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Normalized Design Control of Robust Using Semi-Definite Programming

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ABSTRACT

BACKGROUND: Robust methods aim to achieve robust performance and/or stability in the presence of bounded modeling errors.**OBJECTIVE:** dual problem in which uncertainty enters a filter at the plant output.**RESULTS:** A solution to this problem could be obtained, when parametric uncertainties are involved and a class of static multipliers is used. In view of the fact that the general robust output feedback problem is largely open, it is a challenging research question which kind of interconnections of uncertain systems can be handled by convex optimization in the LMI framework.**CONCLUSION:** It is found that the only structural property needed for a synthesis solution to exist in terms of LMIs, is the fact that the transfer matrix from control input to measurement output is not affected by uncertainty.

INTRODUCTION

Probably the most important example of a robust control technique is H-infinity loop-shaping, which was developed by Duncan McFarlane and Keith Glover of Cambridge University; this method minimizes the sensitivity of a system over its frequency spectrum, and this guarantees that the system will not greatly deviate from expected trajectories when disturbances enter the system (R.A. Meyer, 1975). The theory of robust control began in the late 1970s and early 1980s and soon developed a number of techniques for dealing with bounded system uncertainty. Robust control is a branch of control theory whose approach to controller design explicitly deals with uncertainty. Robust control methods are designed to function properly provided that uncertain parameters or disturbances are found within some (typically compact) set.

Robust methods aim to achieve robust performance and/or stability in the presence of bounded modeling errors. The early methods of Bode and others were fairly robust; the state-space methods invented in the 1960s and 1970s were sometimes found to lack robustness, prompting research to improve them. This was the start of the theory of Robust Control, which took shape in the 1980s and 1990s and is still active today(L. Mirkin, 1997). In contrast with an adaptive control policy, a robust control policy is static; rather than adapting to measurements of variations(Packard, 1993 and F. Paganini, 1993), the controller is designed to work assuming that certain variables will be unknown but bounded. In order to gain a perspective for robust control, it is useful to examine some basic concepts from control theory. Control theory can be broken down historically into two main areas: conventional control and modern control. Conventional control covers the concepts and techniques developed up to 1950.

Modern control methods were developed with a realization that control system equations could be structured in such a way that computers could efficiently solve them. It was shown that any nth order differential equation describing a control system could be reduced to n 1st order equations. These equations could be arranged in the form of matrix equations. This method is often referred to as the state variable method. Modern

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control covers the techniques from 1950 to the present. Each of these is examined in this introduction(P.P. Khargonekar, 1985, Rantzer, 1996, C.W. Scherer, 1997 and J. Sturm, 1999).Conventional control became interesting with the development of feedback theory. One early use of feedback control was the development of the flyball governor for stabilizing steam engines in locomotives. Another example was the use of feedback for telephone signals in the 1920s. The problem was the transmission of signals over long lines. There was a limit to the number of repeaters that could be added in series to a telephone line due to distortion. Harold Stephen Black proposed a feedback system that would use feedback to limit the distortion (V. Balakrishnan, 2002). Even though the added feedback sacrificed some gain in the repeater, it enhanced the overall performance.

Quadratic Constraints Analysis:

Popov first used the notion of 'multiplier' in the context of feedback systems and considered the stability analysis problem for an LTI system with a single non-linearity in the loop (S.G. Dietz, 2008). It is by now well-known that IQC analysis generalizes the Popov criterion, circle criterion and many variations thereof. It includes important stability principles such as those based on small gain or passivity arguments. In this chapter, we discuss the main theory on the analysis with IQCs, and show how to address parametric uncertainties.

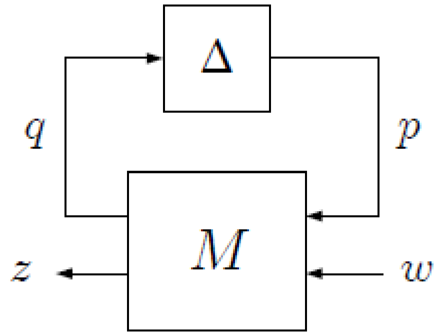

Fig. 1: Interconnection for Stability And Performance Analysis

The quadratic performance criterion is parameterized by the index matrix Pp. It can characterize many different performance measures, of which the induced L2-gain has proven particularly useful. In fact, this quantity equals the H ∞-norm, if applied to an LTI system. The following characterization of robust quadratic performance can be derived.

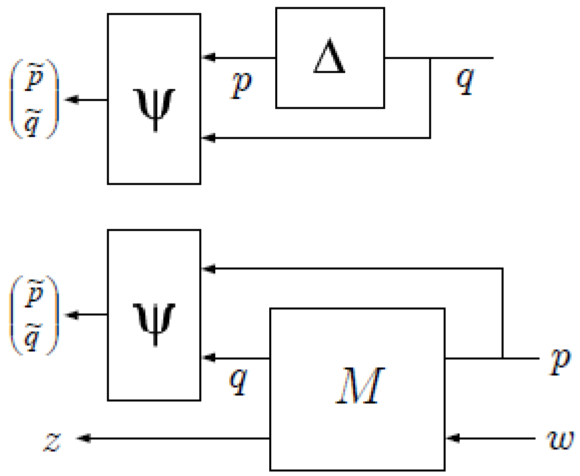

Fig. 2: Dynamic Multipliers

Suppose M is stable then the feedback interconnection of M with *Δ* is robustly stable and satisfies robust quadratic performance in the channel w → z, if there exists a multiplier II.

$$(\,..\,)' \left(\begin{array}{c|c} \Pi(i\omega) & 0 \\ \hline 0 & P_p \end{array} \right) \left(\begin{array}{c|c} I & 0 \\ M_{qp}(i\omega) & M_{qw}(i\omega) \\ \hline 0 & I \\ M_{zp}(i\omega) & M_{zw}(i\omega) \end{array} \right) \prec 0 \quad \forall \omega \in \mathbb{R} \cup \{\infty\}. \tag{1}$$

We can turn equation 1 into an LMI constraint by applying the KYP lemma. In fact, with the realization

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$$M = \left[\begin{array}{cc|c} A & B_p & B_w \\ \hline C_q & D_{qp} & D_{qw} \\ C_z & D_{zp} & D_{zw} \end{array}\right] \quad (2)$$

and the realization of ψ in

$$\left(\begin{array}{c|c} \Psi & 0 \\ 0 & I \end{array}\right)\left(\begin{array}{cc} I & 0 \\ M_{qp} & M_{qw} \\ \hline M_{zp} & M_{zw} \end{array}\right) = \left[\begin{array}{cc|cc} A_\Psi & B_{\Psi 2}C_q & B_{\Psi 1} + B_{\Psi 2}D_{qp} & B_{\Psi 2}D_{qw} \\ 0 & A & B_p & B_w \\ \hline C_\Psi & D_{\Psi 1}C_q & D_{\Psi 1} + D_{\Psi 2}D_{qp} & D_{\Psi 2}D_{qw} \\ 0 & C_z & D_{zp} & D_{zw} \end{array}\right] \quad (3)$$

Which is abbreviated as

$$\left(\begin{array}{ccc} I & 0 & 0 \\ \tilde{A} & \tilde{B}_p & \tilde{B}_w \\ \tilde{C}_q & \tilde{D}_{qp} & \tilde{D}_{qw} \\ \hline 0 & 0 & I \\ \tilde{C}_z & \tilde{D}_{zp} & \tilde{D}_{zw} \end{array}\right)' \left(\begin{array}{ccc|c} 0 & \mathcal{X} & 0 & 0 \\ \mathcal{X} & 0 & 0 & 0 \\ 0 & 0 & Q & 0 \\ \hline 0 & 0 & 0 & P_p \end{array}\right) \left(\begin{array}{ccc} I & 0 & 0 \\ \tilde{A} & \tilde{B}_p & \tilde{B}_w \\ \tilde{C}_q & \tilde{D}_{qp} & \tilde{D}_{qw} \\ \hline 0 & 0 & I \\ \tilde{C}_z & \tilde{D}_{zp} & \tilde{D}_{zw} \end{array}\right) \prec 0.$$

Controller Reconstruction:

An alternative approach is to first compute X from X; T, and then to solve equation 3 for the controller matrices $A_K;B_K;C_K;D_K$, while keeping X fixed. Note that the controller variables enter the closed loop matrices A; B; C;D in an affine fashion.

$$\left(\begin{array}{cc} I & 0 \\ \mathbf{A} & \mathbf{B}_p \\ \hline C_e & \tilde{D}_{qp} \\ \hline \mathbf{C}_z & \mathbf{D}_{zp} \end{array}\right)' \left(\begin{array}{cc|c|c} 0 & I & 0 & 0 \\ I & 0 & 0 & 0 \\ \hline 0 & 0 & Q & 0 \\ \hline 0 & 0 & 0 & I \end{array}\right) \left(\begin{array}{cc} I & 0 \\ \mathbf{A} & \mathbf{B}_p \\ \hline C_e & \tilde{D}_{qp} \\ \hline \mathbf{C}_z & \mathbf{D}_{zp} \end{array}\right) \prec 0,$$

In order to reduce the number of variables, thereby speeding up the numerical computations, one can eliminate (part of) the transformed controller variables K ; L; _M ;N. In the context of the nominal output feedback synthesis problem. Similar arguments allow for elimination in the robust synthesis problem as well. Recall our earlier notation for the augmented plant. The elimination of variables requires that we first turn the synthesis conditions affine in the decision variables X; T; _K ; L; _M ;N and ϒ. Thus, by applying the Schur complement formula, using the index P_p as

$$P_p = \begin{pmatrix} -\gamma & 0 \\ 0 & \frac{1}{\gamma} \end{pmatrix}$$

Which represents an the L_2 induced gain to be bounded by, we arrive at

$$\begin{pmatrix} \mathbf{A} + \mathbf{A}' & \mathbf{B}_p & \mathbf{B}_w & 0 \\ \mathbf{B}_p' & 0 & 0 & 0 \\ \mathbf{B}_w' & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & \mathbf{C}_z' \\ 0 & 0 & 0 & \mathbf{D}_{zp}' \\ 0 & 0 & -\gamma I & \mathbf{D}_{zw}' \\ \mathbf{C}_z & \mathbf{D}_{zp} & \mathbf{D}_{zw} & -\gamma I \end{pmatrix} +$$

$$\begin{pmatrix} C_e' \\ \tilde{D}_{qp}' \\ \tilde{D}_{qw}' \\ 0 \end{pmatrix} Q \begin{pmatrix} C_e & \tilde{D}_{qp} & \tilde{D}_{qw} & 0 \end{pmatrix} \prec 0,$$

In view of the synthesis result for a structured generalized plant with P_{qu} = 0, the natural question arises whether our controller synthesis result can be extended to more general interconnections of uncertain systems. An interconnection which is closely related to our initial configuration.

Uncertainty only enters a filter at the plant output, and a motivation for this problem is given next.

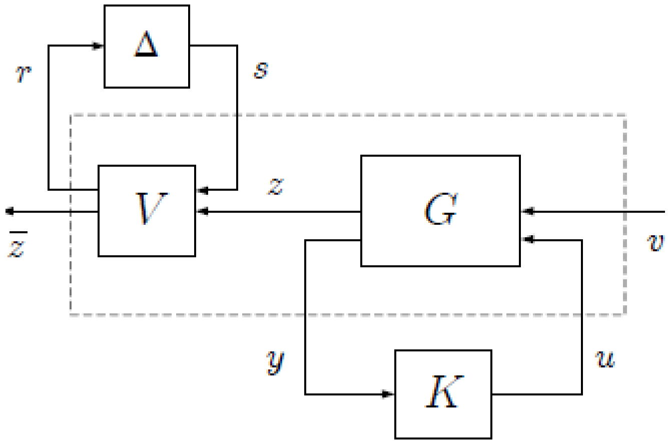

Fig. 3: System interconnection corresponding to generalized plant

Parameter Dependent Disturbance Filter:

The synthesis algorithm is applied to a design example. Here, the focus is on sinusoidal disturbances, which prevail in applications containing rotational mechanics such as helicopters, CD players or disk drives, and references therein. The period/frequency of the sinusoidal disturbance changes in time, which leads to non-stationary sinusoidal signals. As is shown below, such signals can be effectively modeled as the output of a parameter dependent oscillatorand references therein. For the discrete-time case, let δ(t) denote the single time-varying parameter for a continuous-time LPV system. For specified bounds on the parameter δ(t) and its rate-of-variation δ(t), a family of suitable classes of dynamic multipliers have been proposed and references therein. We will show that synthesis result based on static D=G scaling is conservative as compared to synthesis results based on dynamic multipliers. The example thus demonstrates the importance of taking a bound on the rate-of-variation into account.

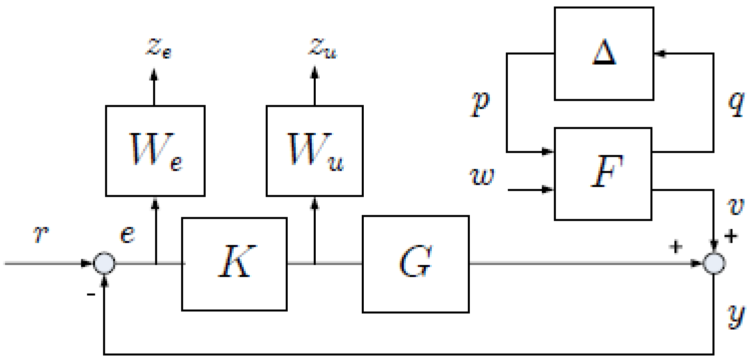

Fig. 4: System interconnection for the disturbance rejection problemConsider the interconnection in Figure 4 in which G is the plant model given as

$$G(s) = \frac{s + 0.1}{(s + 0.2)(s + 0.5)},$$

We;Wu are given weighting functions at the output.

We finally have been able to reduce the conservatism in handling non-stationary sinusoidal disturbance signals by using a class of dynamic multipliers in designs K_1;K_2. As a result, the disturbance rejection performance was improved. In particular, if the parameter is allowed to vary arbitrarily fast, as is the case in KDG, the resulting closed-loop frequency response involves a notch close to the highest frequency in the interval$[\omega_0(1-\bar{\delta}), \omega_0(1+\bar{\delta})]$, which does not effectively account for the fact that the sinusoidal frequency is specified in an interval. Once the rate-of-variation is bounded, this notch shifts to a lower frequency, as shown by design K_1. In case the bound on δ is further reduced, the sharp notch eventually vanishes. Moreover, the design K_2 shows that the closed loop gain from v → e is lower in an average sense, at the cost of a high gain at frequencies greater than 1 rad/s.

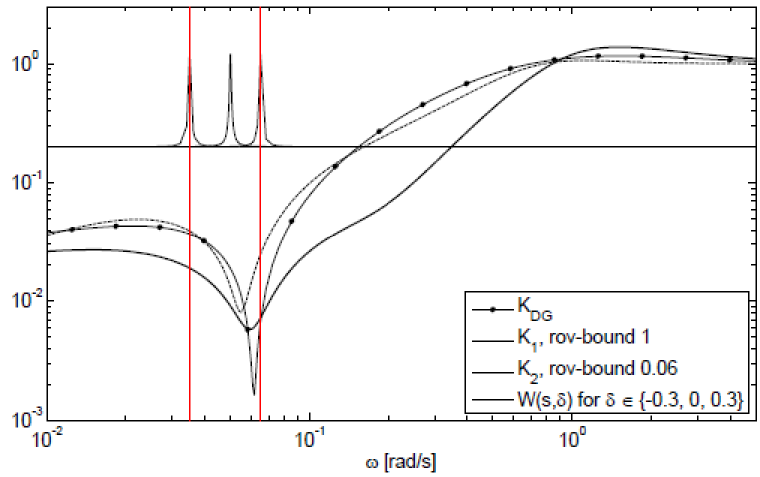

Fig. 5: Closed-loop sensitivity v → e

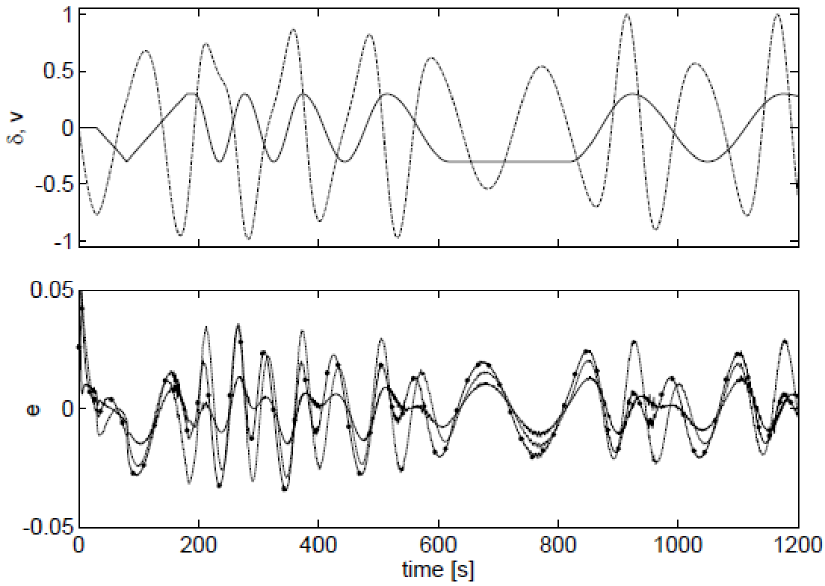

Fig. 6: tracking error e for designs KDG

Conclusion:

An LMI characterization of this property has recently been developed, for which a new elementary proof has been given. The synthesis solution boils down to a combination of two congruence transformations from the existing literature. One of these transformations is well-known, since it solves the nominal output feedback synthesis problem. It is expected that an LMI solution to the robust output feedback synthesis problem exists for many other interconnections of uncertain systems. As a preliminary study, we have considered the dual problem in which uncertainty enters a filter at the plant output. A solution to this problem could be obtained, when parametric uncertainties are involved and a class of static multipliers is used. In view of the fact that the general robust output feedback problem is largely open, it is a challenging research question which kind of interconnections of uncertain systems can be handled by convex optimization in the LMI framework. It is conjectured that the only structural property needed for a synthesis solution to exist in terms of LMIs, is the fact that the transfer matrix from control input to measurement output is not affected by uncertainty. Hence, the interconnections that can be handled in this fashion will cover a specific class of controller design problems only. In particular, it will not include the disturbance rejection problem for a single-input single-output system in which the plant model is affected by an additive uncertain element.

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