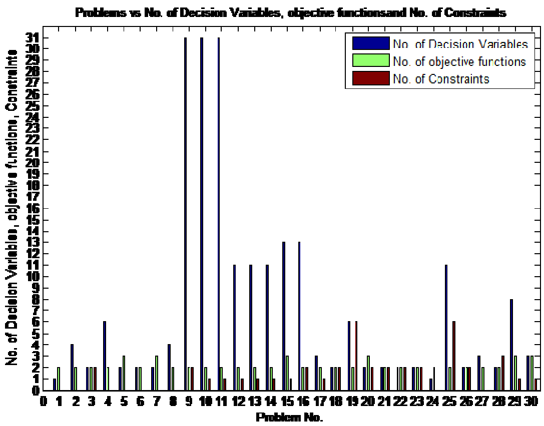

Fig. 6: Problem No. vs.No. of Decision Variables, Objective Functions, Constraints

Figure 6 shows a set of 30 benchmark problems (Kalyanmoy Deb, 2001; Carlos, A. et al., ; Kalyanmoy Deb Kanpur Genetic Algorithms Laboratory, 2001; https://en.wikipedia.org/wiki/Test_functions_for_optimization; Tushar Goel1, Nielen Stander2; http://people.ee.ethz.ch/~sop/download/supplementary/testproblems/dtlz2/index.php".) used for comparative analysis. It shows also the # of variables, the # of objective functions and the # of constraints.

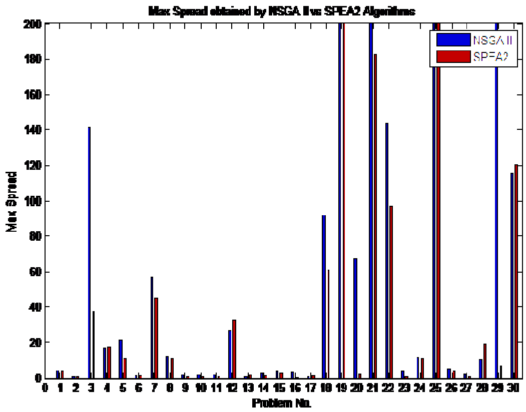

Fig. 7: Maximum Spread

Figure 7 gives the maximum spread using NSGAII and SPEA2 respectively. Generally, NSGAII produces maximum spread larger than SPEA2 for problems 1,3,5,6,7,8,9,10,11,14,15,16,18,19,20,21,22,23,24,26,27, and 29 respectively. In problems 2, 4, 12, 13,17,25,28, and 30 respectively SPEA2 gives higher maximum Spread than NSGA II.

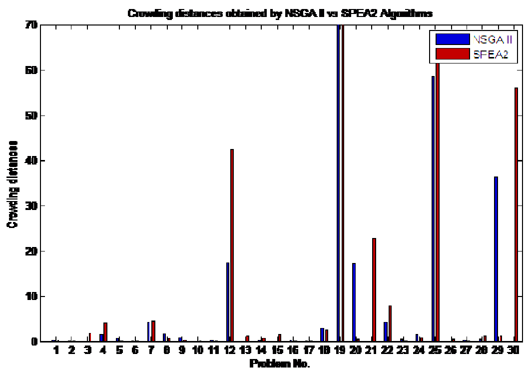

Fig. 8: Crowding distances

Figure 8 shows the Crowding distances using NSGA II and SPEA2 for the 30 set of problems. Generally, the Crowding distances are less using NSGAII than SPEA2 for problems 1,2, 4,9,12,14,22,25,28 respectively. The opposite is true for problems 3, 5, 6, 7, 8,10,11,13,15,16,17,18,19,20,21,23, 24, 26, 27, 29 and 30 respectively.

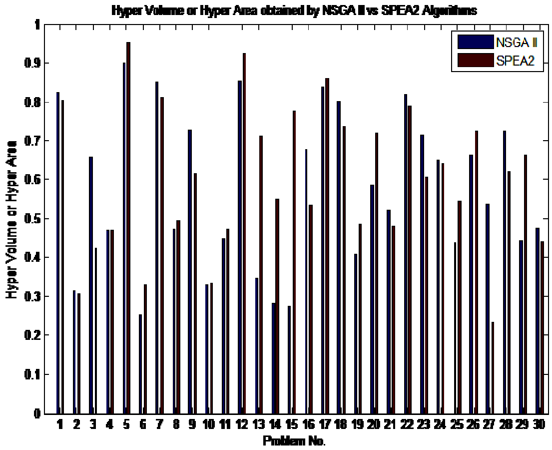

Fig. 9: Hype Volume (Hyper area)

Figure 9 gives the Hyper volume (hyper area) using NSGA II and SPEA 2 for 30 test bed problems. NSGA II algorithm usually gives lower hyper volume than SPEA2 especially for problems 1,2,3,4,7,16,18,20,21,22,23,24,27,28 and 30 respectively. The opposite is true for problems 5,6,8,9,10,11,12,13,14,15,17,19,25,26 and 29 respectively.

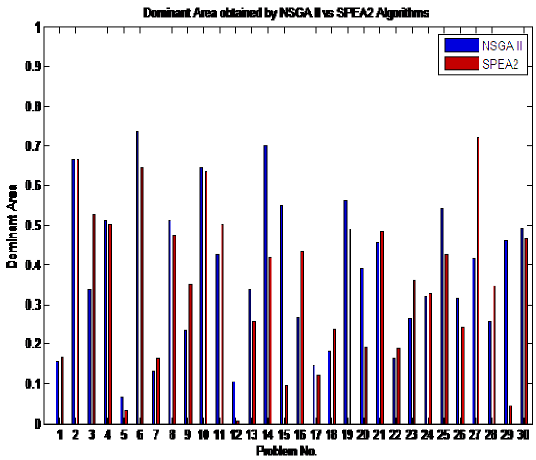

Fig. 10: Dominant Area

Figure 12 shows the dominant area for the 30 test problems using NSGA II and SPEA 2 respectively. The dominant area is lower using NSGA II than SPEA 2 for problems 1, 2, 3, 7, 9, 11, 16, 18, 21, 22, 23, 24, 27 and 28 respectively. The opposite is true for problems 4, 5, 6, 8, 10, 12, 13, 14, 15, 17, 19, 20, 25, 26, 29 and 30 respectively.

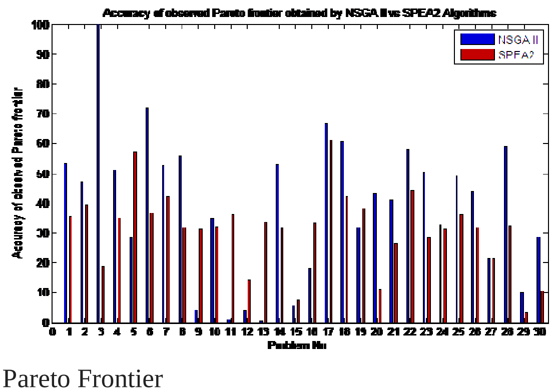

Fig. 11: Accuracy of Observed Pareto Frontier

Figure 11 gives the accuracy of observed Pareto frontier using NSGA II and SPEA 2 Algorithms. Generally, the accuracy of observed Pareto frontier is higher using NSGA II than SPEA 2 for problems 1, 2, 3, 4, 6, 7, 8, 10, 14, 17, 18, 20, 21, 22, 23, 24, 25, 26, 28, 29, and 30 respectively. The opposite is true for problems 5, 9, 11, 12, 13, 15, 16, 19 and 27 respectively.

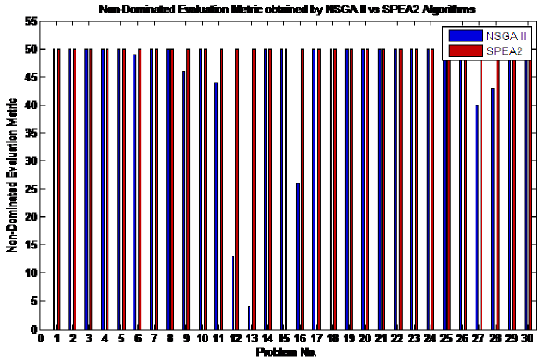

Fig. 12: Non-Dominated Evaluation Metric

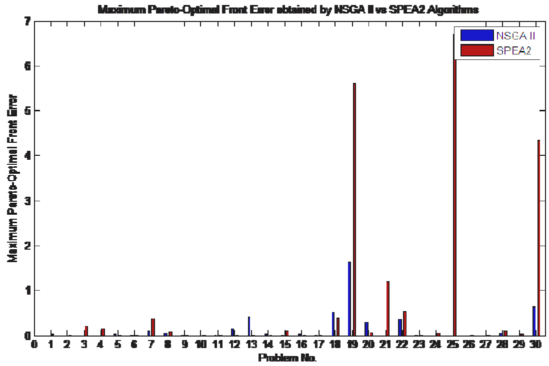

Fig. 13: Maximum Pareto-Optimal Front Error

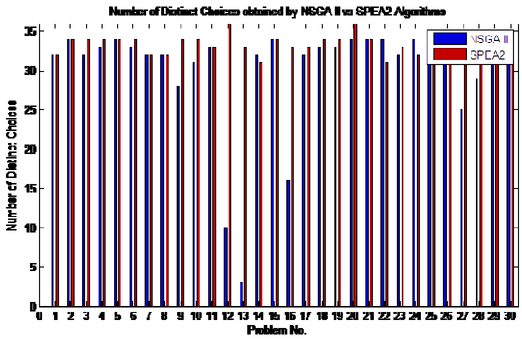

Fig. 14: Number of Distinct Choices

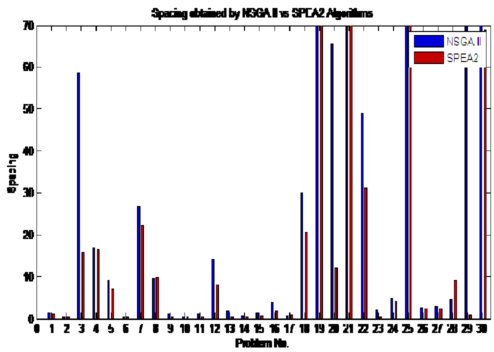

Fig. 15: Spacing

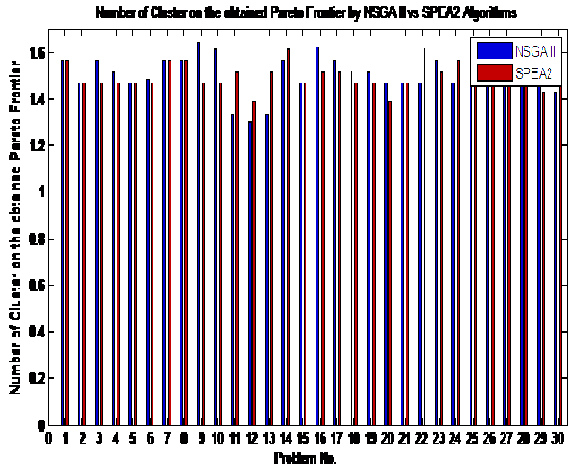

Fig. 16: Number of Cluster on the obtained Pareto Frontier

The main goal of using quality indices with Evolutionary algorithms is to measure diversity and convergence ofalgorithm.

Figure 17shows Pareto optimal sets using NSGAII and SPEA2 for MOP6 (Carlos, A. et al.,).

Handling Constraints in Evolutionary Algorithms:

All the algorithms described previously assumed that the underlying optimization problem is unconstrained. However, this is not the case when solving real-world problems. Now, different MOEAs designed to handle constraints are discussed (Kalyanmoy Deb, 2001). In this paper, the Maximum Constraint Violations Penalty is used to handle the constrained problems (Edward, B.,).

6.1 Ignoring Infeasible Solutions (Kalyanmoy Deb, 2001):

One of the simple ways to handle the constraints is to ignore any solution that violates any constraints (Coello and Christiansen, 1999). By plotting the constraints and problembounds, it is easily to determine the interval in which the problem is infeasible.

6.2 Penalty Function Approach [1]:

In this method, the constraints are normalized as follow:

$$w_j(x^i) = \begin{cases} |\square(\square^{(\square)})|, \square\square\ \square(\square^{(\square)}) < 0, \\ 0, \quad \square\square\square\square\square\square\square\square\square \end{cases} \qquad (26)$$

Thereafter, all constraint violations are added together to get the overall constraint violation:

$$\Omega = \sum_{\square=1}^{\square} wj\ (\square^{(\square)}) \qquad (27)$$

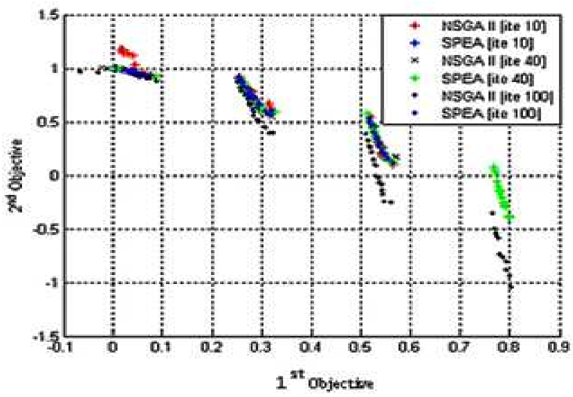

Fig. 17: NSGA II & SPEA2 Convergence and Diversity Comparison at iterations 10, 40, and 100

This constraint violation is then multiplied with a penalty parameter Rm and the product is added to each of the objective function values:

$$F_m(x^{(i)}) = f_m(x^{(i)}) + Rm\ \Omega\ (\square^{(\square)}) \qquad (28)$$

The functions F_m takes into account the constraint violations. For a feasible solution, the corresponding **Ω** term is zero and F_m becomes equal to the original objective function fm. However, for an infeasible solution,

$F_m > f_m$., thereby adding a penalty corresponding to total constraint violation.The penalty parameter Rm. is used to make both terms on the right side of the above equation to have the same order of magnitude. Since the original objective functions could be of different magnitudes, the penalty parameter must vary from one objective function to another.

6.3 Jimenez-Verdegay-Gomez-Skarmeta'sMethod [1]:

Jimenez, Verdegay and Gomez-Skarmeta (1999) suggested a systematic constraint handling procedure for multi-objective optimization. Only inequality constraints of the lesser-than-equal-to type are considered in their study, whereas any other constraints can also be handled using the procedure.

6.4Modification to NSGA II Algorithm:

The proposed steps are listed as follow:

Step 1 Normalize Constraint functions.

Step 2 Define the objective functions so that the objective functions are only calculated when decision variables violatethe constraint functions.

Step 3 Seta condition that, if decision variables do not violate the constraints (go to step 2)

Step 4 Repeat steps (1-3), to construct the initial population.

Step 5 Start Evolutionary algorithms to complete the optimization process on the initial population.

Step 6 Steps 1 to 5 are repeated until the Optimal Pareto set are formed.

Figure 18 shows an Observed Pareto Front for CONSTR_EX Problem obtained above modified NSGAII (US).From figures 18 and 19, it is clear that the developed NSGA II (US) gives better convergence and diversity than Regular NSGAII or in conjunction with penalty functions.in figure 19, quality indices on x Axis are ordered as follow:

1-OS1, 2-OS, 3-Max Spread, 4-Crowding distances, 5-HV, 6-Do, 7- AC, 8- NDEM, 9-MPFE, 10-NDC, 11-Spacing, and 12- CLu (P).

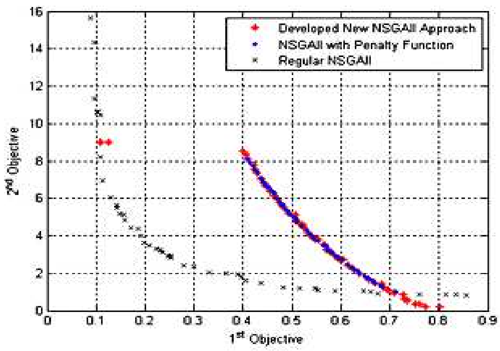

Fig. 18: Pareto Optimal set for Developed NSGA II, vs. NSGA II with Penalty and Regular NSGA II

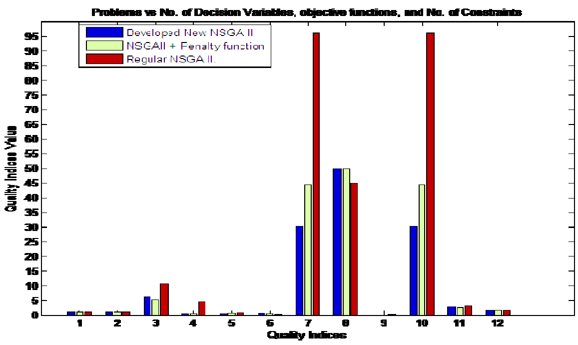

Fig. 19: Comparison between Quality indices

Obtained by the developed NSGA II Algorithm, US. NSGAII + Penalty function, and Regular NSGA II Algorithm:

Conclusions:

a. NSGAII is better than SPEA 2 with respect to diversity and convergence for quadraticobjective functions[MOP1, MOP7, Schaffer2, and OKA2].

b. NSGA II is better than SPEA2 with respect to diversity and worse than SPEA 2 with respect to convergence for Exponential and Sinusoidal objective functions [MOP5, MOP6, and TP_KUR].

c. NSGA II is better than SPEA2 with respect to convergence and worse than SPEA 2 with respect to diversityfor objective functions with exponential functions [MOP2, MOP4].

d. NSGA II is better than SPEA2 with respect to diversity and convergence for non-linearrational objective functions with linear constraints [ZDT1].

e. NSGA II is better than SPEA2 with respect to diversity and worse than SPEA 2 with respect to convergence for non-linear quadratic objective functions[MOPC3, and Comet] and linear rational objective function [CONSTR_EX].

f. NSGA II is better than SPEA2 with respect to diversity and convergence for non-linear quadratic objective functions[MOP3, MOPC1, (Chakong and Haimes), Binh-Korntest function], non-linear rational objective function[DTLZ7] and inverted exponential objective function with a negative power[CTP1].

g. NSGA II is better than SPEA2 with respect to diversity and worse than SPEA 2 with respect to convergence fornon-linear rational objective functions [ZDT2, and ZDT3], non-linear sinusoidal andcosine objective functions [ZDT4, ZDT6, DTLZ2-2, DTLZ2-3 Obj, DTLZ1-3 Obj].

h. SPEA 2 is better than NSGAII with respect to diversity and convergence for non-linear square root objective function [Two Bar Truss].

i. NSGA II is better than SPEA2 with respect to diversity and worse than SPEA 2 with respect to convergence for non-linear quadratic objective functions [MOPC2, and Osyczka&Kundu].

j-12 indices are studied versus a set of 30 benchmark problems. Several issues can be raised:

1- Number of Cluster is almost the same over the 30 test problems. This makes this measure unreliable.

2-The objective spread is always equal 1 for all the test problems. This is the same for the overall spread which makes these measures unreliable.

3- Accuracy of observed Pareto Frontier is dependent on Hyper Volume and Dominant area; therefor it is not a measure for Pareto goodness

3- Crowding Distances sometimes equal to INF, and sometimes have very similar results for problems with very different conditions. It is advisable to discard this measure as well.

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