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Two Suggested Approaches to Estimate Two Parameters Rayleigh Distribution

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ABSTRACT

Rayleigh distribution is a member of exponential family distributions that involved in life time testing machine components that studied mean time to failure. The Distribution has two parameters: shift parameter indicated by (α) and scale parameter (β) described the behavior. Sometimes, the classical methods of estimating parameters of Rayleigh distribution suffered from weakness which results loss of significant information. Moreover, methods of artificial intelligence demonstrated a clear advantage of finding accurate estimation of the probability distribution curves in recent decades. The objective of this research is to introduce a promising estimation ad-hoc method in fast intelligent way that we call it (NEW-Sugg.) method based on nearest neighborhood technique in iterative manner. The (NEW-Sugg.) method was compared with other four estimation methods, three of them were classical methods: Maximum likelihood (**ML**) and Method of Moment (**MOM**).In order to complete the comparison, proposed maximum likelihood (**Prop.ML**) was performed. The optimality obtained from the comparison was investigated by using Mean Absolute Percentage (MAPE) criterion. Nine data sets of distinct empirical experiments were conducted with different values of parameters and different sample sizes. To conclude, **MOM** method got better results gradually as sample size grew for both (α, β).(**MOM**) & (**Prop.ML**) be equivalent in (α) estimation, while (**Prop.ML**) got optimality in (β) estimation. The major conclusion was the superiority of (**NEW-Sugg**.) method associated with all sample sizes for both parameters' estimations.

INTRODUCTION

The English physicist Lord Rayleigh (1880) wrote in this probability distribution firstly as mentioned Taha Anwar (2015). Rayleigh distribution is regarded as a member of exponential family distributions that involved in life time testing machine components that studied mean time to failure. Therefore, Rayleigh distribution is used because the life of the model theory reliability plays an important role in modeling the life of the random phenomenon as Andrew C. Mkolesia referred in his article (2016). Furthermore, Rayleigh distribution can be applied in organism's life expectancy especially in clinical trials in cancer studies and wind velocity.

Some recent contributions had been considered in this field such as: Smail Mahdi et al (2006) in comparison between Rayleigh with Logistic Distribution, Sanku Dey (2012) via introducing Bayesian estimation for inverse version of Rayleigh distribution which is improved by new weighted loss function by Huda A. Rasheed(2017). While Al-Kinani (2014) introduced emphasis comparison between Bayesian approach versus other Non-Bayesian one. The updating paper was formed by Fundi (2017) about estimating Rayleigh parameters under type II censoring data.

The importance of this paper is to introduce new contribution based on selecting combinations of classical estimations of two parameters of Rayleigh distribution, which are: maximum likelihood, method of moments,

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least squares in addition to a proposed modified maximum likelihood. The NEW-Sugg. method regarded the previous estimation as input initial values and then begun to detect the best combination among them in nearest neighbor region (or interval) until satisfying the optimality.

Rayleigh Distribution Characteristics:

Rayleigh distribution is a member of exponential family distributions represented in life time such as: Gamma, Weibull, etc. It is a special case of Generalized Rayleigh Distribution described by A.A. Al-Naqeeb (2009).

Afify (2004) and Walck (2007) described Rayleigh distribution with two parameters α and β has the probability density function below.

$$f(x;\alpha,\beta)=\frac{(x-\alpha)}{\beta^2}\exp\left\{-\frac{(x-\alpha)^2}{2\beta^2}\right\} \quad ;\text{ where is: } x>\alpha \quad , \quad \beta>0 \tag{1}$$

The cumulative probability distribution function of equation (1) above is obtained by:

$$F(x)=\Pr(X\le x)=1-\exp\left\{-\frac{(x-\alpha)^2}{2\beta^2}\right\} \tag{2}$$

Methods of Estimation:

1. ***Method of Moments:***

This method based on assumption that population moments of **r**^th^ order (μ_r) equals to sample moments of the same order (m_r), and then solving the equations to find parameters' estimations, Li (2012) considered the procedure as follows:

$$\mu_r=E(x^r) \tag{3}$$

$$\mu_1=E(x)$$

$$E(x)=\beta\sqrt{\frac{\pi}{2}}+\alpha$$

$$\mu_2=E(x^2)$$

$$E(x^2)=2\beta^2+\sqrt{2\pi}\alpha\beta+\alpha^2$$

While the sample moment of order **r** is:

$$m_r=\frac{1}{n}\sum_{i=1}^{n}(x_i^r) \tag{4}$$

$$\therefore m_1=\frac{\sum_{i=1}^{n}(x_i)}{n}=\bar{x}$$

By putting : $\mu_1=m_1$

$$\hat{\beta}\sqrt{\frac{\pi}{2}}+\hat{\alpha}=\bar{x}$$

$$m_2=\frac{\sum_{i=1}^{n}x_i^2}{n}$$

By putting : $\mu_2=m_2$

$$2\hat{\beta}^2 + \sqrt{2\pi}\hat{\alpha}\hat{\beta} + \hat{\alpha}^2 = \frac{\sum_{i=1}^{n} x_i^2}{n}$$

But; $\frac{\sum_{i=1}^{n} x_i^2}{n} = Var(x) + [E(x)]^2$

$$S^2 = \left(2 - \frac{\pi}{2}\right)\hat{\beta}^2$$

$$\therefore S^2 = \left(\frac{4-\pi}{2}\right)\hat{\beta}^2$$

Where is : $\pi = \frac{22}{7}$

By solving the last equation above we get the moment estimation of (α) & (β) are:

$$\hat{\beta}_{mom} = \sqrt{\frac{2}{4-\pi}}S \quad (5)$$

$$\hat{\alpha}_{mom} = \bar{x} - \sqrt{\frac{\pi}{2}}\hat{\beta}_{mom} \quad (6)$$

S: the standard deviation of the data.

2. Maximum Likelihood Method:

The maximum likelihood method represents is one of the most significant method that maximize the likelihood for a r.v. follows Rayleigh distribution with location parameter (α) and scale parameter (β). This method had a well discussion by Yahya (2013) and Ahmed (2013). We can get their estimates by following steps:

$$L(x_i;\alpha,\beta) = \pi_{i=1}^{n}\left[\frac{(x_i-\alpha)}{\beta^2}e^{-(xi-\alpha)/2\beta^2}\right] \quad (7)$$

$$L(x_i;\alpha,\beta) = \frac{1}{\beta^{2n}}\pi_{i=1}^{n}(x_i-\alpha)\exp\left\{-\frac{\sum_{i=1}^{n}(xi-\alpha)^2}{2\beta^2}\right\}$$

But, after taking logarithm:

$$\ln L(x;\alpha,\beta) = -2n\ln(\beta) + \sum_{i=1}^{n}\ln(x_i-\alpha) - \frac{\sum_{i=1}^{n}(x_i-\alpha)^2}{2\beta^2}$$

with partial derivatives with respect to (α) and (β) and equating them to zero respectively, we will get:

$$\frac{\partial \ln L}{\partial \alpha} = 0 \quad (8)$$

$$-\sum_{i=1}^{n}\left(\frac{1}{x_i-\hat{\alpha}}\right)+\frac{\sum_{i=1}^{n}x_i-n\hat{\alpha}}{\hat{\beta}^2}=0$$

Since, $x_i > \alpha \quad \rightarrow \quad \min(x_i) \geq \hat{\alpha}$

Hence, α estimation will be merely the first ordered statistics of (x_i) after sort them in ascending manner, i.e.,

$$\hat{\alpha}_{ML}=x_{(1)} \tag{9}$$

Similarly, we can get (β) estimation by:

$$\frac{\partial \ln L}{\partial \beta}=0 \tag{10}$$

$$-\frac{2n}{\hat{\beta}}+\frac{\sum_{i=1}^{n}(x_i-\hat{\alpha})^2}{\hat{\beta}^2}=0$$

$$\hat{\beta}_{ML}=\sqrt{\frac{\sum_{i=1}^{n}(x_i-\hat{\alpha}_{ML})^2}{2n}}$$

$$\therefore \hat{\beta}_{ML}=\sqrt{\frac{\sum_{i=1}^{n}\left(x_i-x_{(1)}\right)^2}{2n}} \tag{11}$$

3. *Proposed Maximum Likelihood:*

We propose the following modification upon the classical Maximum Likelihood described above. We do this by using median value in two instead of mean of data.

i. Beginning the procedures with input data of (x_i).
ii. Calculate μ_e as the median of (x_i).
iii. Find the Mean Absolute of Deviation of μ_e which is given by:

$$S_A=\frac{1}{n}\sum_{i=1}^{n}\left|x_i-\mu_e\right| \tag{12}$$

iv. Initializing two parameters with values:

$$\beta_0=S_A\cdot\sqrt{\frac{2}{4-\pi}} \tag{13}$$

$$\alpha_0=\bar{x}-\beta_0\sqrt{\frac{\pi}{2}} \tag{14}$$

v. Enter iterative procedure with the following:

$$\hat{\beta}_{i.}=\sqrt{\frac{\sum_{i=1}^{n}(x_i-\hat{\alpha}_{i-1.})^2}{2n}} \tag{15}$$

$$\hat{\alpha}_{i.}=\bar{x}-\hat{\beta}_{i-1}\sqrt{\frac{\pi}{2}} \tag{16}$$

vi. Construct the stopping rule (TD) interms of total deviation as below.

$$TD = \frac{\left|\hat{\alpha}_i - \hat{\alpha}_{i-1}\right| + \left|\hat{\beta}_i - \hat{\beta}_{i-1}\right|}{2} \tag{17}$$

vii. Stop if $TD \leq 0.0001$ and obtain: $\left(\hat{\alpha}_{i.} \quad , \quad \hat{\beta}_i\right)$ at final.

4. The NEW-Sugg. suggested Method:

In seeking a new promising hybrid approach by obtaining the estimation of parameters by indicating the nearest neighbor strategy in iterative manner in six-sigma pattern.

The essential thing is to achieve maxima (or minima) in wide scope of point estimations comes originally from initial values and their neighbors as selected samples and then resampling alternative points iteratively.

1st Stage: Initialize procedure with values of the couple parameters (α , β) by choosing the classical estimations that got in advance: Ml & Mom in addition to the Prop.ML as "seeds".

2nd Stage: Select the best combination estimation (α_0 , β_0) according to (MAPE) criterion.

3rd Stage: Construct an area performed by points (say; α_{0i} , β_{0i}); i=1,2,3, …, 200 that belonging to the window {$\alpha_{0i} \in (\alpha_{0i} \pm Sig(\alpha)$, $\beta_{0j} \in (\beta_{0i} \pm Sig(\beta)$}, where is:

$$Sig(\varphi) = 6 * \sqrt{\frac{\sum_{j=1}^{r}\left(\varphi_{0ij} - \varphi_{0i}\right)^2}{r}} \tag{18}$$

Where is **(φ)** represents **(α)** or **(β)**.

r : represents number of replicates.

(i): is one of the four methods.

4th Stage: Make comparisons among the selected best method (α_0 , β_o) got in step(2) against the (α_{0i} , β_{0i}) to get the required , regarded as candidate estimations in successive comparisons among them against (α_0 , β_o) via (MAPE) until getting settle down in convergence stability.

5th Stage: Going on NEW-Sugg. estimations consequently in alternative approach until obtain $\left(\hat{\alpha}_{sugg.}, \hat{\beta}_{sugg.}\right)$ finally.

Comparison Criteria:

Suppose (φ) be one of the two parameters of Rayleigh distribution (λ) or (θ). Let (**r**) be the replications.

Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{1}{rep}\sum_{i=1}^{rep}\left|\frac{\varphi_i - \hat{\varphi}_i}{\varphi_i}\right| \tag{19}$$

Rashid et al (2011) used Total Deviations (TD)

$$TD = \frac{1}{rep}\sum_{i=1}^{rep}\left(\left|\frac{\hat{\alpha}_i - \alpha_i}{\alpha_i}\right| + \left|\frac{\hat{\beta}_i - \beta_i}{\beta_i}\right|\right) \tag{20}$$

Empirical Simulations:

The model represents the system itself, while the simulation represents the operation of the system over time. Computer simulations have become a useful part of mathematical modeling of many natural systems in sciences .So , the simulation is a type of sampling techniques as had been referred by Taha (2015).

Simulations for both parameters (α and β) had been done as follows:

α = (0.50, 0.75, 1.0)
β = (1, 2, 3.5)
n = (10, 25, 50, 200)

The numerical results in the following tables: (1, 2, 3, 4, 5, 6 and 7) as below.

Australian Journal of Basic and Applied Sciences, 11(16) December 2017, Pages: 1-10

Discussion and Conclusion:

On the light of experimental results got above, both parameters (α , β) of Rayleigh distribution are estimated using five methods that compared by (MAPE). We can list the following conclusions:

a) In general, it can be noticed that (NEW-SUGG.) suggested method the best than the other methods for both parameters with respect to (MAPE) criterion and for all sample sizes.

b) (MOM) method got better gradually as sample size grew for MAPE(α) compared with (ML) and (Prop.ML), while it got closer to (Prop.ML) as sample sizes (10 , 25), i.e. both of (MOM) & (Prop.ML) be equivalent and better than the other two methods (ML). On other hand, (MOM)and (Prop.ML) got more closeness at the sample sizes (50, 200).

c) (Prop.ML) is the second best method in estimation after (NEW-SUGG.) for MAPE(β) in Table (3) compared with the rest four methods with all sample sizes.

d) (MOM) method got better results gradually as sample size grew for MAPE(β) in Table (4) compared with (ML) while it got closer to (Prop.ML) as sample sizes (200).

e) The Replicates in Table (6) showed that (MOM) had the major contribution that got the best sharing in (NEW-Sugg.) method optimization.

f) The Replicates in Table (7) reflect the competition in global superiority among the five methods that represented as: (MOM and Prop.ML) when ordered in descending respectively. While the (NEW-SUGG.) Method beats them all.

Table 1: Estimated (α)

			Methods of Estimation			
n	α	β	MLE	MOM	Prop.ML	NEW-SUGG.
10	1	0.5	1.197849	1.017038	1.049020	1.001720
		0.75	1.300470	1.028457	1.075651	1.004197
		1.0	1.404631	1.044674	1.107472	1.005660
	2	0.5	2.203064	2.025092	2.056488	2.003233
		0.75	2.302654	2.028737	2.075870	2.003785
		1.0	2.393647	2.037441	2.100209	2.004668
	3.5	0.5	3.704836	3.523060	3.554418	3.502553
		0.75	3.803400	3.541135	3.587641	3.502497
		1.0	3.887730	3.518989	3.582646	3.503420
25	1	0.5	1.123654	1.005349	1.018341	1.001293
		0.75	1.188789	1.014976	1.034021	1.001956
		1.0	1.251342	1.023856	1.049107	1.002061
	2	0.5	2.123805	2.004390	2.017383	2.000744
		0.75	2.189956	2.017507	2.036540	2.001239
		1.0	2.250384	2.013691	2.039182	2.003011
	3.5	0.5	3.625259	3.506241	3.519171	3.501171
		0.75	3.689612	3.506058	3.525326	3.501168
		1.0	3.755704	3.511482	3.536859	3.501957
50	1	0.5	1.088491	1.003633	1.010283	1.000790
		0.75	1.130233	1.004031	1.013838	1.001594
		1.0	1.176157	1.008075	1.020961	1.002179
	2	0.5	2.089131	2.003728	2.010399	2.001151
		0.75	2.132597	2.007699	2.017480	2.000812
		1.0	2.180145	2.011511	2.024385	2.001568
	3.5	0.5	3.589863	3.505828	3.512485	3.500435
		0.75	3.631736	3.503279	3.513070	3.501379
		1.0	3.677461	3.510005	3.522902	3.501785
200	1	0.5	1.045197	1.001855	1.003821	1.000685
		0.75	1.065182	0.998670	1.001425	1.000802
		1.0	1.091314	1.005998	1.009526	1.001591
	2	0.5	2.043725	2.000494	2.002462	2.000676
		0.75	2.065665	1.998602	2.001349	2.000724
		1.0	2.088025	1.999082	2.002620	2.001117
	3.5	0.5	3.544377	3.499869	3.501839	3.500693
		0.75	3.566449	3.500818	3.503569	3.500690
		1.0	3.587769	3.500062	3.503592	3.500871

Table 2: Estimated (β)

			Methods of Estimation			
n	α	β	MLE	MOM	Prop.ML	NEW-SUGG.
10	1	0.5	0.377184	0.491037	0.465588	0.498817
		0.75	0.556336	0.727566	0.689979	0.747367
		1.0	0.743547	0.970168	0.920131	0.997600
	2	0.5	0.369864	0.481962	0.456980	0.498321
		0.75	0.554138	0.726598	0.689061	0.748137
		1.0	0.745146	0.969738	0.919724	0.997961
	3.5	0.5	0.367003	0.481348	0.456397	0.498771
		0.75	0.551552	0.716872	0.679834	0.748351
		1.0	0.751685	0.983523	0.932801	0.997415
25	1	0.5	0.423268	0.497285	0.486988	0.499720
		0.75	0.627664	0.736369	0.721241	0.749202
		1.0	0.839030	0.981357	0.961279	0.998681
	2	0.5	0.422662	0.497275	0.486977	0.499751
		0.75	0.627978	0.735842	0.720725	0.749495
		1.0	0.842942	0.990888	0.970618	0.998530
	3.5	0.5	0.420525	0.494833	0.484584	0.499407
		0.75	0.630584	0.745195	0.729890	0.749358
		1.0	0.833948	0.986332	0.966153	0.998423
50	1	0.5	0.443294	0.496271	0.491033	0.499707
		0.75	0.668108	0.746990	0.739233	0.749035
		1.0	0.886128	0.991177	0.980964	0.999084
	2	0.5	0.444528	0.497860	0.492607	0.499588
		0.75	0.666810	0.744834	0.737098	0.749491
		1.0	0.885065	0.990387	0.980183	0.999337
	3.5	0.5	0.444220	0.496772	0.491529	0.499794
		0.75	0.665352	0.745593	0.737849	0.748885
		1.0	0.887361	0.992084	0.981862	0.999223
200	1	0.5	0.471719	0.498812	0.497312	0.499593
		0.75	0.709350	0.750894	0.748764	0.749499
		1.0	0.943962	0.997314	0.994567	0.999086
	2	0.5	0.472607	0.499617	0.498116	0.499618
		0.75	0.706727	0.748614	0.746490	0.749536
		1.0	0.945170	1.000723	0.997968	0.999326
	3.5	0.5	0.472523	0.500317	0.498814	0.499608
		0.75	0.708040	0.749060	0.746934	0.749602
		1.0	0.942745	0.997532	0.994785	0.999266

Table 3: MAPE(α)

			Methods of Estimation			
n	α	β	MLE	MOM	Prop.ML	NEW-SUGG.
10	1	0.5	0.197849	0.119418	0.121266	0.002376
		0.75	0.300470	0.173852	0.175242	0.005128
		1.0	0.404631	0.223424	0.229109	0.006904
	2	0.5	0.101532	0.056979	0.058440	0.001967
		0.75	0.151327	0.084946	0.086552	0.002388
		1.0	0.196823	0.111563	0.114022	0.003006
	3.5	0.5	0.058524	0.032776	0.033202	0.000930
		0.75	0.086686	0.048437	0.050153	0.001028
		1.0	0.110780	0.066383	0.066369	0.001332
25	1	0.5	0.123654	0.068344	0.068852	1.001293
		0.75	0.188789	0.104860	0.105594	0.002595
		1.0	0.251342	0.142847	0.145363	0.002895
	2	0.5	0.061903	0.036243	0.036353	0.000579
		0.75	0.094978	0.052516	0.053464	0.000956
		1.0	0.125192	0.070742	0.071099	0.001931
	3.5	0.5	0.035788	0.020672	0.020911	0.000463
		0.75	0.054175	0.030727	0.030824	0.000517
		1.0	0.073058	0.041470	0.041616	0.000811
50	1	0.5	0.088491	0.049824	0.050138	0.001097
		0.75	0.130233	0.075687	0.076047	0.002037
		1.0	0.176157	0.101285	0.101745	0.002778
	2	0.5	0.044565	0.025724	0.025829	0.000719
		0.75	0.066298	0.037782	0.038071	0.000632
		1.0	0.090072	0.048317	0.048659	0.001069
	3.5	0.5	0.025675	0.013890	0.013995	0.000204
		0.75	0.037639	0.021794	0.021851	0.000520
		1.0	0.050703	0.028018	0.028107	0.000684
200	1	0.5	0.045197	0.025420	0.025593	0.000834
		0.75	0.065182	0.036612	0.036633	0.000997

		1.0	0.091314	0.049340	0.049621	0.001869
	2	0.5	0.021863	0.012815	0.012818	0.000412
		0.75	0.032833	0.018721	0.018647	0.000477
		1.0	0.044012	0.025148	0.025143	0.000716
	3.5	0.5	0.012679	0.007445	0.007454	0.000248
		0.75	0.018985	0.010513	0.010529	0.000262
		1.0	0.025077	0.013952	0.013910	0.000341

Table 4: MAPE(β)

			Methods of Estimation			
n	α	β	MLE	MOM	Prop.ML	NEW-SUGG.
10	1	0.5	0.260908	0.192968	0.192520	0.003506
		0.75	0.271121	0.198755	0.200265	0.004868
		1.0	0.266304	0.189710	0.191589	0.003739
	2	0.5	0.277043	0.203761	0.207237	0.004801
		0.75	0.273563	0.200675	0.201414	0.003883
		1.0	0.269265	0.190792	0.192747	0.003294
	3.5	0.5	0.277166	0.196948	0.200039	0.003928
		0.75	0.275967	0.194247	0.198483	0.003474
		1.0	0.265556	0.200740	0.200986	0.003930
25	1	0.5	0.163592	0.119489	0.119500	0.001416
		0.75	0.173834	0.120406	0.121578	0.001867
		1.0	0.170010	0.123030	0.124163	0.002152
	2	0.5	0.165025	0.121839	0.121616	0.001345
		0.75	0.172264	0.119182	0.120795	0.001483
		1.0	0.165845	0.120618	0.121123	0.002358
	3.5	0.5	0.165848	0.119060	0.120117	0.002042
		0.75	0.168212	0.121224	0.121022	0.001785
		1.0	0.173143	0.122012	0.122530	0.002408
50	1	0.5	0.119837	0.083771	0.084348	0.001164
		0.75	0.115321	0.085811	0.085804	0.001898
		1.0	0.121528	0.086457	0.086793	0.001509
	2	0.5	0.117872	0.083751	0.084027	0.001343
		0.75	0.119692	0.086217	0.086404	0.001320
		1.0	0.121137	0.080039	0.080866	0.001238
	3.5	0.5	0.118180	0.080749	0.081172	0.001044
		0.75	0.119398	0.087160	0.087158	0.002129
		1.0	0.118580	0.085152	0.085395	0.001442
200	1	0.5	0.060076	0.041494	0.041591	0.001142
		0.75	0.057827	0.040752	0.040785	0.000970
		1.0	0.060138	0.041744	0.041801	0.001209
	2	0.5	0.059552	0.042715	0.042717	0.001073
		0.75	0.061148	0.042057	0.042017	0.000965
		1.0	0.058743	0.041810	0.041819	0.000968
	3.5	0.5	0.058795	0.043542	0.043489	0.001110
		0.75	0.059638	0.042113	0.042162	0.000840
		1.0	0.061578	0.042900	0.042960	0.001052

Table 5: TD(α, β)

			Methods of Estimation			
n	α	β	MLE	MOM	Prop.ML	NEW-SUGG.
10	1	0.5	0.458758	0.312386	0.313786	0.005882
		0.75	0.571592	0.372608	0.375508	0.009996
		1.0	0.670936	0.413134	0.420698	0.010642
	2	0.5	0.378576	0.260740	0.265678	0.006768
		0.75	0.424890	0.285622	0.287966	0.006272
		1.0	0.466088	0.302356	0.306770	0.006300
	3.5	0.5	0.335690	0.229724	0.233242	0.004858
		0.75	0.362652	0.242684	0.248636	0.004502
		1.0	0.376336	0.267122	0.267354	0.005262
25	1	0.5	0.287246	0.187832	0.188352	0.003152
		0.75	0.362622	0.225266	0.227172	0.004462
		1.0	0.421352	0.265878	0.269526	0.005048
	2	0.5	0.226928	0.158082	0.157968	0.001924
		0.75	0.267242	0.171698	0.174258	0.002438
		1.0	0.291036	0.191360	0.192222	0.004288
	3.5	0.5	0.201636	0.139732	0.141028	0.002504
		0.75	0.222388	0.151952	0.151846	0.002302
		1.0	0.246202	0.163482	0.164146	0.003220
50	1	0.5	0.208328	0.133594	0.134486	0.002260
		0.75	0.245554	0.161498	0.161850	0.003934
		1.0	0.297686	0.187742	0.188538	0.004286

	2	0.5	0.162438	0.109474	0.109856	0.002062
		0.75	0.185990	0.124000	0.124474	0.001952
		1.0	0.211210	0.128356	0.129526	0.002306
	3.5	0.5	0.143856	0.094640	0.095168	0.001248
		0.75	0.157036	0.108954	0.109008	0.002648
		1.0	0.169284	0.113170	0.113502	0.002126
200	1	0.5	0.105272	0.066914	0.067184	0.001976
		0.75	0.123010	0.077364	0.077418	0.001968
		1.0	0.151452	0.091084	0.091422	0.003078
	2	0.5	0.081414	0.055530	0.055536	0.001484
		0.75	0.093980	0.060778	0.060664	0.001442
		1.0	0.102756	0.066958	0.066962	0.001684
	3.5	0.5	0.071474	0.050986	0.050942	0.001358
		0.75	0.078624	0.052626	0.052690	0.001102
		1.0	0.086654	0.056852	0.056870	0.001392

Table 6: Replicates of the number of Best Method Occurred

			Methods of Estimation		
n	α	β	MLE	MOM	Prop.ML
10	1	0.5	177	156	233
		0.75	155	145	253
		1.0	138	175	262
	2	0.5	163	145	207
		0.75	151	146	245
		1.0	146	150	249
	3.5	0.5	170	145	209
		0.75	164	165	214
		1.0	198	161	211
25	1	0.5	204	125	264
		0.75	168	120	275
		1.0	167	136	285
	2	0.5	227	95	249
		0.75	196	135	229
		1.0	194	144	256
	3.5	0.5	217	120	231
		0.75	220	117	248
		1.0	186	119	264
50	1	0.5	218	114	263
		0.75	201	85	299
		1.0	183	96	311
	2	0.5	216	90	264
		0.75	207	83	278
		1.0	192	108	267
	3.5	0.5	228	107	255
		0.75	239	90	254
		1.0	237	87	243
200	1	0.5	227	61	293
		0.75	234	74	302
		1.0	193	83	308
	2	0.5	259	61	275
		0.75	244	54	306
		1.0	259	77	279
	3.5	0.5	273	59	273
		0.75	244	67	278
		1.0	262	69	275

Table 7: Replicates of the number of Best Method Occurred

			Methods of Estimation			
n	α	β	MLE	MOM	Prop.ML	NEW-SUGG.
10	1	0.5	0	0	0	1000
10		0.75	0	0	0	1000
10		1.0	0	0	0	1000
10	2	0.5	0	0	0	1000
10		0.75	0	0	1	1000
10		1.0	0	0	0	1000
10	3.5	0.5	0	1	1	1000
10		0.75	0	0	0	1000
10		1.0	0	0	0	1000
25	1	0.5	0	1	0	1000
25		0.75	0	1	0	1000
25		1.0	0	0	0	1000
25	2	0.5	0	0	0	1000

25		0.75	0	1	0	1000
25		1.0	0	0	0	1000
25	3.5	0.5	0	0	0	1000
25		0.75	0	0	2	1000
25		1.0	0	0	0	1000
50	1	0.5	0	0	0	1000
50		0.75	0	0	0	1000
50		1.0	0	0	0	1000
50	2	0.5	0	0	0	1000
50		0.75	0	1	0	1000
50		1.0	0	0	0	1000
50	3.5	0.5	0	0	0	1000
50		0.75	0	0	1	1000
50		1.0	0	0	0	1000
200	1	0.5	0	1	0	1000
200		0.75	0	0	0	1000
200		1.0	0	0	0	1000
200	2	0.5	0	0	0	1000
200		0.75	0	0	0	1000
200		1.0	0	0	0	1000
200	3.5	0.5	0	0	0	1000
200		0.75	0	0	0	1000
200		1.0	0	0	0	1000

REFERENCES

AL-Naqeeb, A.A. and A.M. Hamed, 2009. "Estimation of the two parameters for Generalized Rayleigh Distribution Function using simulation Technique", Ibn Al- Haitham J. for Pure & Appl. SCI., 22.

Afify, E.E., 2004. "Comparison of Estimators of Parameters for the Rayleigh Distribution", Faculty of Eng. Shibeen El KomMenoufia University.

Ahmed, A., S.P. Ahmed and J.A. Reshi, 2013. "Bayesian Analysis of Rayleigh Distribution",International Journal of Scientific and Research Publications, 3(10): 1-9.

Al – Kanani, Iden Hassan and Jasim, Shaima Abbas, 2014. "Non-Bayesian and Bayesian Estimation for Generalized Rayleigh Distribution", International Journal of Modern Mathematical Sciences, 10(2): 103-115.

Dey , Sanku, 2012. "Bayesian Estimation of the Parameter and Reliability Function of an Inverse Rayleigh Distribution", Malaysian Journal of Mathematical Sciences, 6(1): 113-124.

Fundi, Murithi Daniel, Njenga, Edward Gachangi and Keitany, Kemboi George, 2017. "Estimation of Parameters of the Two-Parameter RayleighDistribution Based on Progressive Type-II Censoring Using Maximum Likelihood Method via the NR and the EM Algorithms", American Journal of Theoretical and Applied Statistics, 6(1): 1-9.

http://dx.doi.org/10.1155/2016/8246390.

http://interstat.statjournals.net/YEAR/2003/articles/0302004.pdf.

Li, Yaowu and Li, Meihua, 2012. "Moment Estimation of the Parameters in Rayleigh Distribution with Two Parameters", Communications in statistics theory and methods, 41(15): 2643-2660.

Mahdi, Smail and Cenac, Myrtene, 2006. "Estimating and Assessing the Parameters of the Logistic and Rayleigh Distributions from Three Methods of Estimation",Caribb. J. Math. Comput. Sci., 13: 25-34.

Mkolesia, A.C., C.R. Kikawa and M.Y. Shatalov, 2016. " Estimation of the Rayleigh Distribution Parameter",Transylvanian Review: XXIV(8): 1158-1163.

Rasheed, Huda A. and ArefRaghdaKh., 2017. "Bayes Estimator for Inverse Rayleigh Distribution under Generalized Weighted Loss Function",Mathematics and Statistics Journal, 3(1): 18 -24.

Rashid, M. Zahid and Akhter, A. Saeed, 2011. "Estimation Accuracy of Exponential Distribution Parameters", Pakistan Journal of Statistics and Operation Research, VII(2): 217-232.

Seo, Jung-In,Jeon, Jae-Woo and Kang , Suk-Bok, 2016. "Exact Interval Inference for the Two-Parameter Rayleigh Distribution Based on the Upper Record Values", Journal of Probability and Statistics, pp. (1-5).Article ID 8246390.Hindawi Publishing Corporation.

Taha, Taha Anwar, 2015. "On Reliability Estimation for the Rayleigh Distribution Based on Monte Carlo Simulation", International Journal of Science and Research (IJSR), 6(8): 1163-1167. DOI: 10.21275/ART20176044.

Walck, Christian, 2007. "Hand-book on Statistical Distributions for experimentalists",Internal Report SUF–PFY/96–01,University of Stockholm.

Yahya Mahdi Al Mayali and rtifaaAbulKadhumAl_Shaibani, 2013. "A Comparison for Some of the Estimators of Rayleigh Distribution with Simulation", Journal of Karbala University, 11(4): 104-112.