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The Revenue Generating Function for Non Regular Fixed Lifetime Inventory System, a Case Study of Hotel Rooms

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ABSTRACT

Hotel rooms are classified as non-regular fixed lifetime inventory systems because rooms that outdate in one period become helpful at the start of the next period. Over the years, hotel managers in Nigeria have found it difficult to determine how much money they make and trace the number of rooms in demand or not in demand. This study develops a model to determine how much hotel managers make in a period trace the number of rooms in demand and the number of rooms available to meet demand. The revenue generated from each subunit of the hotel, the outdates and the shortages in a period are determined. Finally, the demand matrix for hotel rooms is derived and used to compare the performance of the two hotels.

Keywords: Hotel, rooms, demand, revenue, outdate, shortage

INTRODUCTION

The perishable inventory system is divided into the fixed lifetime inventory system and the non-fixed lifetime inventory system. The fixed lifetime inventory system is further divided into the regular fixed lifetime inventory and non-regular fixed lifetime inventory systems(Izevbizua and Apanapudor, 2019). In the regular fixed lifetime inventory system, items not used to meet demand at the end of their helpful inventory period are outdated and discarded (Ulku and Emre, 2021). On the other hand, the non-regular fixed lifetime inventory system is one in which items not used to meet demand at the end of a period are outdated but are not discarded from inventory. Instead, they become helpful at the start of the next period. Examples of such inventory include the rooms in a hotel, any room(s) not used to meet demand at the end of a day is considered outdated but become useful at the start of the next day or period. Other examples include; seats on the airplane, any seat(s) not used to meet demand in a particular flight outdated and useful by the next flight. Advert space(s) in the daily newspaper, spaces not used on a specific day, outdate and can be used the next day. Advertising on TV, periods not used for advertising/or paid programming in one period can be used up in the next period. The same applies to seats on a bus and a train, where seats not used in a period can be used in the next period.

Hotels are significant in the growth of any economy (www.intellectsoft.net (2021)) as they provide a home away from home for businessmen and women. They also play an instrumental role in the tourism business growth in any country. Many authors have studied the hotel business. Zhang *et al*. (2011) investigated how location and situation factors influence hotel room costs in Beijing. Their study offers a complete reference table of empirical information linked to hotel room pricing analysis using various research methods such as consumer behavioural analysis, conjoint analysis, and hedonic price analysis. Guenter (2012) conducted research on hotel room rate modeling in a small market. Using a hedonic model, they calculated the willingness to pay for various hotel qualities in a small market scenario. Bing *et al*. (2012) conducted a study utilizing search engine data to estimate hotel room demand. They looked at the utility of search query volume data in anticipating hotel room demand and tried to find the best econometric forecasting model. Liuyi *et al*. (2015) conducted a study on hotel room availability collaborating with online travel agents. They introduced a new way for hotels to collaborate with an OTA on booking services by managing hotel room availability for the partner OTA. During the sales time, customers can make bookings directly through the hotel's distribution channel or indirectly through the OTA, if necessary. After receiving adequate room bookings, the hotel forecasts room demand based on distribution information and optimizes room availability for maximum income by determining if on-hand rooms are available for the OTA. Craig and Frank (2018) researched hotel demand. From 2005 to 2014, they examined the impact of a wide range of political and sporting events on the hotel room market in Charlotte, North Carolina, using daily hotel occupancy data. NASCAR auto races and two political conventions were linked to significant hotel occupancy, prices, and revenue increases. Guanqun et al. (2020) looked at a room allocation problem for a hotel with several channels for room reservations. Traditional room allocation models presume that hotels can predict demand function and/or arrival process features to some extent. Minimizing running costs while fixing an initial profit target is the goal of hotel managers, Lofti and Salah (2021). For as much as possible, hotel managers try to avoid the vacation of the server, a situation where there are no customers demanding rooms(Radhamani et al., 2021).

Hotels are classified based on several factors such as size, target market, location, available facilities, type of service, affiliation and ownership. Besides, hotels can also be rated according to the crown, star or diamond system, which depends on their location. Hotel rooms are the main focus of this work. The total number of rooms in the hotel is fixed and are classified into subunits based on the type of bed, number of occupants, number of beds, décor, specific furnishing or features, views and service provided.

Some common subunits in hotels, according to setupmyhotel.com, are;

Single Room: A room with one or more beds that are allocated to one person. The size of the room or area measures $37m^2$ to $45m^2$.

Double Room. A room with one or more beds that are allocated to two people. The size of the room or area measures $40m^2$ to $45m^2$.

Triple Room: A room with three twin beds or one double bed and one twin bed to accommodate three people. The measurement is mostly $45m^2$ to $65m^2$.

Quad Room: A room that accommodates four people and measures $70m^2$ to $85m^2$.

Queen Room: A room that accommodates one or more people with a queen-sized bed and measures $32m^2$ to $50m^2$.

King Room: A room that accommodates one or more people with a king-sized bed and measures $32m^2$ to $50m^2$.

Double-double Room: A room that accommodates two to four people with two double beds and measures $50m^2$ to $70m^2$.

Suite or Executive Suite: One or more bedrooms that are linked by a parlour or a living room and measures $70m^2$ to $100m^2$.

Mini-Suite or Junior Suite: This single room has a bed and a sitting area. The sleeping area is sometimes in a bedroom separated from the parlour or the living room and measures $60m^2$ to $80m^2$**.**

Presidential Suite: This is the most expensive room in a hotel. Like the normal suites, a presidential suite always has one or more bedrooms and a living area, emphasizing magnificent in-room decorating, high-quality facilities and supplies, and custom-tailored services. This measures $80m^2$ to $350m^2$.

Apartments: This room type is familiar to service apartments and hotels that cater for long-term visitors. Things usually available in the room are open kitchens, cooking equipment, dryers, washers, and other amenities. This measures $96m^2$ to $250m^2$.
Accessible Room or Disabled Room: This room type is mainly designed for disabled guests. This measures $30m^2$ to $42m^2$.
Cabana Room: This type of room is either located next to a swimming pool or has a private pool attached to it. This measures $30m^2$ to $45m^2$.

Villa Room: This is a specific accommodation type found in some hotels. It's a kind of stand-alone residence that allows hotel guests more privacy and space. This measures $100m^2$ to $150m^2$**.**

Smoking or Non-smoking Room: To reduce the effects of second-hand smoke exposure on non-smoking guests, several hotels offer both smoking and non-smoking rooms.

DESCRIPTION OF THE MODEL

The total number of rooms K in a hotel is fixed and subdivided into subunits $k_1, k_2, ... k_n$. The number of rooms in each subunit is also fixed. The price for a room in each subunits $k_1, k_2, ... k_n$. is $x_1, x_2, .. x_n$. The rooms in each subunit is further divided into rooms in demand and rooms available to meet demand. The total revenue for the day is the sum of revenue from all the subunits. Table 1.1 shows the subunits, price of rooms in each subunit, rooms in demand and rooms available to meet demand.

Table 1: Total number of rooms, rooms in demand and rooms available to meet demand.

Subunits	No of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits
k_1	c_1	x_1	A_1	b_1	R_1
k_2	c_2	x_2	A_2	b_2	R_2
k_3	c_3	x_3	A_3	b_3	R_3
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
k_n	c_n	x_n	A_n	b_n	R_n

Assumptions of the Model

1) Any room not used to meet demand is outdated but becomes useful in the next period. Therefore, an outdated cost is incurred for any room that outdates in a period.
2) The total number of rooms in the hotel k is fixed, and the number of rooms in each subunit $k_1, k_2, ... k_n$ is fixed.
3) Shortages occur when there are no rooms available to meet demand and a shortage charge is incurred for every demand not satisfied.
4) There is a fixed operating cost P.
5) Total revenue is the sum of revenue from all subunits.
6) The total number of outdates and shortages are finite, but the demand is infinite.

Notation of the Model

k = Total number of rooms in the hotel.
$k_i, i = 1 \dots n$ = Subunits in the hotel.
$c_i, i = 1 \dots n$ = Number of rooms in each subunit.
$x_i, i = 1 \dots n$ = Price per room in each subunit.
$A_i, i = 1 \dots n =$ Number of rooms in demand from each subunit
$a_{i,j}, i = 1 \dots n, j = 1 \dots n$ = number of rooms in demand from each subunit and the period in demand.
$b_i, i = 1 \dots n$ = Number of rooms available to meet demand from each subunit.
R = Total revenue generated per period.
$R_i, i = 1 \dots n$ = Revenue generated per period from each subunit.
P = Fixed operating cost of the hotel.
θ = Outdate cost per room.
v = Shortage cost per room.

Derivation of Revenue Function

The components of our revenue function are revenue generated per period, shortage cost per period, outdate cost per period and the fixed operating cost. That is
Revenue generated = revenue generated per period+ fixed operating cost – shortage cost per period – outdated cost per period.

Revenue Generated Per Period

The revenue generated per period is the sum of revenue from all subunits of the hotel. Now the rooms in demand from a given subunit are further divided into the number of periods (or days) they are/ or will be in demand.
For example, if the number of rooms in demand from the subunit k_1 is, A_1, then A_1 is subdivided into rooms in demand for one period, two periods, three periods and so on. i.e

$$A_1 = a_{11} + a_{12} + a_{13} + \cdots$$

where a_{11} represents the number of rooms in demand for one period from subunit k_1, a_{12} represents the number of rooms in demand for two periods from subunit k_1 and a_{13} represents the number of rooms in demand for three periods from subunit k_1 and so on. Next, we give a numerical example.
If 20 rooms are in demand from K_1, i.e $A_1 = 20$ and 2 of them are in demand for 1 period, 10 of them are in demand for 2 periods, 5 of them are in demand for 3 periods and 3 of them are in demand for 4 periods, then

$$A_1 = a_{11} + a_{12} + a_{13} + a_{14} \equiv 20 = 2 + 10 + 5 + 3$$

Similarly, for subunits from $k_1 \dots k_n$

$A_1 = a_{11} + a_{12} + a_{13} + \cdots$

$A_2 = a_{21} + a_{22} + a_{23} + \cdots$

$A_3 = a_{31} + a_{32} + a_{33} + \cdots$

. . . .

. . . .

. . . .

$A_n = a_{n1} + a_{n2} + a_{n3} + \cdots$

This give rise to the matrix of demand,

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ . & . & . & \\ . & . & . & \\ . & . & . & \\ a_{n1} & a_{n2} & a_{n3} & \cdots \end{bmatrix}$$

Hotel management will always want the entries in the demand matrix to be non-zero entries. Having zero entries would mean that demand is low and a null matrix implies no patronage.

Now, revenue from subunits $k_1 \dots k_n$ will be

$R_1 = t_1 a_{11} x_1 + t_2 a_{12} x_1 + t_3 a_{13} x_1 + \cdots$

$R_2 = t_1 a_{21} x_2 + t_2 a_{22} x_2 + t_3 a_{23} x_2 + \cdots$

$R_3 = t_1 a_{31} x_3 + t_2 a_{32} x_3 + t_3 a_{33} x_3 + \cdots$

. . . .

. . . .

. . . .

$R_n = t_1 a_{n1} x_n + t_2 a_{n2} x_n + t_3 a_{n3} x_n + \cdots$

So that total revenue $R = R_1 + R_2 + R_3 + \cdots + R_n$.

where t_i is the period(s) rooms are in demand. Therefore

Revenue per period $= \sum_{i,j=1}^{n} t_i a_{i,j} x_i$ (1)

Shortage Cost

For every demand not satisfied by the hotel management, a shortage cost is incurred. A shortage occurs when the demand in a period exceeds the total number of rooms available to meet the demand. If demand in a period $(d) > k$ then, the shortage quantity in a period is given by

Shortage quantity $= \int_k^\infty (d-k) f(d) dd$.

with a shortage cost v per room, the shortage cost is given (2)

Shortage Cost $= v \int_k^\infty (d-k) f(d) dd$ (2)

Outdate Cost

For every room not used to meet demand in a period, an outdate cost θ is charged against the hotel management. Outdate occurs if the number of rooms available to meet demand exceeds the demand in a period. If $k > d$ and the outdated cost for a room is θ, then the outdate cost per period is given by (3)

Outdate Cost $= \theta \int_0^k (k-d) f(d) dd$ (3)

Finally, there is a fixed operating cost P

Combining equations 1, 2 and 3, the periodic revenue generating function for the hotel is

Revenue function $= \sum_{i,j=1}^{n} t_i a_{i,j} x_i - v \int_k^\infty (d-k) f(d) dd - \theta \int_0^k (k-d) f(d) dd + P$ (4)

NUMERICAL EXAMPLE

The model was applied to two hotels, A and B. The data in Tables 2 and 3 shows the subunits, the number of rooms in each subunit and the price per room in each subunit.

Table 2: Room types and prices in hotel A.

Room type	Number of rooms	Price per room (₦)
Single room	10	4000
Double room	10	8000
Queen room	8	16000
King room	12	16000
Presidential	1	28000

Total number of rooms K in hotel A = 41

Table 3: Room types and prices in hotel B.

Room type	Number of rooms	Price per room (₦)
Single room	15	4000
Double room	10	7500
Queen room	10	15000
King room	15	15000
Presidential	1	32000

Total number of rooms K in hotel A = 41

The record obtained from the hotels for a week (7 days) was analyzed with our model and the result is shown in Tables 4 and 5.

Table 4: Revenue generated from hotel A.

Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
1	Single room	10	4000	2(1) 8 4(2) 2(3)	2	64000	144000
	Double room	10	8000	2(1)	8	16000	
	Queen room	8	16000	2(2)	6	64000	
	King room	12	16000	0	12	0	
	Presidential	1	28000	0	1	0	
2	Single room	10	4000	4(1) 2(2) 2(2)	2	16000	56000
	Double room	10	8000	3(1)	7	24000	
	Queen room	8	16000	2(1) 0	6	0	
	King room	12	16000	1(1)	11	16000	
	Presidential	1	28000	0	1	0	
3	Single room	10	4000	2(1) 2(1) 6(1)	0	24000	120000
	Double room	10	8000	4(1)	6	32000	
	Queen room	8	16000	1(1)	7	16000	
	King room	12	16000	3(1)	9	48000	
	Presidential	1	28000	0	1	0	
Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
4	Single room	10	4000	5(1) 10 4(2) 1(3)	0	64000	176000
	Double room	10	8000	6(1)	4	48000	
	Queen room	8	16000	4(1)	4	64000	
	King room	12	16000	0	12	0	

	Presidential	1	28000	0	1	0	
5	Single room	10	4000	4(1) 1(2) 5(1)	0	20000	124000
	Double room	10	8000	7(1)	3	56000	
	Queen room	8	16000	0	8	0	
	King room	12	16000	3(1)	9	48000	
	Presidential	1	28000	0	1	0	
6	Single room	10	4000	1(1) 6(1)	3	24000	64000
	Double room	10	8000	5(1)	5	40000	
	Queen room	8	16000	0	8	0	
	King room	12	16000	0	12	0	
	Presidential	1	28000	0	1	0	
7	Single room	10	4000	8(1)	2	32000	160000
	Double room	10	8000	6(1)	4	48000	
	Queen room	8	16000	2(1)	6	32000	
	King room	12	16000	3(1)	9	48000	
	Presidential	1	28000	0	1	0	

In Table 4, the number of rooms in demand on day1 is 12 rooms. 8 from single rooms, 2 from double rooms and 2 from queen rooms. Of the 8 rooms in demand from single rooms, 2 are in demand for 1 day, 4 are in demand for 2days and 2 are in demand for 3days.
2 rooms are in demand for 1 day, from the double rooms and 2 rooms are in demand for two days from the queen rooms.

Also, from Table 4, the sum of revenue generated by hotel A for a period of seven days is
$144000 + 56000 + 120000 + 176000 + 124000 + 64000 + 160000 = 844000$

The total number of outdates (that is, rooms not used to meet demand) over the same period is
$29 + 27 + 23 + 21 + 21 + 29 + 22 = 172$, so that the outdated cost is given as
Outdate cost $=172 \times 1000 = 172000$

No shortages were recorded in the period under review. Therefore revenue generated in the period by hotel A is,
Revenue for hotel A $= 844000 - 172000 + 20000000 = 20672000$

Table 5: Revenue generated from hotel B.

Day	Subunits	Number of rooms in subunits	Price per room	Rooms in demand from subunits	Rooms available to meet demand from subunits	Revenue from subunits	Total Revenue for the Period
1	Single room	15	4000	2(1) 5 1(2) 2(3)	10	40000	90000
	Double room	10	7500	1(2)	9	15000	
	Queen room	10	15000	2(1)	8	20000	
	King room	15	15000	1(1)	14	15000	
	Presidential	1	32000	0	1	0	
2	Single room	15	4000	1(1) 2(2) 8(1)	4	32000	114500
	Double room	10	7500	3(1)	7	22500	
	Queen room	10	15000	2(2)	8	60000	
	King room	15	15000	0	15	0	
	Presidential	1	32000	0	1	0	
3	Single room	15	4000	2(1) 4(1) 9 2(2) 3(3)	4	68000	173000
	Double room	10	7500	2(1)	8	15000	
	Queen room	10	15000	0	10	0	
	King room	15	15000	3(2)	12	90000	
	Presidential	1	32000	0	1	0	

From Table 5. The number of rooms in demand on day 1 is 9 rooms. 5 rooms from the single rooms, 1 room from double rooms, 2 rooms from queen rooms and 1room from the king rooms.

From Table 5, the total revenue generated by hotel B for a period of seven days is
$90000 + 114500 + 173000 + 125000 + 955000 + 152000 + 167500 = 1777000$

The total number of outdates over the same period is
$42 + 35 + 35 + 36 + 35 + 37 + 33 = 253$, so that the outdated cost is
Outdate cost $= 253 \times 1000 = 253000$

No shortages were recorded in the period under review and revenue generated is
Revenue function for hotel B $= 1777000 - 253000 + 20000000 = 21524000$.

MATRIX OF DEMAND

Hotel managers use the matrix of demand as a tool to determine their level of patronage or customers inflow. Hotel managers desire that the matrix of demand entries should be non-zero entries, as a null matrix or almost null matrix will indicate no patronage or low patronage. Also, the matrix of demand can be used to compare the level of patronage between two hotels. The hotel with the most zero entries in its matrix of demand in a period has lower patronage, when compared with the hotel with the least zero entries in its matrix of demand. From Tables 4 and 5, we computed the demand matrix for hotel A and hotel B for day 1.

$$\text{Day 1} \quad A = \begin{bmatrix} 2 & 4 & 2 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{Day 1} \quad B = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Comparing the level of patronage for hotels A and B, hotel B has more demand than hotel A because the number of zero entries in matrix A is more than the number of zero entries in matrix B.

CONCLUSION

The revenue-generating function derived in this work allows the hotel manager to determine the amount of money generated over a period of time. The model enables the hotel manager to know the number of shortages in a period and the number of outdates. The model also allows the manager to trace the rooms in demand and know the revenue generated from each subunits. Finally, the matrix of demand is a tool for testing the level of patronage in the hotels.

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AUTHOR CONTRIBUTION

All the authors contributed equally to this work.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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